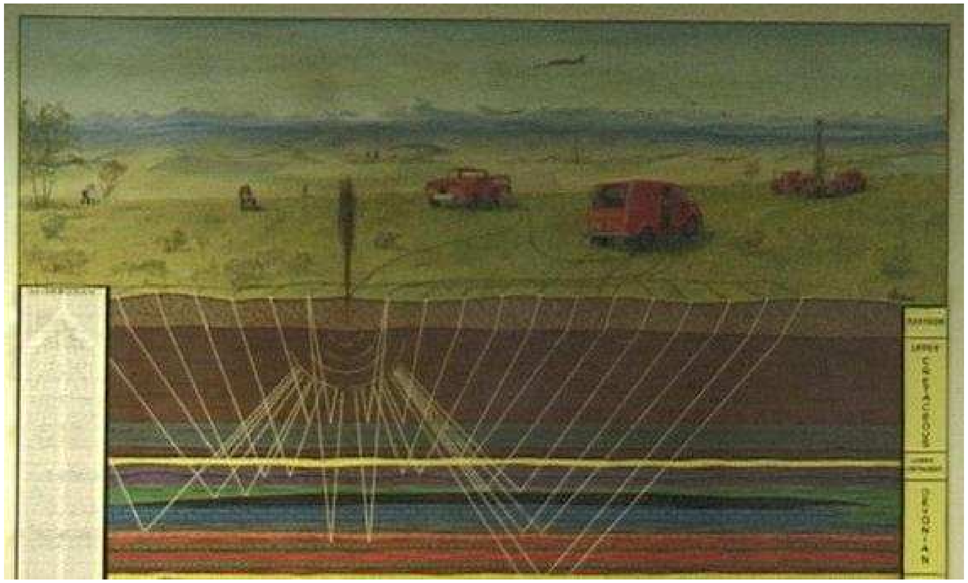
Seeing through rock: the mathematics of Reflection Seismology

William W. Symes

Reflection seismology is...

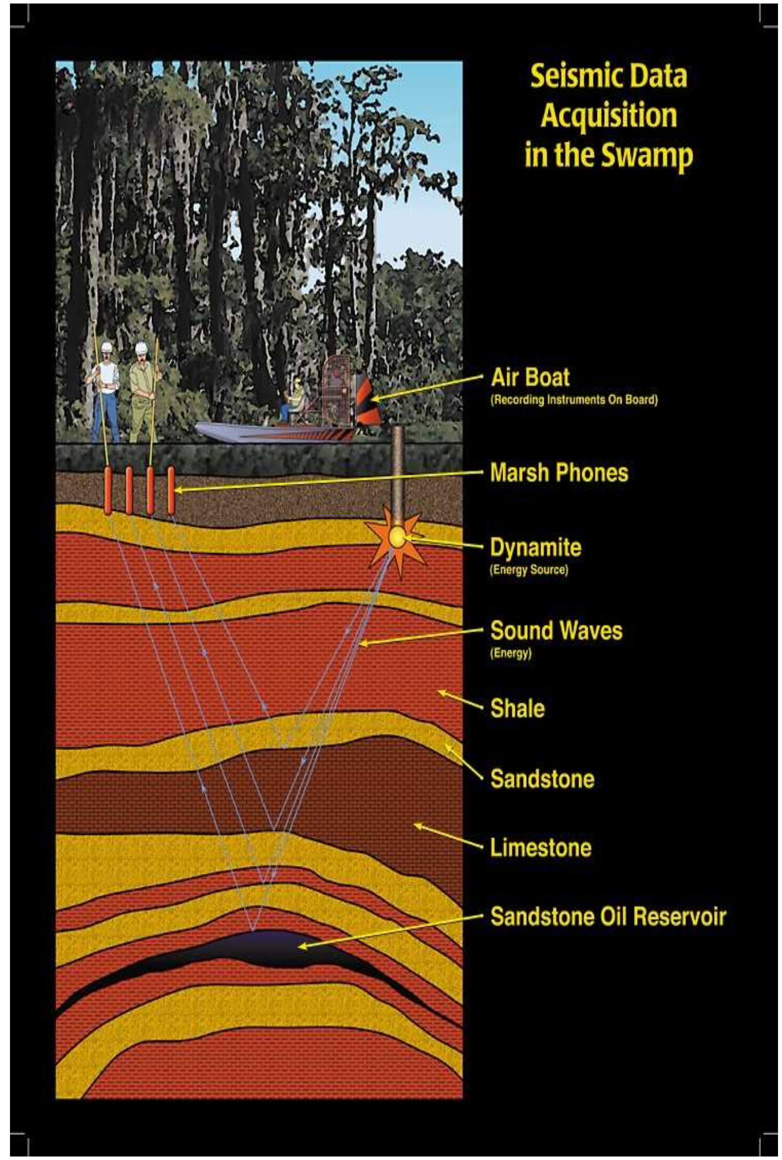
- the main prospecting tool of the oil and gas industry
- also useful in civil/environmental engineering and academic earth science
- a $4 billion / year industry
- a big deal in Houston
- *rich* in mathematical ideas and questions

The basic idea: collect echoes...



and - through various mathematical operations - convert them to maps of the Earth’s structure. Infer location of oil and/or gas from structural maps, sometimes from character of echoes also.

Land Operations:
undertaken worldwide,
From the Louisiana swamps...



