2012 SEG Dicussion @ TRIP

Seismic imaging in the (image || data) domain

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What is seismic imaging?

What is image domain? What is data domain?

How to combine them?

What is seismic imaging?

What is image domain? What is data domain?

How to combine them?

Answer:

W. W. Symes, 2008, Migration velocity analysis and waveform inversion: Geophysical Prospecting, 56, 765–790

W. W. Symes, 2009, The seismic reflection inverse problem: Inverse Problems, 25, 1–39

13 of 48 oral sessions

2 workshops

1. Introduction

$$\boxed{\text{Model Space}} \xrightarrow{\textit{forward map}} \boxed{\text{Data Space}}$$

Assumption of Constant-density acoustics:

Space def: $M := \{v(\mathbf{x})\}$ $\quad D := \{d(t, \mathbf{x}_{\mathbf{r}}; \mathbf{x}_{\mathbf{s}})\}$

Forward map: CDA equation + Sampling operator

Linearized Approximation:

Space def: $M := \{v_0(\mathbf{x}); \delta v(\mathbf{x})\}$ $\quad D := \{d(t, \mathbf{x}_{\mathbf{r}}; \mathbf{x}_{\mathbf{s}})\}$

Forward map: Born scattering operator + Sampling operator

1. Introduction

$$\boxed{\text{Model}} \xleftarrow[\textit{imaging}]{\textit{inversion}} \boxed{\text{Seismic Data}}$$

Assumption of Constant-density acoustics:

Problem: $$\min_v J(v) = \|F[v] - d\|_2^2$$

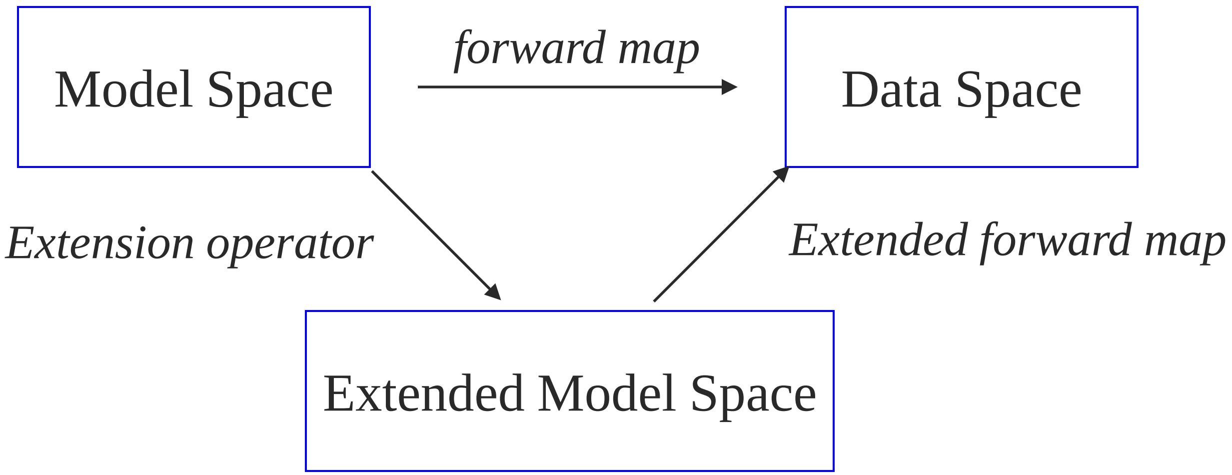
Imaging: Output least-squares inversion->FWI

Linearized Approximation:

Problems: $$\min_{v_0, \delta v} J(v_0; \delta v) = \|DF[v_0]\delta v - (d - F[v_0])\|_2^2$$

Imaging: background velocity building->MVA
velocity perturbation building->migration

1. Introduction



Space def: $M := \{v(\mathbf{x},\mathbf{h})\}$ $\quad D := \{d(t,\mathbf{x}_\mathbf{r};\mathbf{x}_\mathbf{s})\}$

Extended forward map: Extended CDA equation + Sampling operator

Problem: $\min_v J(v) = \left\| F[v(\mathbf{x},\mathbf{h})] - d \right\|_2^2 ; s.t. A\left[v(\mathbf{x},\mathbf{h})\right] = 0$

Imaging: Extended inversion

Agenda

2.1 Least-squares migration

2.2 Migration velocity analysis

2.3 Waveform inversion

2.4 Combine MVA and WI

Agenda

2.1 Least-squares migration

2.2 Migration velocity analysis

2.3 Waveform inversion

2.4 Combine MVA and WI

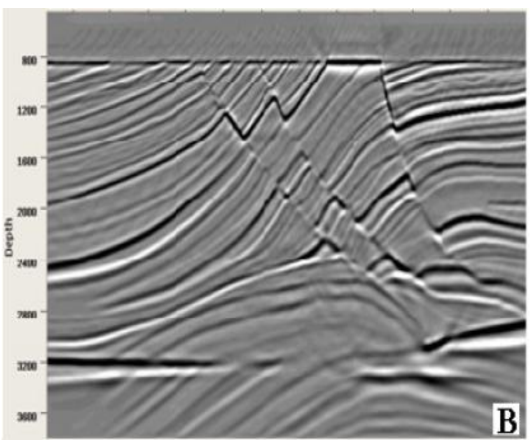
2.1 LSM

$$\min_{\delta v} J\left(v_0;\delta v\right) = \left\| DF[v_0]\delta v - \left(d - F[v_0]\right)\right\|_2^2$$

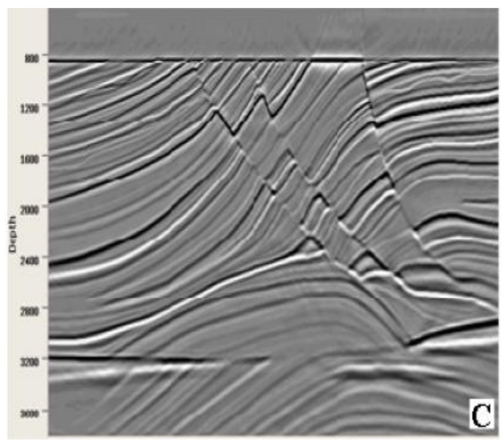
Questions:

1. How to define *DF*?

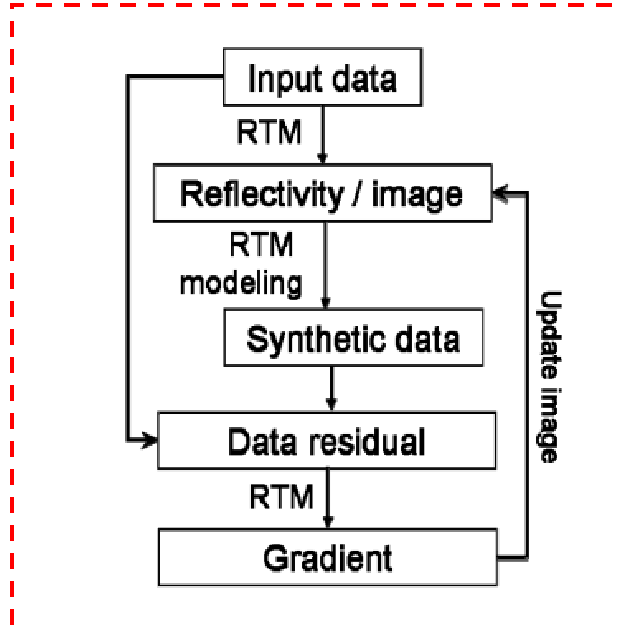
LSRTM (D. Sun et al; G. Yao et al; X. Li; W. Dai)



