Full waveform inversion by deconvolution gradient method

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November 14, 2012

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Introduction

- FWI is still very expensive.
- Make it cheaper either by reducing the cost of seismic modeling (source encoding), or by reducing the number of iterations by improving the convergence rate of the inversion scheme.
- Most current industrial-scale applications of FWI use algorithms based on the calculation and some kind of preconditioning of the gradient of the objective function.

Introduction

- Pratt et al. (1998) noted that the effects of the application of the inverse Hessian to the gradient included the correction of the effects of the (limited) acquisition aperture and spatially-varying illumination, and source deconvolution.
- It has been widely observed that the calculation of the gradient of a least-squares FWI objective function corresponds to a RTM of the residuals using the correlation imaging condition;
- This inspired them to instead consider the application of a deconvolution imaging condition to calculate the update direction.
- This is trivial in the frequency domain, where we perform our FWI gradient calculation, and automatically compensates for the source-side illumination as well as performing the source deconvolution. That is, it applies a partial inverse Hessian correction to the gradient.

Method

- Objective function (square of $L2$-norm):

$$E = \delta d^t \delta d^*$$

- Velocity update (steepest descent method):

$$p^{(k+1)} = p^k - \alpha \nabla_p E$$

where $\nabla_p E$ is the gradient of the objective function with respect to the model parameter. It is computed as:

$$\nabla_p E = -Re \sum_{shots} \left(u^t \frac{\partial S^t}{\partial p} v \right)$$

where $v = S^{-t} \delta d^*$ is the back-propagated residual wave field and S is the wave propagator: $Su = f$.

Method

- The Gauss-Newton approximation to the Hessian can be expressed as:

$$(H_{GN})_{ij} = \sum_{shots} \sum_{r} \left(w^t \frac{\partial S}{\partial p_i} u(r) \right)^t \left(w^t \frac{\partial S}{\partial p_j} u(r) \right)^*$$

  where $w^t = S^{-1}$ is the back-propagator corresponding to the receiver-side illumination pattern.

- The update direction corresponds to the deconvolution imaging condition:

$$\nabla_p E = -Re \sum_{shots} \left(u^t \frac{\partial S^t}{\partial p} vW \right)$$

  where W is a second-order diagonal matrix defined by:

$$W = diag([1/(u_1 u_1^* + \varepsilon) \cdots 1/(u_n u_n^* + \varepsilon)])$$

Implementation

- Define the damping parameter as water level:

$$\varepsilon = \lambda^* avg([u_1 u_1^* \cdots u_n u_n^*])$$

where λ is a constant and *avg* denotes average over the elements of the vector, following the similar approach by Valenciano and Biondi (2003) in their study on the deconvolution imaging condition for wave-equation migration.

Model geometry

- velocity: $1.5 - 5.5km$
- shots: 93 (interval: 100 m)
- receiver: 185 (interval: 50 m)
- 12 iterations at three frequencies: 1, 2.5, and 4 Hz.

Marmousi model

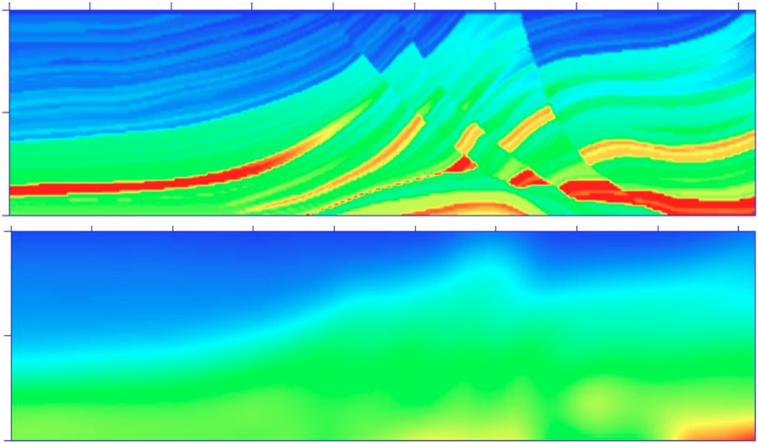

Figure : The true Marmousi model and the initial model used in the FWI tests in this study. The model is 9.2km wide and 2km deep.