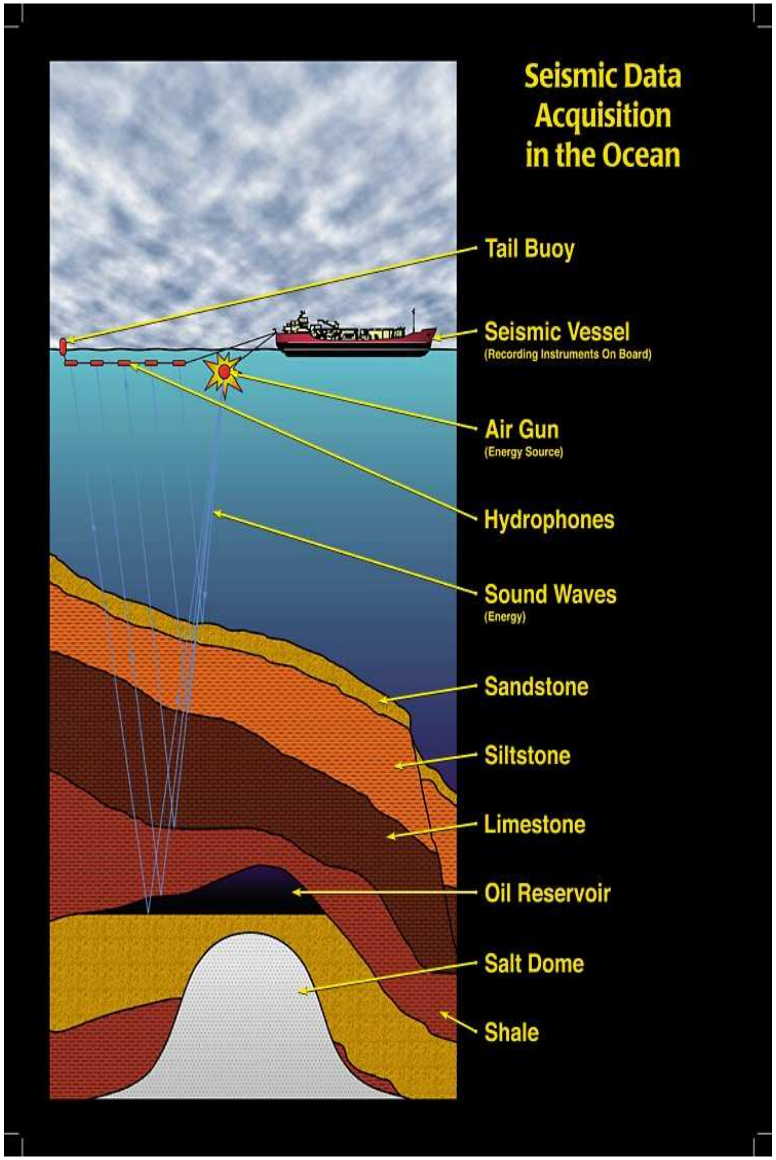
to the jungles of Papua-New Guinea...

to the Canadian Rockies...

to the Arabian desert...

to the hills of West Texas...

Marine Operations
90% of all data
collected worldwide



Data, data everywhere

The end result of all of these seismic surveys: *vast* quantities of data!

- each geo/hydro phone collects perhaps 1500 digitized samples per experiment ("shot").
- each experiment deploys 100's of geo/hydro phones.
- each survey consists of 100's (1980's) to 100,000's (today) of shots.
- upshot: 750 MB = small, old; 150 GB = modest, contemporary
- equivalent: 1 - 300 CD's. Immediate future: 100's of DVD's!

How to see raw sound...

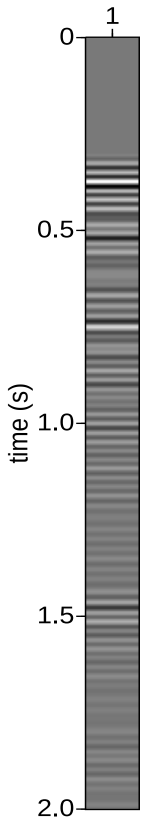
You don't want to hear it - like 10 million stereos going at once! You don't even want to "see" all of it at once!

Basic facts:

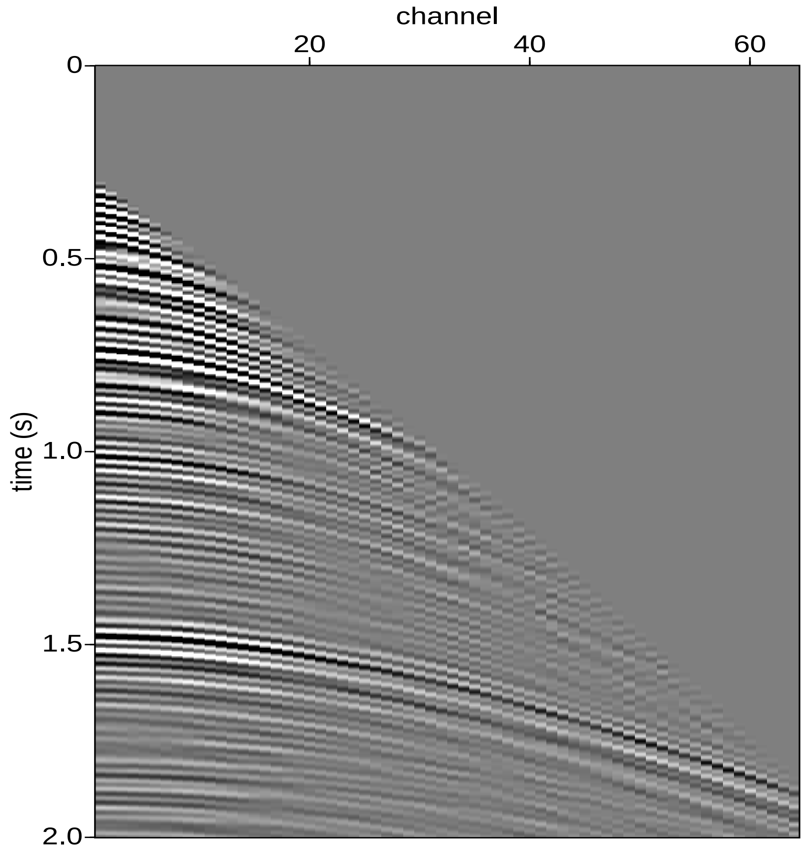
- sound is *pressure* fluctuations - quantify in lb/ft^2 or N/m^2 or ..., i.e. *numbers*
  - *structure of ear, drum*
- sound is *oscillatory* - rapid highs, lows of pressure following upon each other
  - *A below middle C = 440 cycles/sec ("Hertz", Hz)*
- $\Rightarrow$ represent pictorially as grey - dark = higher pressure, light = low pressure, plotted vs. time.