RTM



LSRTM



Workflow of LSRTM

(D. Sun et al)

2.1 LSM

$$\min_{\delta v} J\left(v_0;\delta v\right)=\left\|DF[v_0]\delta v-\left(d-F[v_0]\right)\right\|_2^2$$

Questions:

2. How to increase efficiency?

Dynamic phase encoding (X. Wang et al; W. Dai)

Sparse promotion & message passing (Herrmann & X. Li)

BP: $\underset{\mathbf{x}}{\text{minimize}} \frac{1}{2}\|\mathbf{x}\|_1 \quad \text{s.t.} \quad \underline{\delta\mathbf{D}} = \nabla\mathscr{F}[\mathbf{m};\underline{\mathbf{Q}}]\mathbf{S}^H\mathbf{x}$

LASSO: $\underset{\mathbf{x}}{\text{minimize}} \frac{1}{2}\|\underline{\delta\mathbf{D}} - \nabla\mathscr{F}[\mathbf{m};\underline{\mathbf{Q}}]\mathbf{S}^H\mathbf{x}\|_F^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \le \tau$

Result: Estimate for the model $\tilde{\mathbf{x}}$

$\mathbf{x}_0 \longleftarrow \mathbf{0}$; `// initial model`

$\mathbf{k} \longleftarrow \mathbf{0}$; `// initial counter`

$\{\underline{\delta\mathbf{D}}, \underline{\mathbf{Q}}\} := \{\mathbf{DW}, \mathbf{QW}\}$; `// random shots`

while $\|\mathbf{x}_0 - \tilde{\mathbf{x}}\|_2 \ge \varepsilon$ **do**

$k \longleftarrow k+1$; `// increase counter`

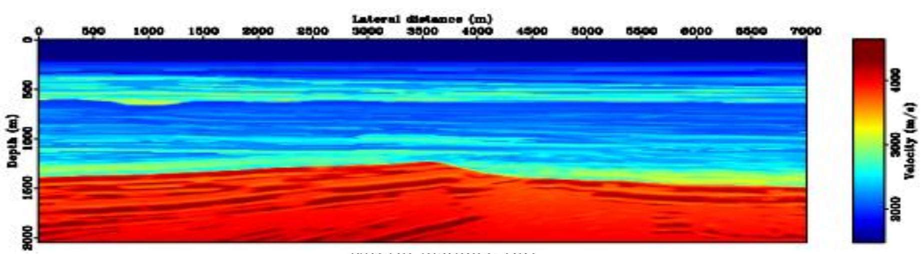
$\tilde{\mathbf{x}} \longleftarrow \mathbf{x}_0$; `// update warm start`

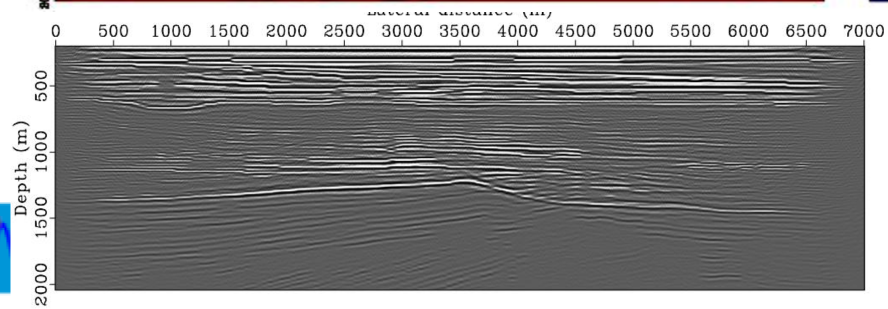
$\mathbf{x}_0 \longleftarrow \text{Solve}(\text{minimize}_{\mathbf{x}} \frac{1}{2}\|\underline{\delta\mathbf{D}} - \nabla\mathscr{F}[\mathbf{m};\underline{\mathbf{Q}}]\mathbf{S}^H\mathbf{x}\|_F^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \le \tau)$; `// solve the subproblem`

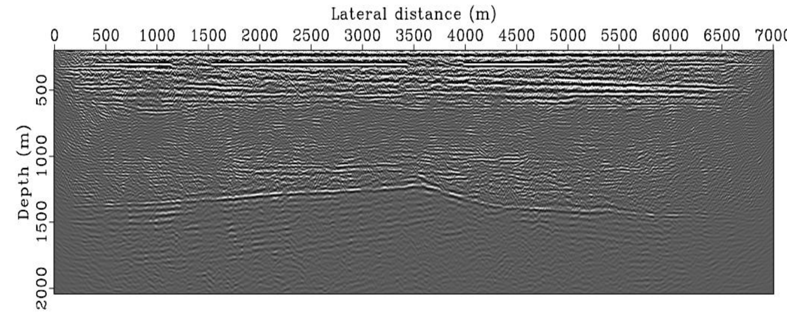
$\boxed{\{\underline{\delta\mathbf{D}}, \underline{\mathbf{Q}}\} := \{\mathbf{DW}, \mathbf{QW}\}}$; `// do redraws`

end

Algorithm 1: sparsity promoting recovery with approximated message passing







2.1 LSM

$$\min_{\delta v} J(v_0;\delta v) = \left\| \left(DF^T[v_0]DF[v_0]\right)\delta v - DF^T[v_0]\delta d \right\|_2^2 = \left\| H[v_0]\delta v - \delta v_{mig} \right\|_2^2$$

Questions:

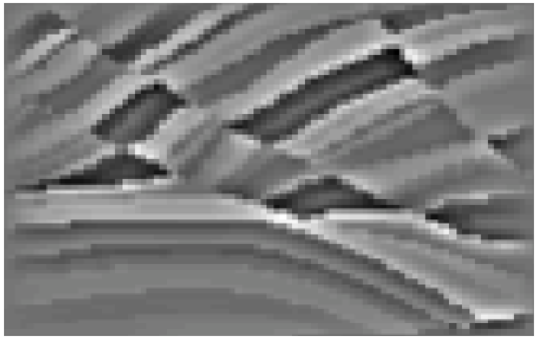
How to calculate Hessian efficiently?