Convergence at $1Hz$

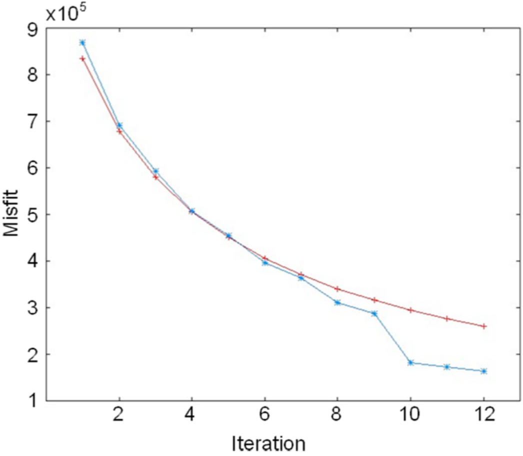

Figure : The convergence history of FWI inverting $1Hz$ frequency component by the conventional gradient method (red) and the de-convolution gradient method (blue).

Convergence 2.5Hz

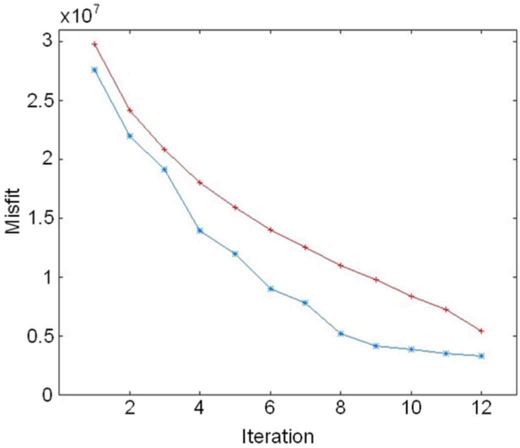

Figure : The convergence history of FWI inverting 2.5Hz frequency component by the conventional gradient method (red) and the de-convolution gradient method (blue).

Convergence $4Hz$

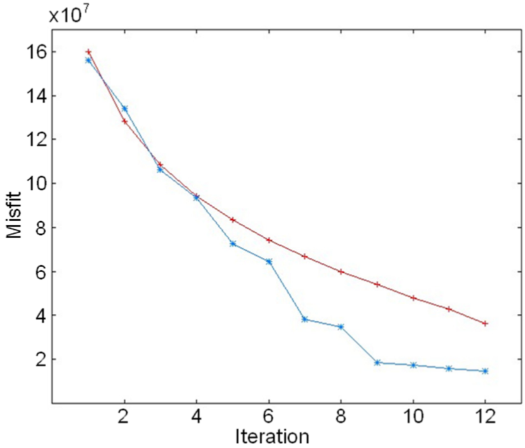

Figure : The convergence history of FWI inverting $4Hz$ frequency component by the conventional gradient method (red) and the de-convolution gradient method (blue).

Example

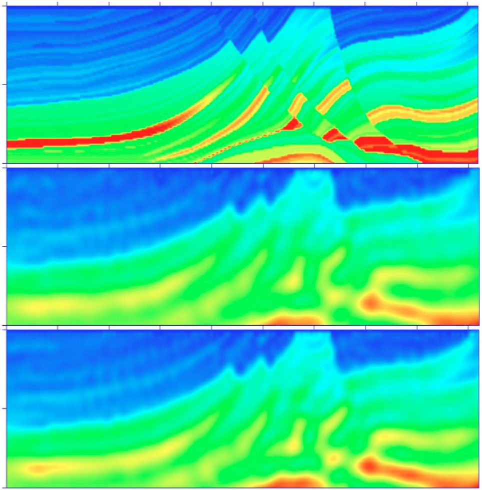

Figure : True model (top), regular FWI and FWI by deconvolution gradient (bottom).

Conclusions

- We have presented a simple, inexpensive way to accelerate convergence in FWI by partial compensation of illumination. The method involves applying a deconvolution-type RTM "imaging condition" to calculate the update direction, rather than the cross-correlation one which yields the classical gradient direction.
- This corresponds to preconditioning the gradient by a factor of the diagonal of the Gauss-Newton approximation to the hessian matrix.
- The method seems to have a greater impact in later iterations of a given series, and at higher frequencies;
- This may be because the variation in illumination becomes more pronounced in these cases. The irregularity in the convergence may be linked to the estimation of the damping parameter in the deconvolution, which is known to be a delicate issue in applying the analogous imaging condition in migration.