Recording of single hydrophone

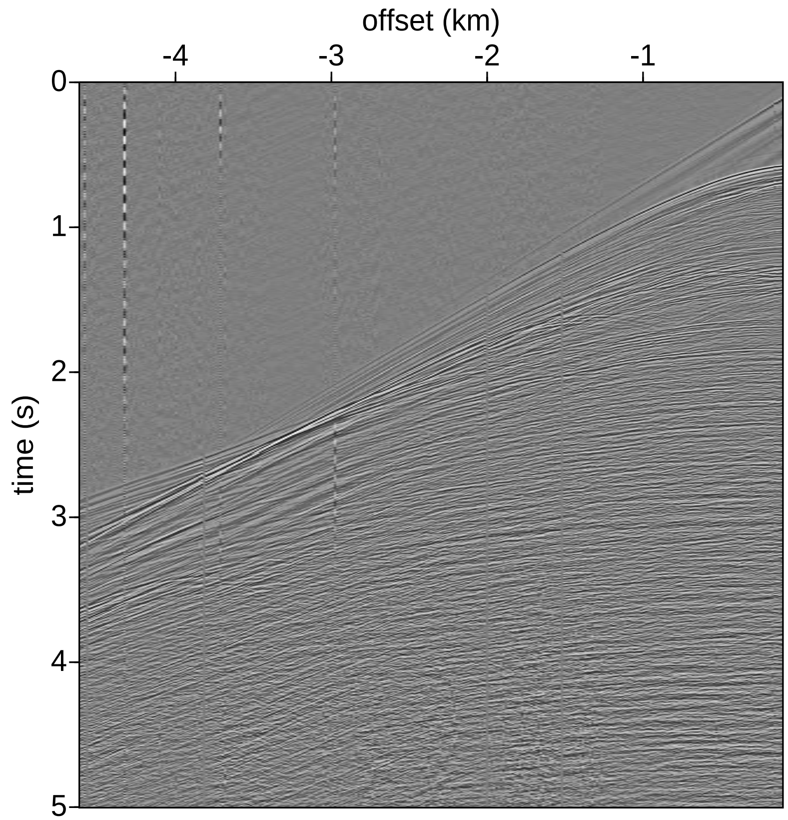
North Sea survey (thanks: Shell)



Shot record = recording of
single experiment, 64 hydrophones
North Sea survey (thanks: Shell)

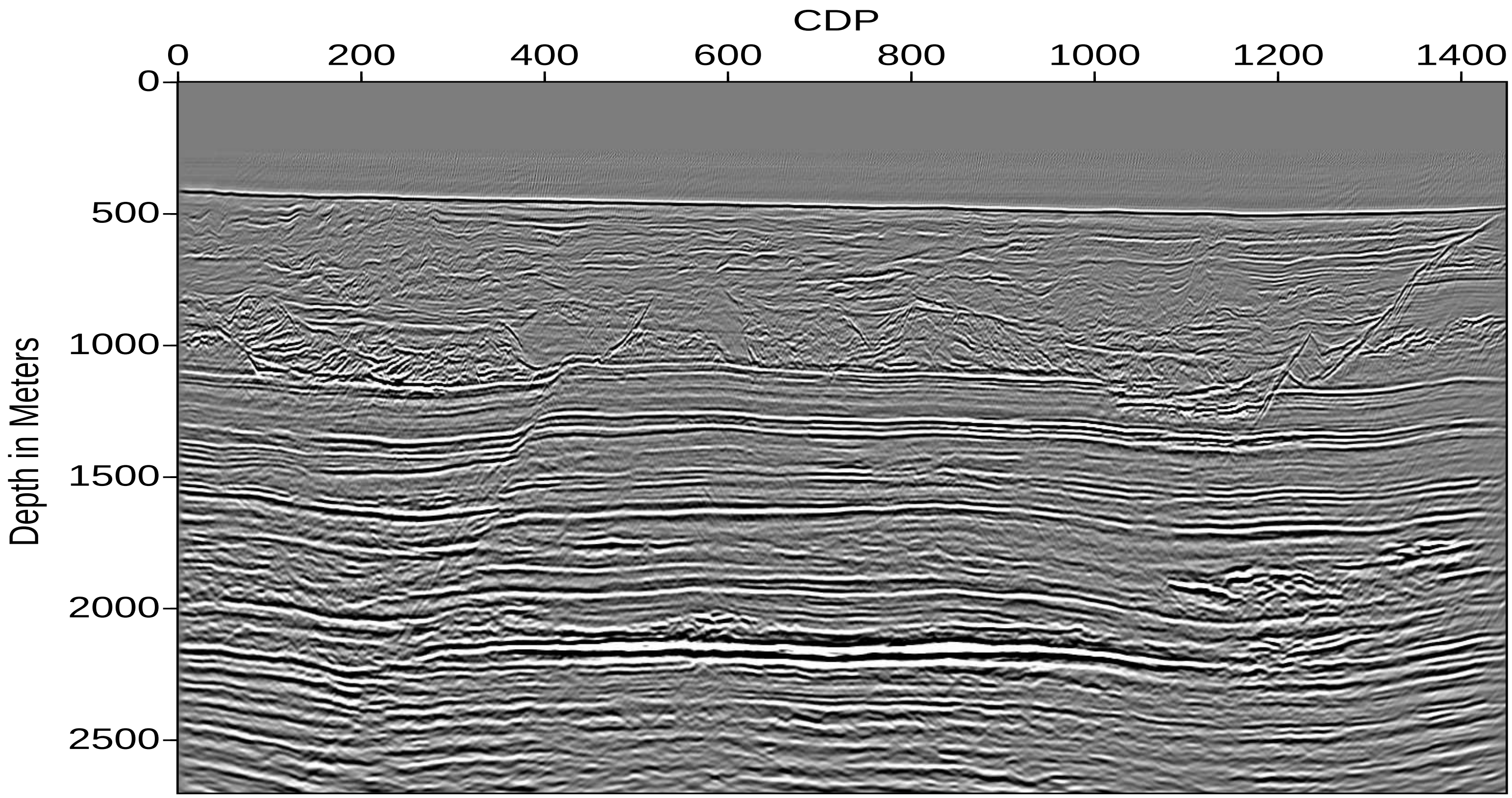


How do you turn lots of this...



(field seismogram from the Gulf of Mexico - thanks: Exxon.)

into this?