(P. Letourneau et al)

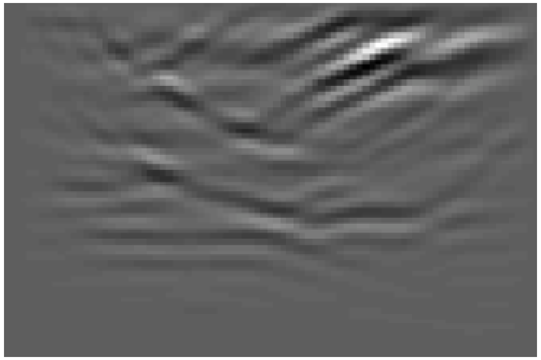
$$H^{-1} \simeq \sum_{i=1}^{p} c_i B_i,$$

$$m_{1,k} = \sum_{i=1}^{p} c_i B_i m_{2,k}$$

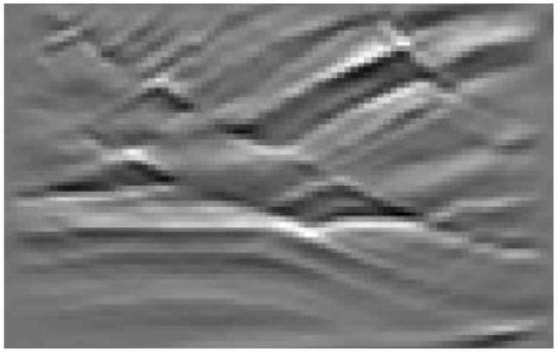
1. Create a random model m (white noise);
2. Take a forward fast curvelet transform Candes et al. (2006);
3. Use ray-tracing to remove elements of the null-space (misaligned local reflectors);
4. Apply the inverse fast curvelet transform to get the filtered random model m_1.



Reference model



migrated image



preconditioned migrated image

Agenda

2.1 Least-squares migration

2.2 Migration velocity analysis

2.3 Waveform inversion

2.4 Combine MVA and WI

2.2 MVA

1. Objective Function

(1) Moveout-based WEMVA(Y. Zhang et al.);

$$J_{Sm}(\rho(s)) = \frac{1}{2}\sum_x \sum_z \frac{\int dz_w \left(\int d\gamma I(\gamma, z+z_w+\rho\tan^2\gamma, x; s_0)\right)^2}{\int dz_w \int d\gamma I^2(\gamma, z+z_w+\rho\tan^2\gamma, x; s_0)}$$

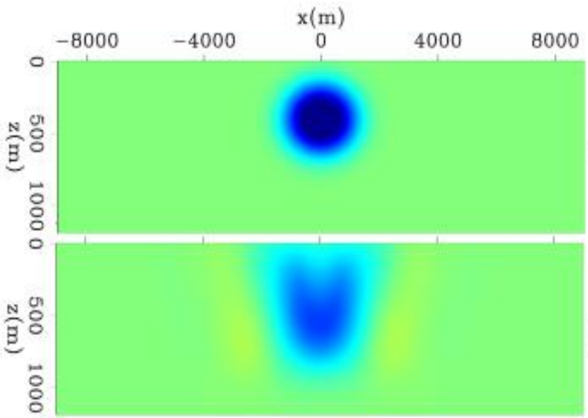
$$g= -\int dz_w \int d\gamma \sum_{z,x} \frac{\partial I(z+z_w,\gamma,x;s)}{\partial s}(F_{11}\tan^2\gamma + F_{12})\dot{I}(z+z_w,\gamma,x;s_0)\frac{\partial J_{Sm}}{\partial \rho}(z,x)$$

$$\frac{\partial^2 J_{\mathrm{aux}}}{\partial \rho^2} = \int dz_w \int d\gamma \ddot{I}(z+z_w+\theta,\gamma,x;s_0)\tan^4\gamma I(z+z_w,\gamma,x;s) = E_{11}(z,x)$$

$$\frac{\partial^2 J_{\mathrm{aux}}}{\partial \rho \partial \beta} = \int dz_w \int d\gamma \ddot{I}(z+z_w+\theta,\gamma,x;s_0)\tan^2\gamma I(z+z_w,\gamma,x;s) = E_{12}(z,x)$$

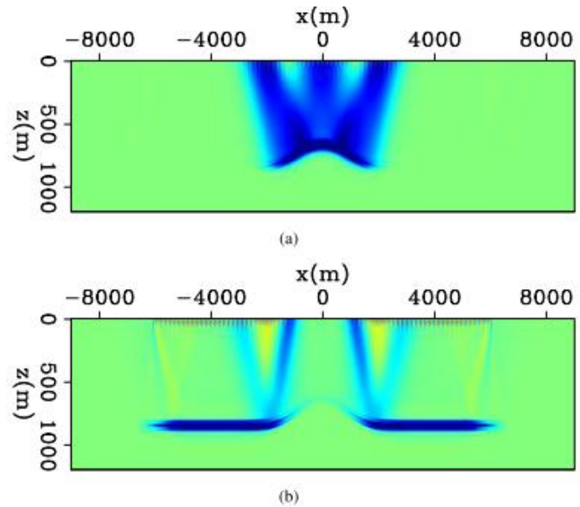
$$\frac{\partial^2 J_{\mathrm{aux}}}{\partial \beta^2} = \int dz_w \int d\gamma \ddot{I}(z+z_w+\theta,\gamma,x;s_0) I(z+z_w,\gamma,x;s) = E_{22}(z,x)$$

$$F = \begin{bmatrix} F_{11} & F_{12} \\ F_{12} & F_{22} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix}^{-1}$$



(a) True model

(b) Inverted model



Gradient of Proposed Method

Gradient of SPO

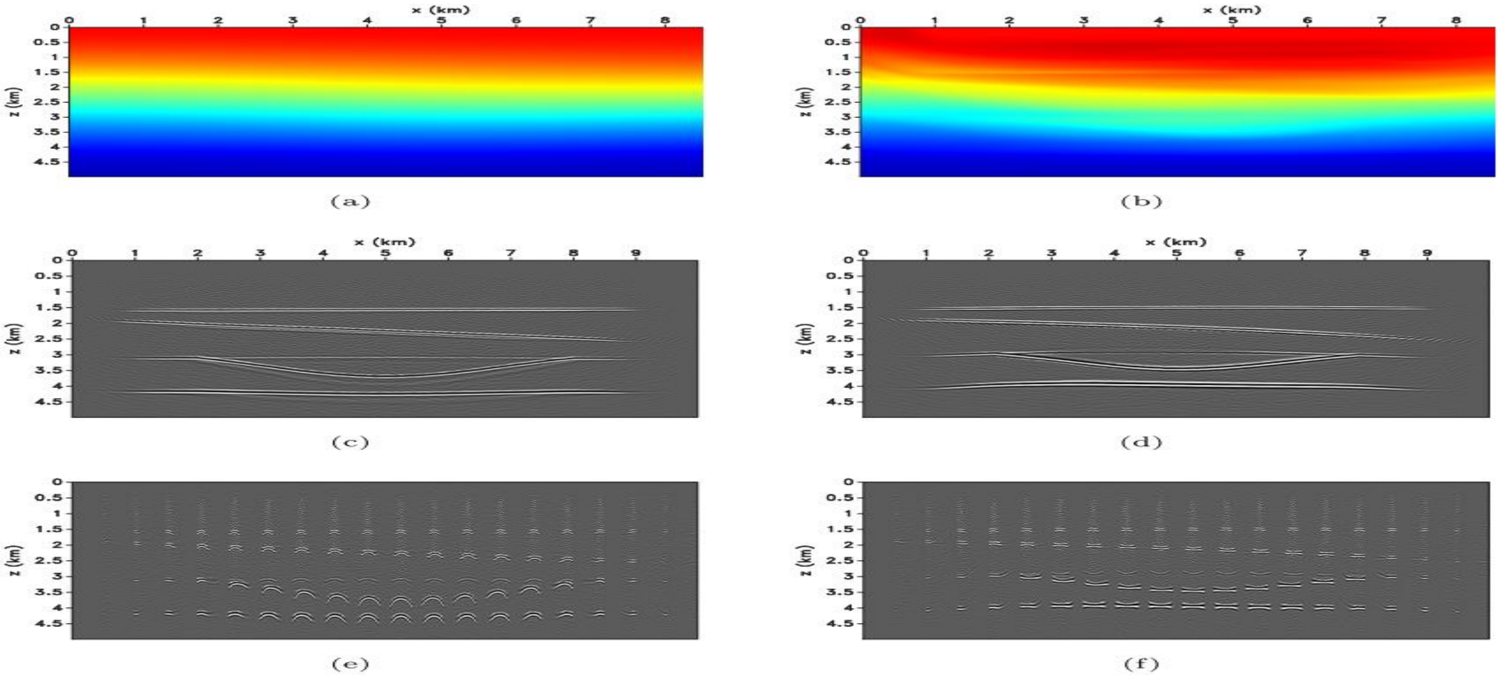
2.2 MVA

1. Objective Function

(2) Local image correction based WEMVA(F. Perrone et al.);

$$\mathcal{J}(m) = \frac{1}{2}\sum_{i} \| \sum_{\boldsymbol{\lambda}} P(\mathbf{x}, \boldsymbol{\lambda})\, c_i(\mathbf{x}, \boldsymbol{\lambda}) \|^2_{\mathbf{x},\boldsymbol{\lambda}}$$

$$c_i(\mathbf{x}, \boldsymbol{\lambda}) = \int_{w(\mathbf{x})} R_{i+1}\left(\boldsymbol{\xi} - \tfrac{\boldsymbol{\lambda}}{2}\right) R_i\left(\boldsymbol{\xi} + \tfrac{\boldsymbol{\lambda}}{2}\right) d\boldsymbol{\xi}$$



Initial model

Inverted model

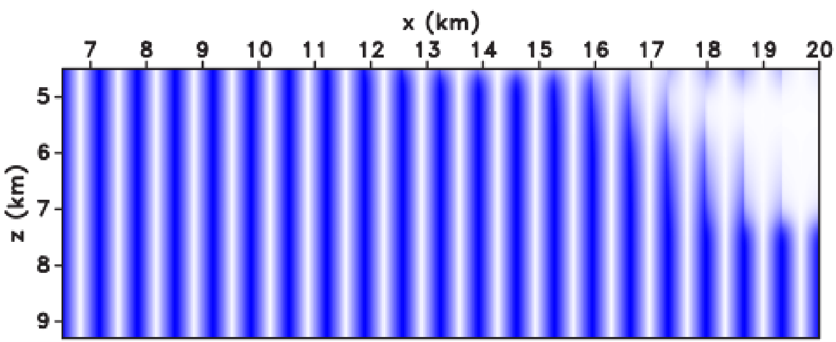
2.2 MVA

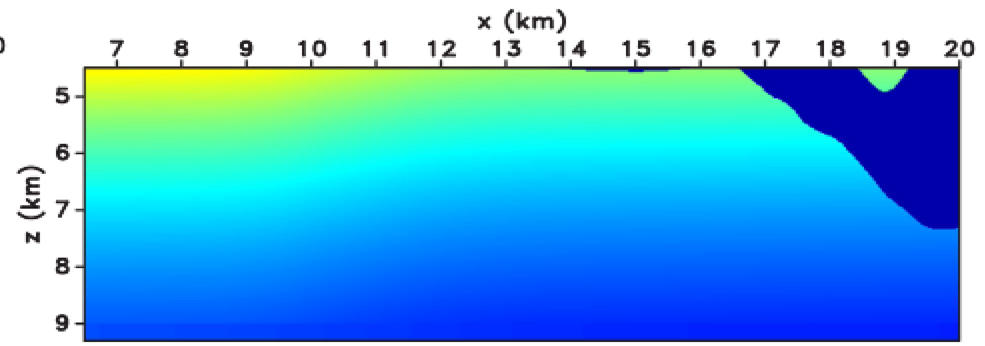
1. Objective Function

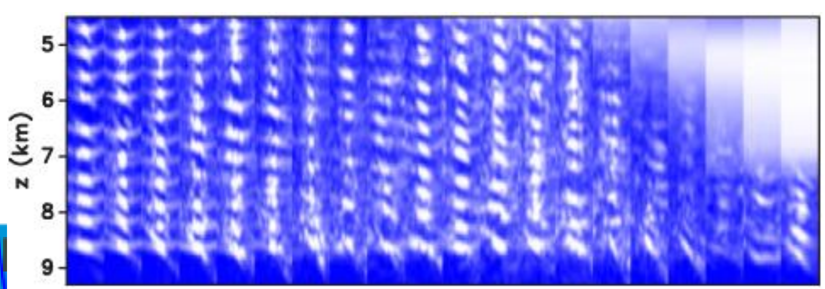
(3) Illumination compensation WEMVA(T. Yang et al.)

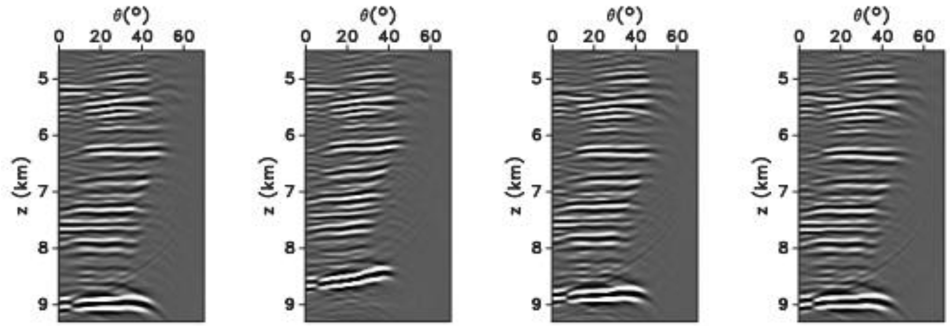
$$\mathcal{H}_{\boldsymbol{\lambda}} = \frac{1}{2} \| K_I(\mathbf{x}) \, P(\boldsymbol{\lambda}) \, r(\mathbf{x}, \boldsymbol{\lambda}) \|^2_{\mathbf{x},\boldsymbol{\lambda}}$$

$$P(\mathbf{x}, \boldsymbol{\lambda}) = \frac{1}{E[r_e(\mathbf{x}, \boldsymbol{\lambda})] + \epsilon} \qquad r_e(\mathbf{x}, \boldsymbol{\lambda}) = \mathcal{M}^* \mathcal{M} r(\mathbf{x}, \boldsymbol{\lambda})$$