(structural image, Mississippi Canyon, offshore Louisiana)

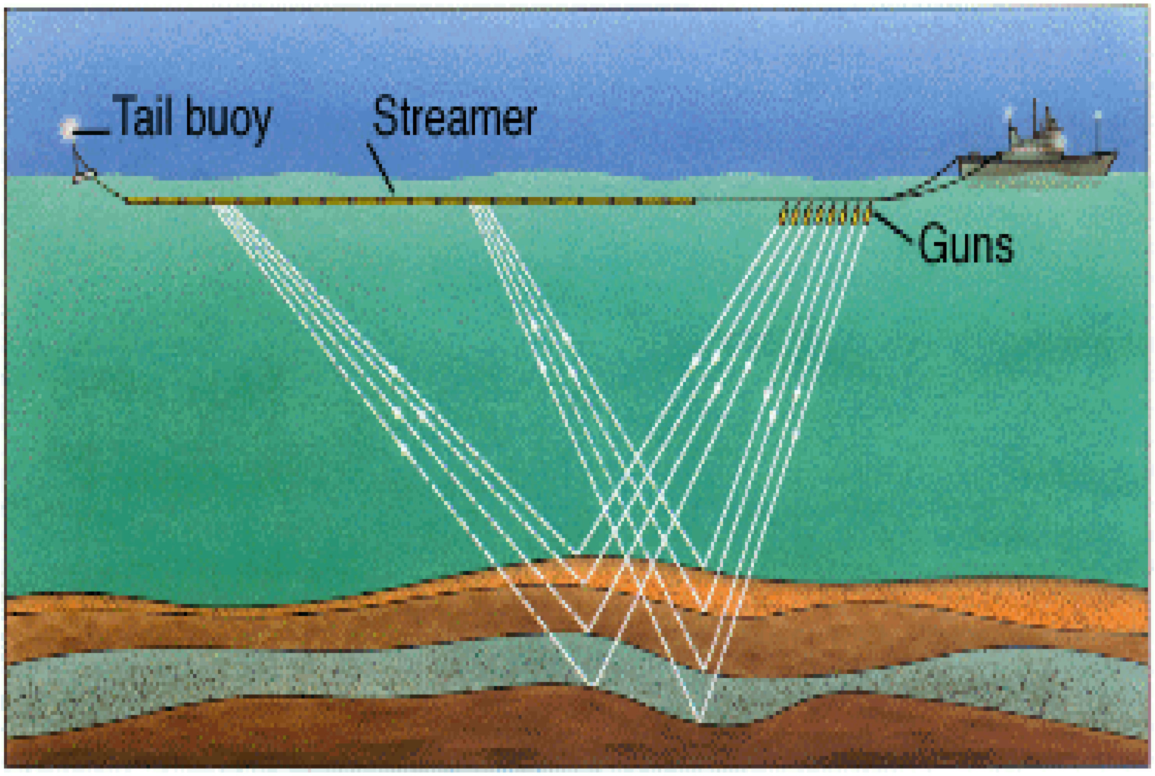
Outline

- Acoustic fundamentals
- Zero offset imaging
- Multioffset imaging
- Velocity via redundancy
- Refraction of sound

Acoustic fundamentals

- sound (in rock or in air) consists of *oscillatory pressure fluctuations*
- sound travels at a definite speed, characteristic of material
  - *sound speed in air* $\sim$ *330 m/s* $\sim$ *1000 ft/s; so count seconds after lightning flash until thunder arrives, multiply by 1000 to get distance in feet to lightning strike*
- reflection seismographs record *echoes* $\Rightarrow$ sound must travel from *sound source* (dynamite, air gun) to subsurface structure *and back* to geo/hydro phone.
- echoes (or *reflections*) occur when sound reaches a change in mechanical characteristics (eg. a change in sound speed!) - air to marble, sandstone to limestone, water-saturated sandstone to gassy sandstone,...

Acoustic fundamentals

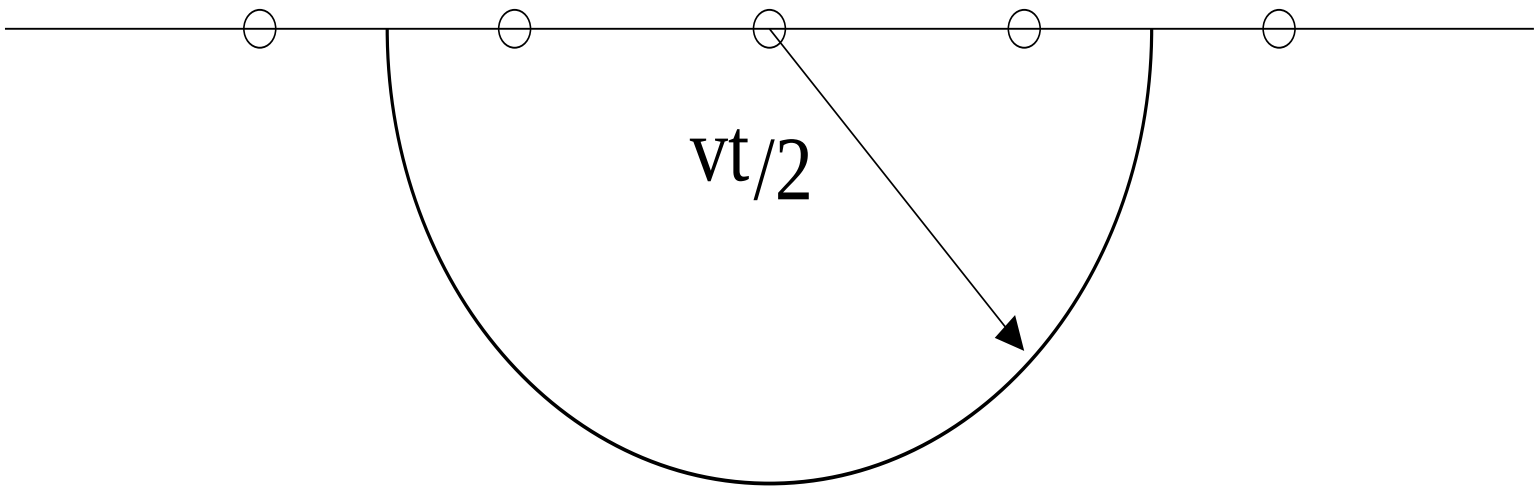


Offset = distance from source to receiver. Picture shows "near" and "far" offsets.

Zero Offset

- source and receiver in same place
- can’t really acquire (dynamite!)
- but can fake it by standard techniques
- easiest to understand how to build image...

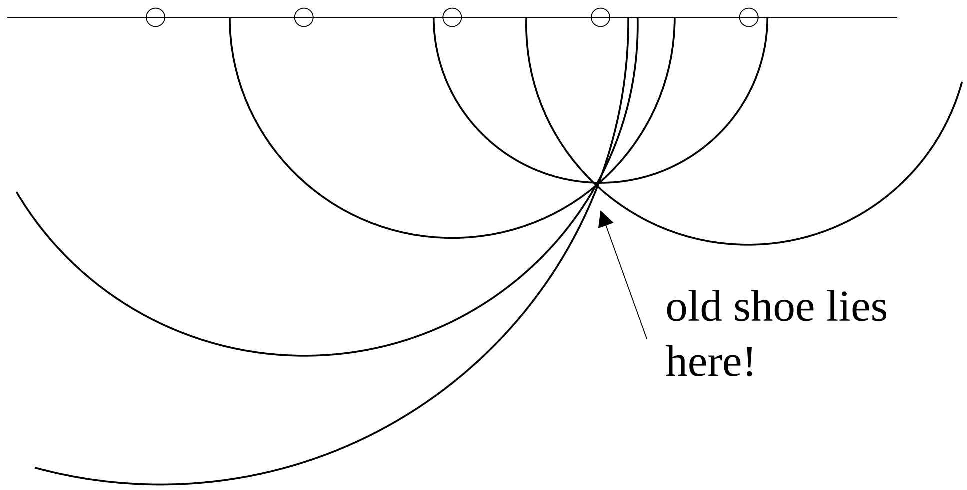
Out and back...



v = sound speed (or *velocity*) - 330 m/s in air, 1500 m/s in water, 2000 - 5000 m/s in sedimentary rock.

If you hear an echo at time t after seismic wave initiation at zero offset, then the source of the echo must lie somewhere on a circle of diameter vt.

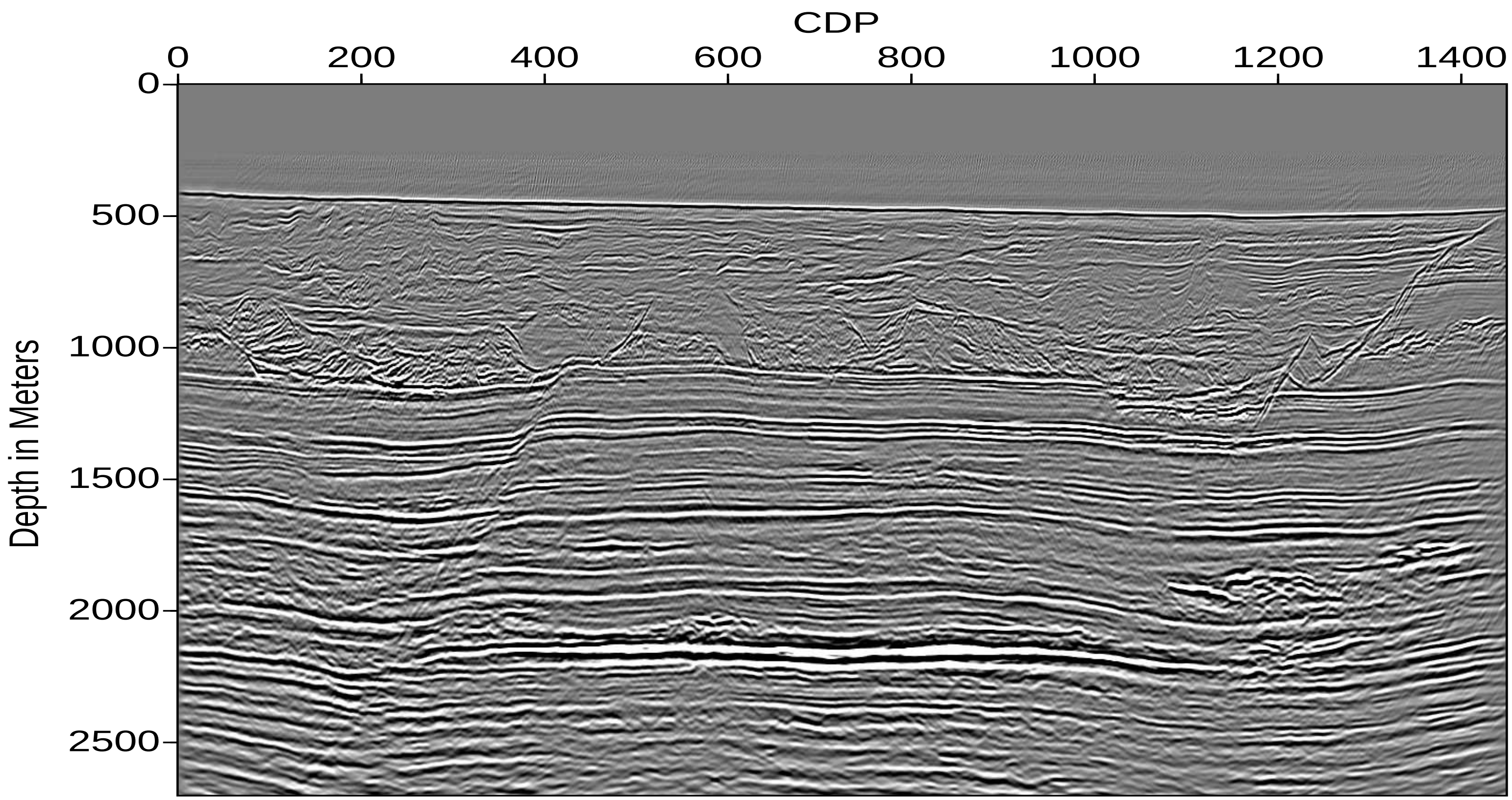
Finding and old shoe...



Time the echo from the old shoe... “hello!” from each source/receiver point, listent at same place, record time.

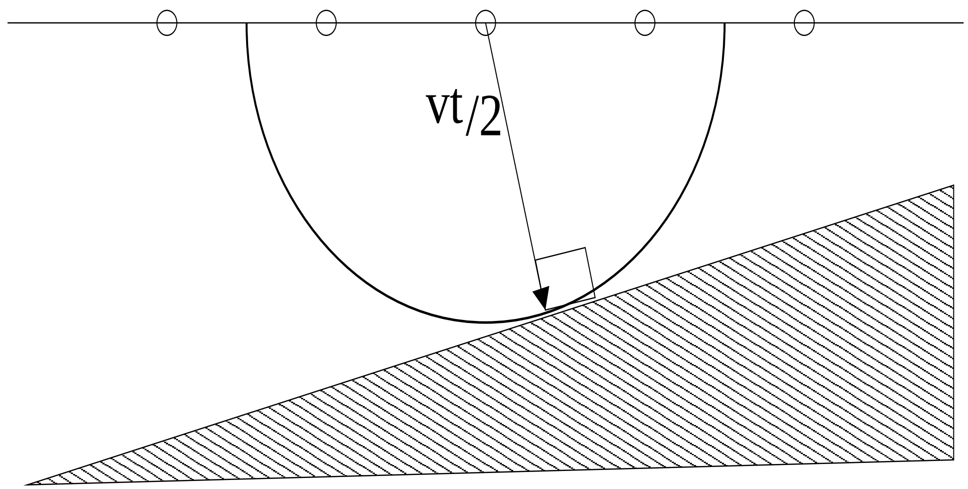
Shoe must lie on circles of various radii, computed as in previous slide - therefore on their intersection!