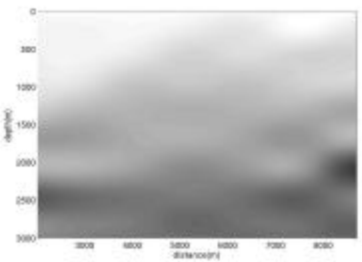
(a) (b) (c) (d)

2.2 MVA

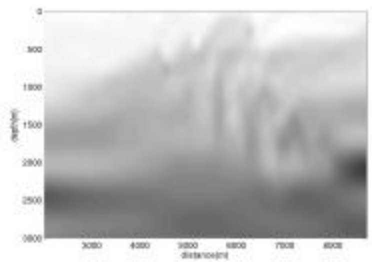
2. RTM based WEMVA (P. Shen; W. Weibull)

$$J_1 = \frac{1}{2}\int dxdh|hI|^2 \qquad J_2 = \int dxdh|I|^2.$$

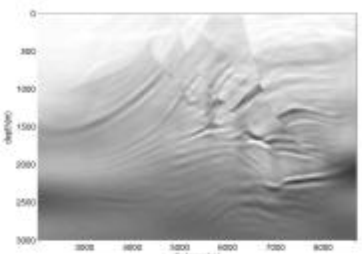
$$J = J_1 - \lambda J_2. \qquad J = J_1/J_2.$$



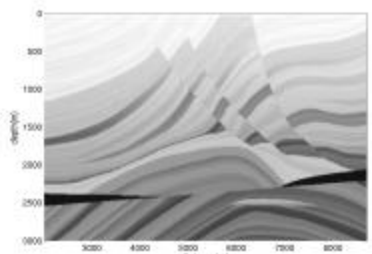
(a)



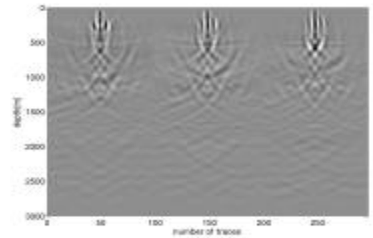
(b)



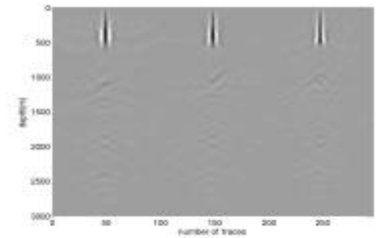
(c)



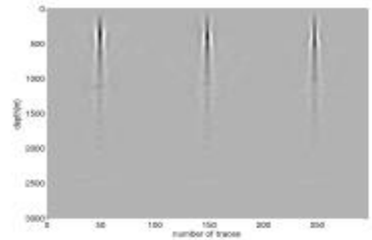
(d)

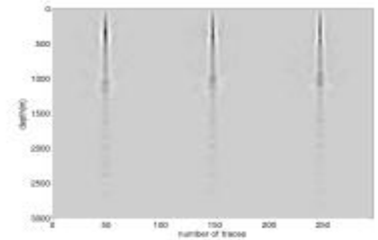


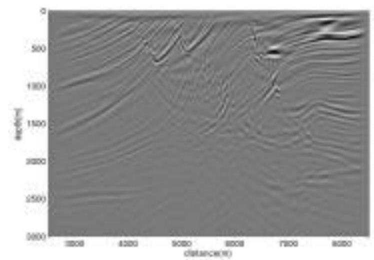
(a)



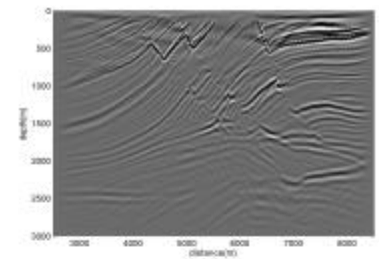
(b)







(a)



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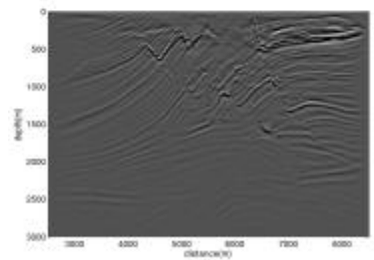
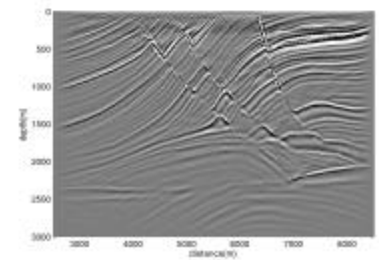


(c)



(d)

Agenda

2.1 Least-squares migration

2.2 Migration velocity analysis

2.3 Waveform inversion

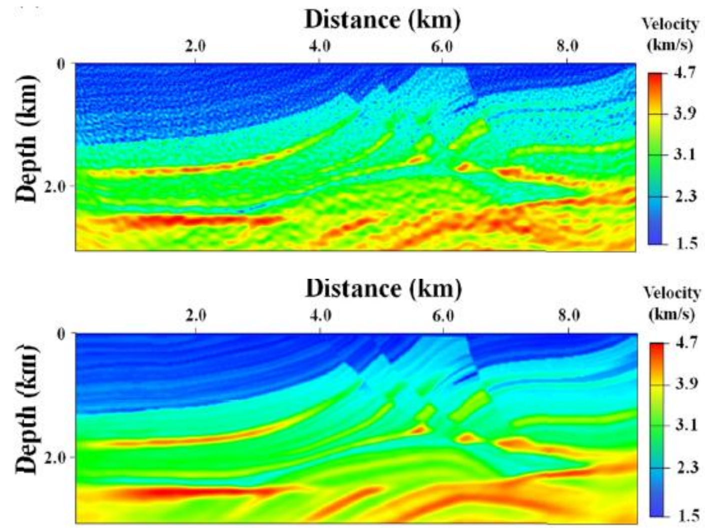
2.4 Combine MVA and WI

2.3 FWI

1. Objective Function

(1) Using denoise function (J. Oh et al.);

$$\nabla_{p_k} E = \sum_{\omega} g(\omega) \frac{\sum_s \left[\left(\mathbf{f}_s^{p_k}\right)^T \left(\mathbf{S}^{-1}\right)^T \left(\mathbf{u}_s - \mathbf{d}_s\right)^* \right]}{\left[\sum_s \left[\left(\mathbf{f}_s^{p_k}\right)^T \left(\mathbf{f}_s^{p_k}\right)^* \right] \right] + \beta} \qquad g(\omega) = \left[\frac{\sum_r \left| \sum_s u_{s,r}^{\text{model}} \right|}{\sum_r \left| \sum_s \left(d_{s,r}^{\text{model}} + d_{s,r}^{\text{noise}} \right) \right|} \right]^e \approx \frac{1}{b}$$