However...



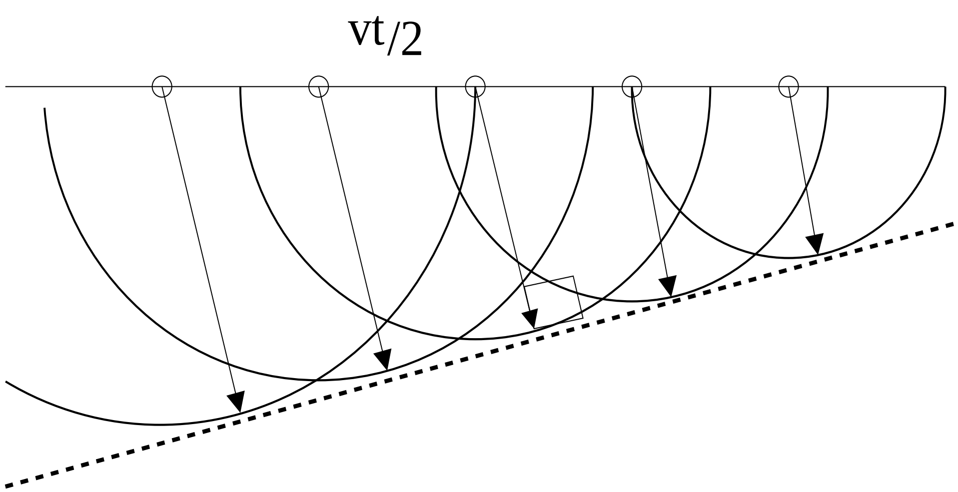
Earth structure does not resemble an old shoe - *extended*, *distributed*

Physics of sound reflection - Zero Offset



Reflection occurs at the place and time at which the wavefront is *tangent* to the reflecting surface - the *specular* (mirrorlike) reflection principle. Radius from source/receiver point is *normal* (perpindicular) to surface.

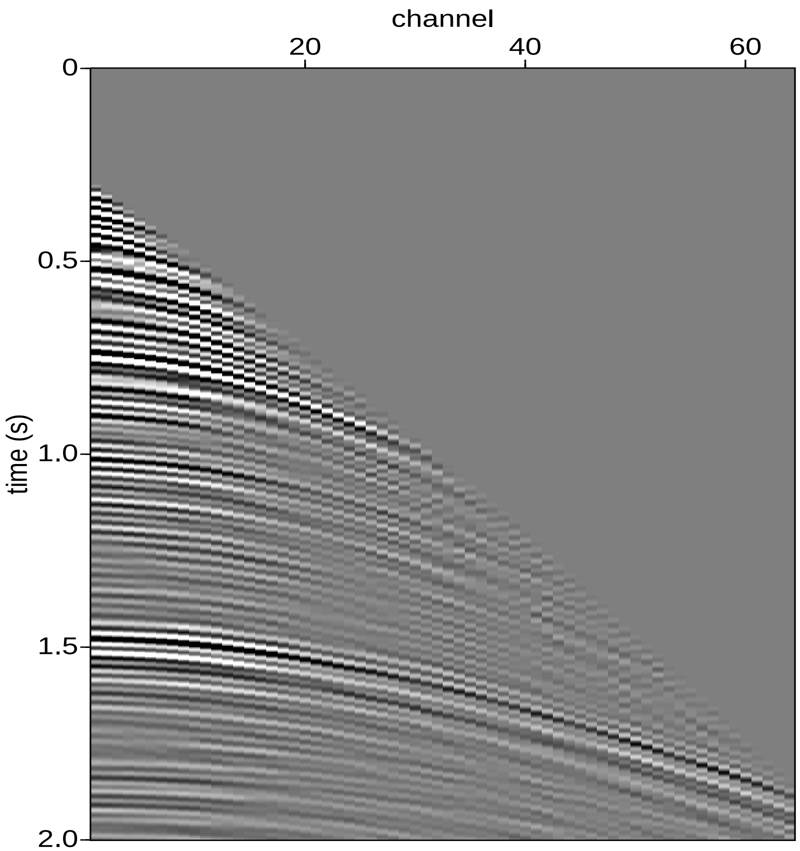
Locating a reflecting surface - zero offset



Reflecting surface must be tangent to each wavefront circle - *envelope* of circles.

Another issue

Do you really want to find the times of all the echoes?



Direct construction of images - *ZO Migration*

Concept:

(1) for each source/receiver point each sample time t (usually multiples of 2 ms or 4 ms), *place* the sample value at all possible image points at distance $vt/2$ (circle).

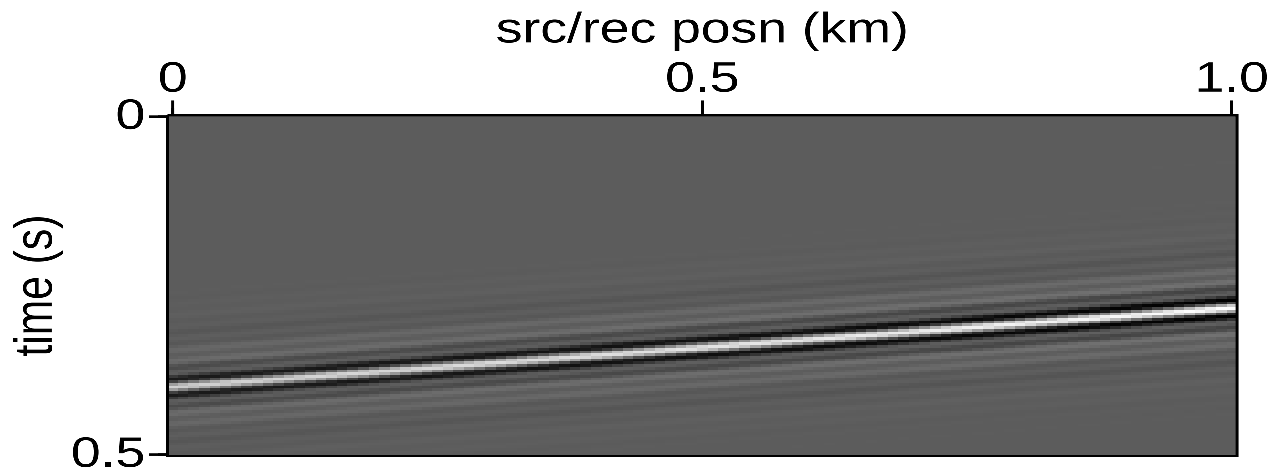
(2) then sum the resulting images, one for each source/receiver point.

Each image will exhibit circular wavefront.

Principle of Stationary Phase: Sum of oscillatory signals *cancels*, *except* along envelope of wavefronts.

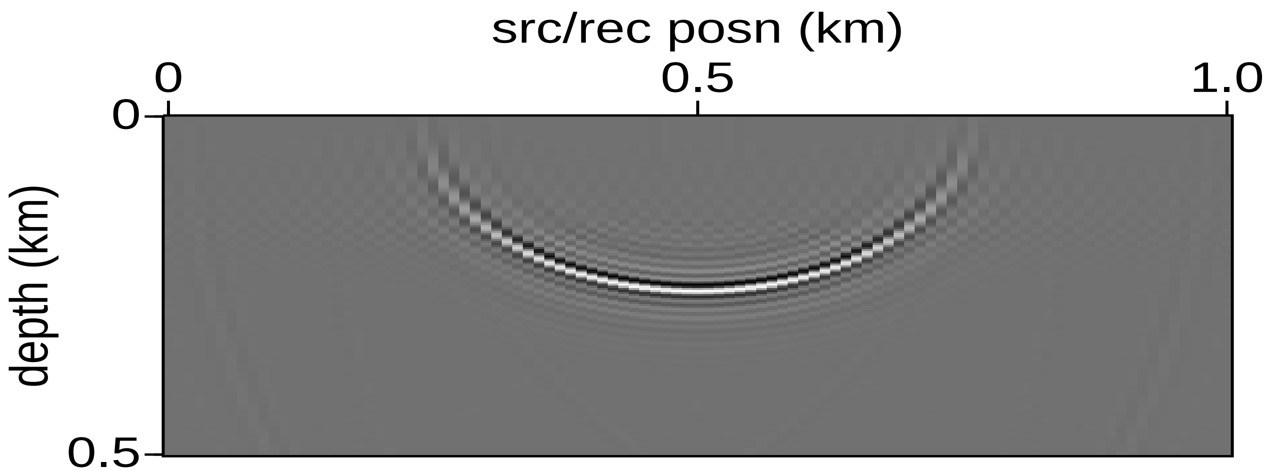
Consequence: after the summation, *only the envelope is visible!!!*.

ZO Migration Example (1/4)



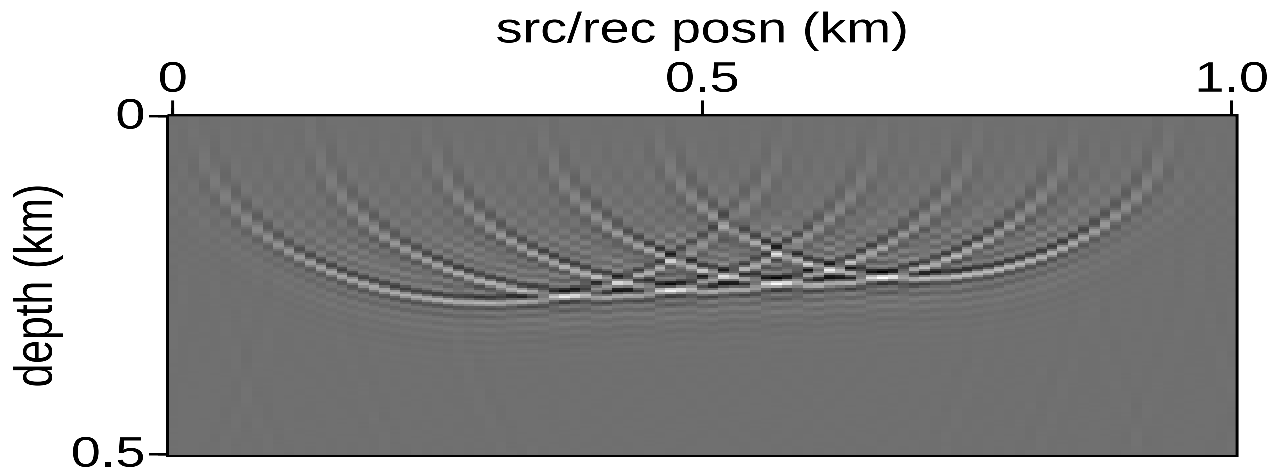
Zero offset simulated data for reflecting surface at depth $z = 0.3$ km at left edge ($x = 0$ km), sloping upwards to the right at 5 deg. 101 source/receiver points, spaced 10 m apart, from left to right.

ZO Migration Example (2/4)



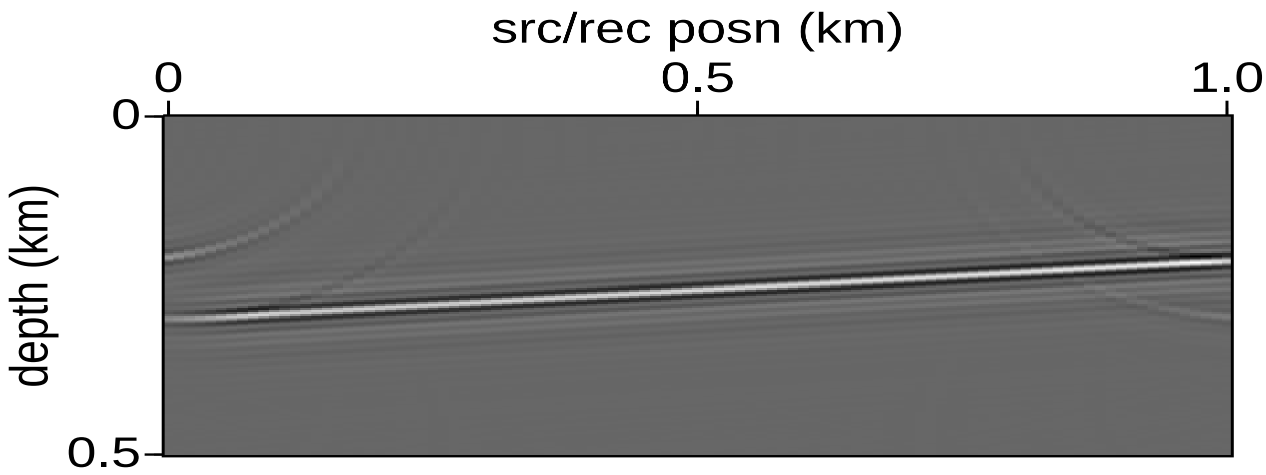
(1) for each source/receiver point each sample time t (usually multiples of 2 ms or 4 ms), *place* the sample value at all possible image points at distance $vt/2$ (circle).