(2) Combine WET and FWI (H. Wang et al.);

$$H = \alpha d_{real} * d_{syn} + (1-\alpha) \left\| d_{real} - d_{syn} \right\|^2$$

(3) Interferometric WI (L. Demanet et al.);

$$\min_m \sum_{i,j} | d_i \overline{d_j} - (Fm)_i \overline{(Fm)_j} |^2$$

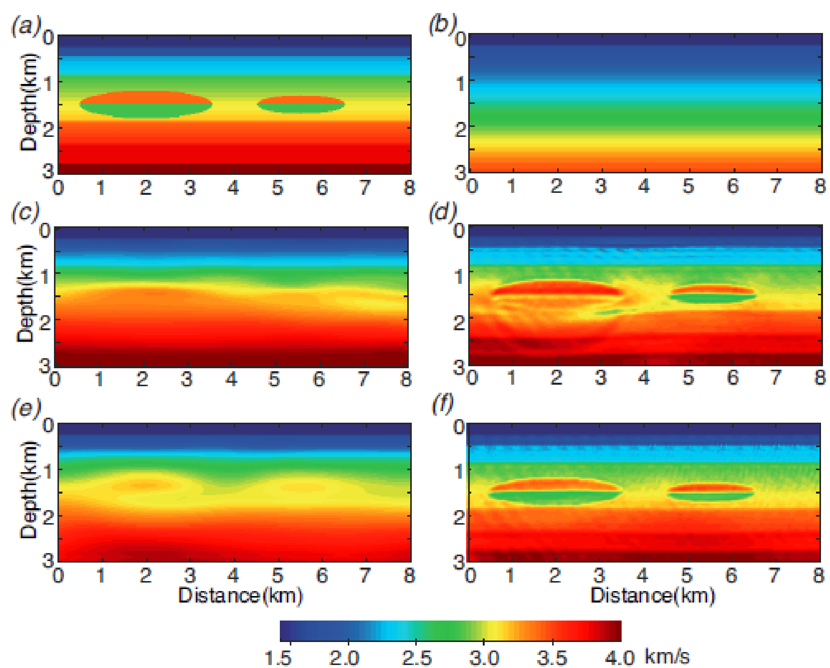


Figure 2: Comparison of wave-equation tomography and ray-based tomography results. (a) real model. (b) sarting model. (c) RTT result. (d) FWI result after 25 iterations using (c) as starting model. (e) WET result after 30 iterations. (f) FWI result after 25 iterations using (e) as starting model.

2.3 FWI

2. Decrease computational cost

(1) Encoding

a. Random phase encoding & L1norm (W. Son)

b. Random phase encoding & full Newton method (A. Anagaw)

c. Cyclic shot sampling (W. Ha)

(2) Sparse representation

a. Wavelet (Y. Lin et al)

b. Image-guided (Y. Ma et al)

(3) Algorithm

a. Truncated Newton (L. Metiver et al)

(4) Preconditioning

a. wave-energy-based preconditioning (T. Fei et al.);

b. Deconvolution gradient (F. Gao et al.);

(5) GPU (J. Mao)

3. FWI based on reflected wave (S. Xu; H. Zhou)

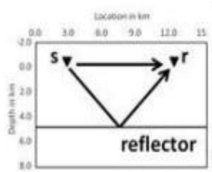
$$-\omega^2 \boldsymbol{m} G - \Delta G = \delta(\boldsymbol{x} - \boldsymbol{s})$$

$$C(\boldsymbol{m}) = -\iiint ds\, dr d\omega G_{obs} G_{cal}^{*} \quad (\boldsymbol{m})$$

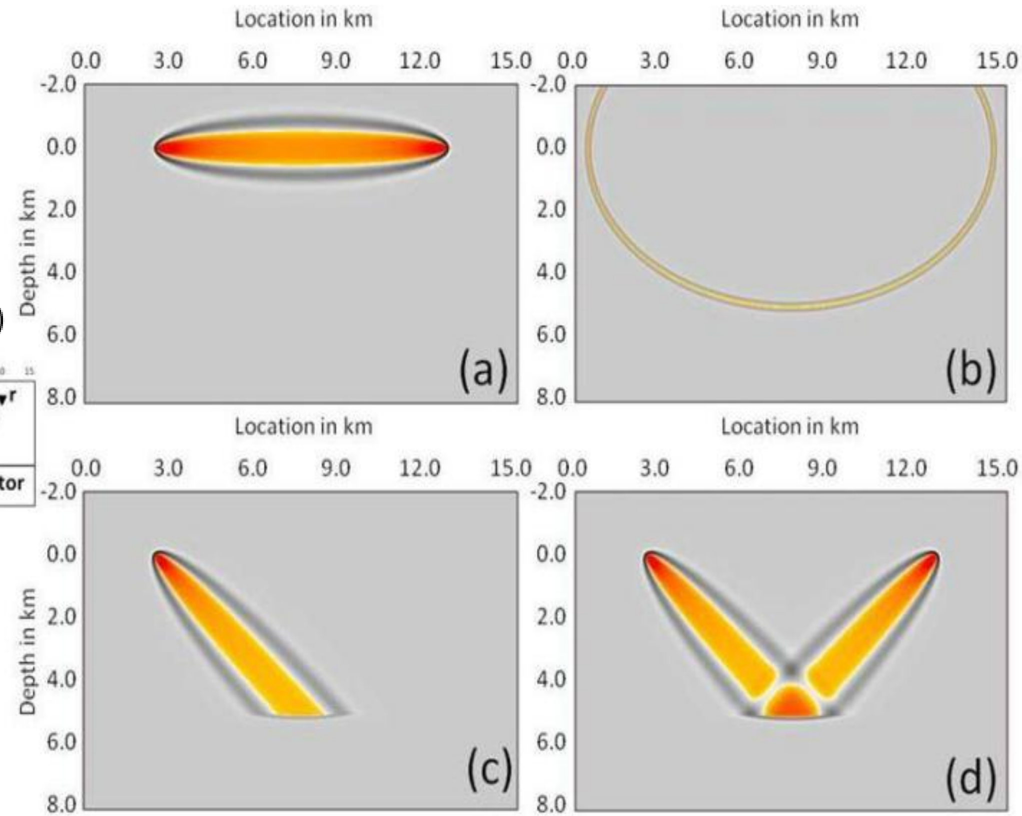
$$\left.\frac{\partial C}{\partial m_0(\boldsymbol{x})}\right|_{\delta m} = -\iint ds dr \int d\omega \left.\frac{\partial \delta G}{\partial m_0(\boldsymbol{x})}\right|_{\delta m}^{*} G_{obs}$$

$$\left.\frac{\partial \delta G}{\partial m_0(\boldsymbol{x})}\right|_{\delta m} = \omega^2 \big(G_0(\boldsymbol{r},\boldsymbol{x})\delta G(\boldsymbol{x},\boldsymbol{s}) + \delta G(\boldsymbol{r},\boldsymbol{x}) G_0(\boldsymbol{x},\boldsymbol{s})\big)$$

$$\delta G(\boldsymbol{m}_0, \delta \boldsymbol{m})(\boldsymbol{r},\boldsymbol{s}) = \omega^2 \int d\boldsymbol{x}\, G_0(\boldsymbol{r},\boldsymbol{x}) G(\boldsymbol{x},\boldsymbol{s}) \delta m(\boldsymbol{x})$$



1) True amplitude prestack depth migration of the reflected wavefield for the initial velocity model m_0^{init};
2) From the true amplitude migration result and the initial velocity model m_0^{init} simulation of the perturbed wave field, δG;
3) Computation of the residual data;
4) Computation of the gradient of the cost function;
5) Update of the velocity model to $m_0^{updated}$.



a) Contribution of the direct wave to the gradient of FWI; b) Contribution of the reflected wave to the gradient of FWI; c) Source-reflector contribution of reflected wave to the gradient of SRFWI; c) Source-reflector-receiver contribution of reflected wave to the gradient of SRFWI

Agenda

2.1 Least-squares migration

2.2 Migration velocity analysis

2.3 Waveform inversion

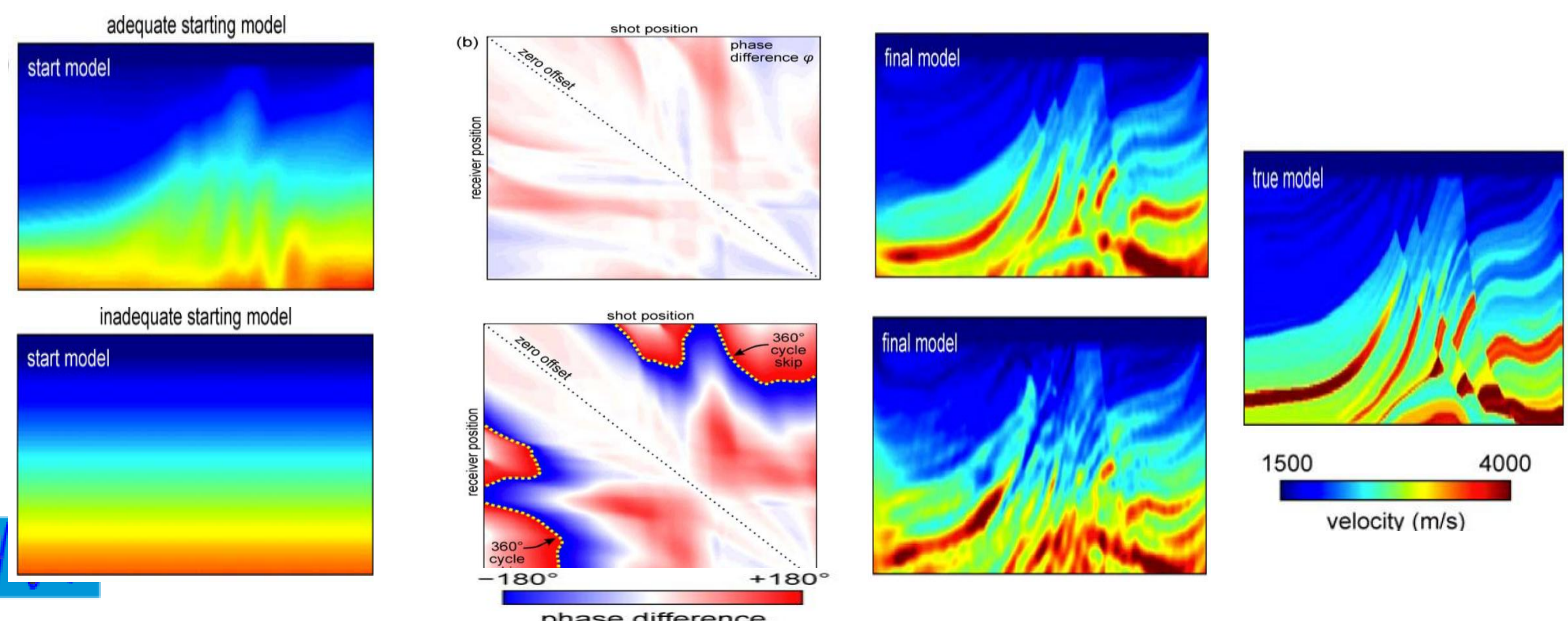
2.4 Combine MVA and WI

2.4 Combine MVA and FWI

1. Two-stage method (W. Weibull; N. Shah)

Q: How to ensure starting model adequacy?

A: *calculate the phase difference between the observed and predicted data, at the lowest useable frequency present in the field data, after windowing the data in time using a Gaussian window centered on the early arrivals. In practice, we do not pick first-break times from the field data; rather we use arrival times calculated from the start model to control the position of the Gaussian window.*

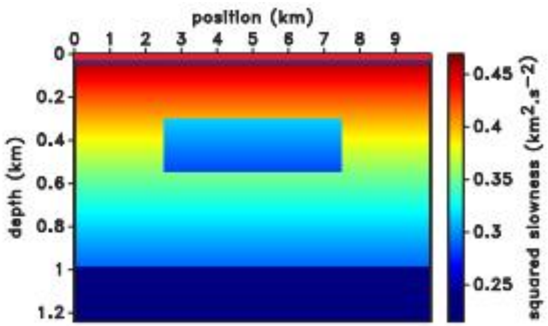


2.4 Combine MVA and FWI

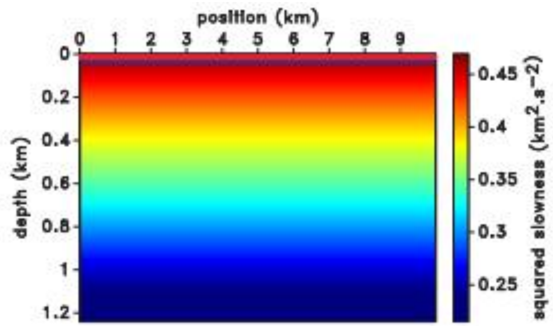
2. Bi-objective optimization method (C. Fleury)

$$J = (1-\alpha) w_{FWI} J_{FWI} + \alpha w_{MVA} J_{MVA},$$

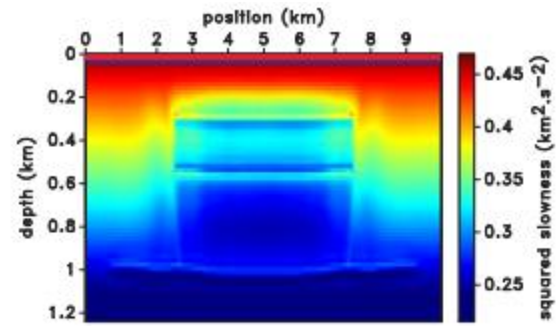
$$J_{FWI} = \frac{1}{2} \sum_{\mathbf{x}_S} ||\mathbf{d} - \delta_{\mathbf{x}_R} \mathbf{w}||_2^2 \qquad w_{FWI} = \sum_{\mathbf{x}_S} ||\mathbf{d}||_2^2 \qquad J_{MVA} = \frac{1}{2} ||\mathbf{A} \nabla_{\mathbf{m}_{ext}} J_{FWI}||_2^2 \qquad w_{MVA} = ||\nabla_{\mathbf{m}_{ext,0}} J_{FWI}||_2^2$$



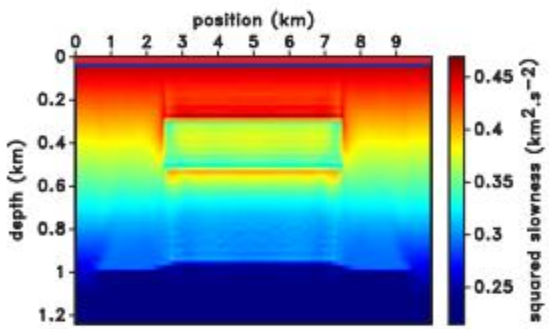
(a) Model $\mathbf{m}_{true}$



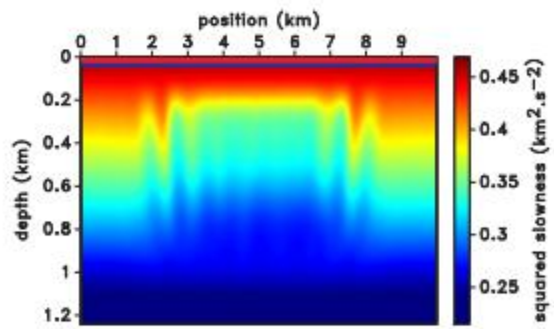
(b) Model $\mathbf{m}_0$



(c) Model $\mathbf{m}_{opt}$, $\alpha = 0.7$



(d) Model $\mathbf{m}_{opt}$, $\alpha = 0.0$



(e) Model $\mathbf{m}_{opt}$, $\alpha = 1.0$

2.4 Combine MVA and FWI

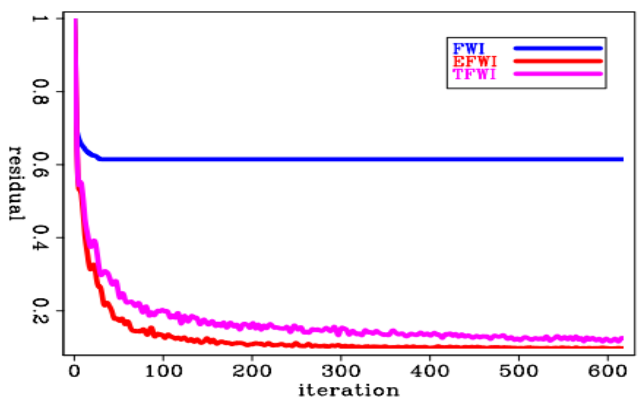
3. “TFWI” (D. Sun, Symes; Biondi, Almoni)

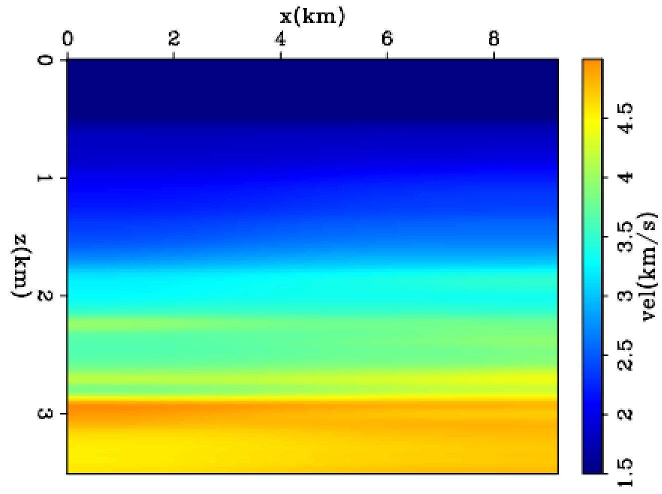
$$J_{\mathrm{EFWI}}\left(\mathbf{v}(\mathbf{h})\right) = \left\|\mathbf{d}\left(\mathbf{v}(\mathbf{h})\right) - \mathbf{d}_{\mathrm{obs}}\right\|_2^2$$

$$J_{\mathrm{DSO}}(\mathbf{v}(\mathbf{h})) = \||\mathbf{h}|\mathbf{v}(\mathbf{h})\|_2^2.$$

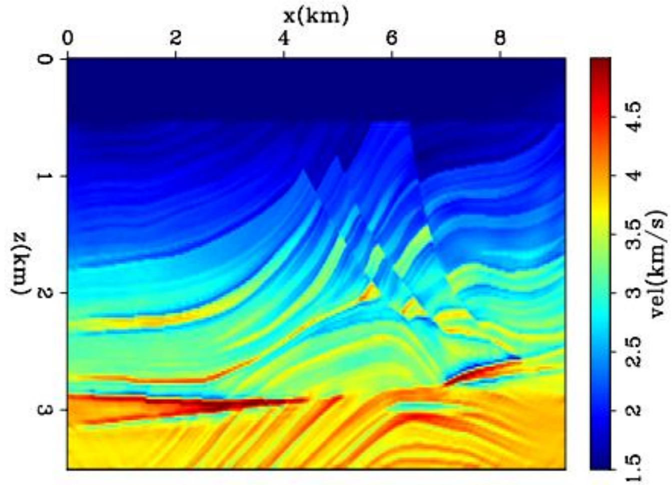
$$\left(v^2(\mathbf{x},\mathbf{h}) *^{-1} \omega^2 + \nabla^2\right) G(\mathbf{x}_s,\mathbf{x},\omega;\mathbf{v}) = \delta(\mathbf{x}_s - \mathbf{x})$$

$$\boxed{\begin{aligned} g_d(\mathbf{x},\mathbf{h}) = & \\ & \sum_{\mathbf{x}_s,\mathbf{x}_r,\omega} \nabla^2 f(\mathbf{x}_s,\omega) G(\mathbf{x}_s,\mathbf{x}-\mathbf{h},\omega;\mathbf{v}_0(\mathbf{h})) \\ & G(\mathbf{x}_r,\mathbf{x}+\mathbf{h},\omega;\mathbf{v}_0(\mathbf{h}))\ \Delta d^*(\mathbf{x}_s,\mathbf{x}_r,\omega;\mathbf{v}_0(\mathbf{h})) \end{aligned}}$$





Initial velocity



Inverted velocity by TFWI

Thinking…

1. What's the best sparse promotion operator? Curvelet may be good for reflectivity imaging, how about FWI?

2. Multiple is still a difficult problem in seismic imaging.

3. It is also possible to converge globally in data domain without low frequency if we choose OF carefully.

4. Extended model is the bridge of MVA and WI, but how to decrease the computational cost of extended inversion? Data compressing + sparse representation + preconditioning + GPU ?

Thank **CSC** for supporting my visit in TRIP!

Thank **TRIP** for supporting me to join SEG!

Thank **YOU** for your attention!