ZO Migration Example (3/4)



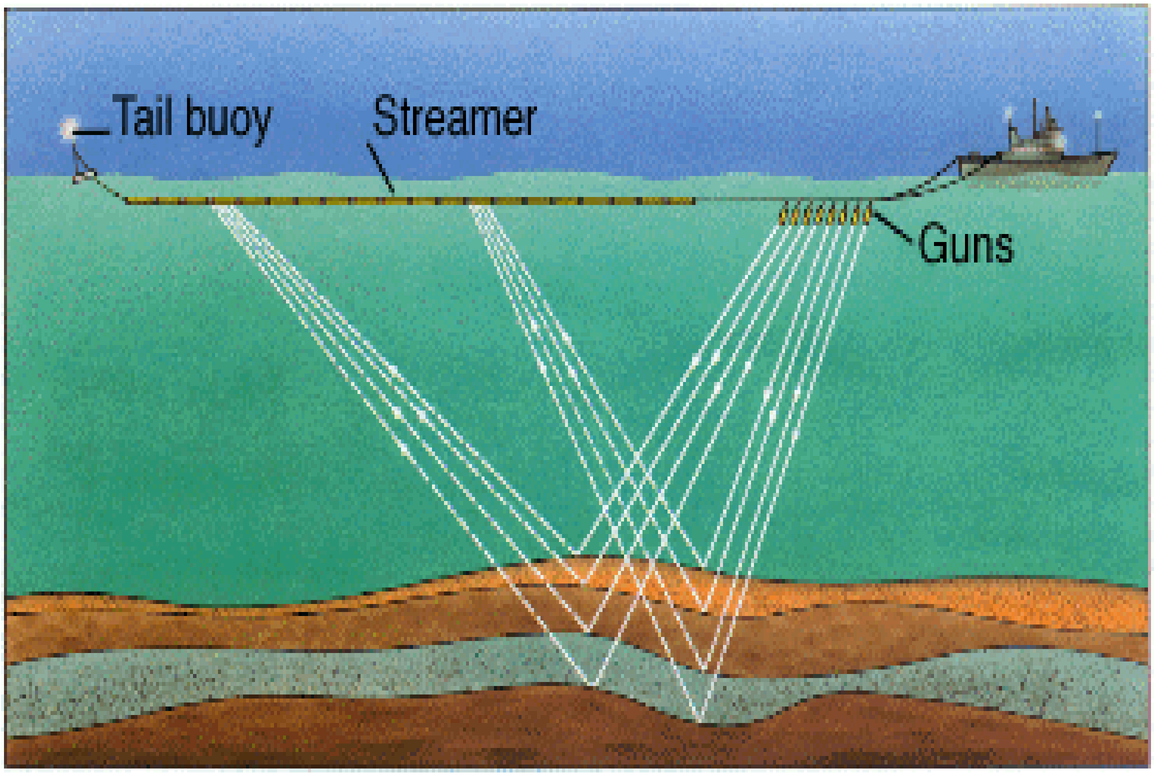
(2) then sum the resulting images, one for each source/receiver point (this figure shows 5 of the 101 source/receiver images added up, at 0.3, 0.4, 0.5, 0.6, 0.7 km from left edge).

ZO Migration Example (4/4)



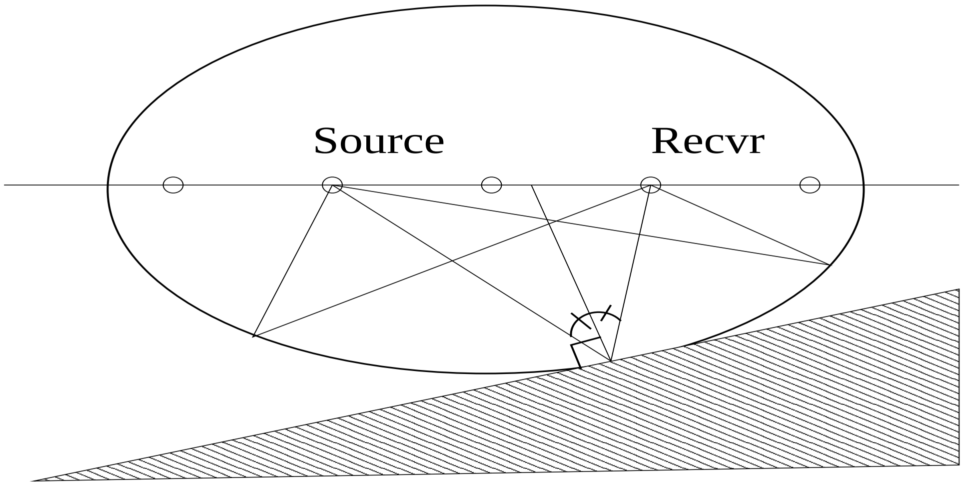
(2) then sum the resulting images, one for each source/receiver point (this figure shows all 101 source/receiver images added up - the wavefront envelope is recovered, sitting exactly on the reflecting surface!)

Multioffset Migration



Multiple Offsets = data that's actually measured. Can either *fake* ZO data from multioffset data (1970's technology) or directly use MO data - routine today.

Physics of sound reflection - Nonzero Offset



Reflection occurs at the place and time t at which ellipse with focii at source, recvr, diam = vt is *tangent* to the reflecting surface - the *specular* (mirrorlike) reflection principle. Bisector of lines from source and receiver points is *normal* (perpindicular) to surface (*Snell's law*) $\Rightarrow$ surface is *envelope* of ellipses.

Direct construction of images - *MO Migration*

Concept:

(1) for each source/receiver point each sample time t (usually multiples of 2 ms or 4 ms), *place* the sample value at all possible image points on ellipse of diameter vt.

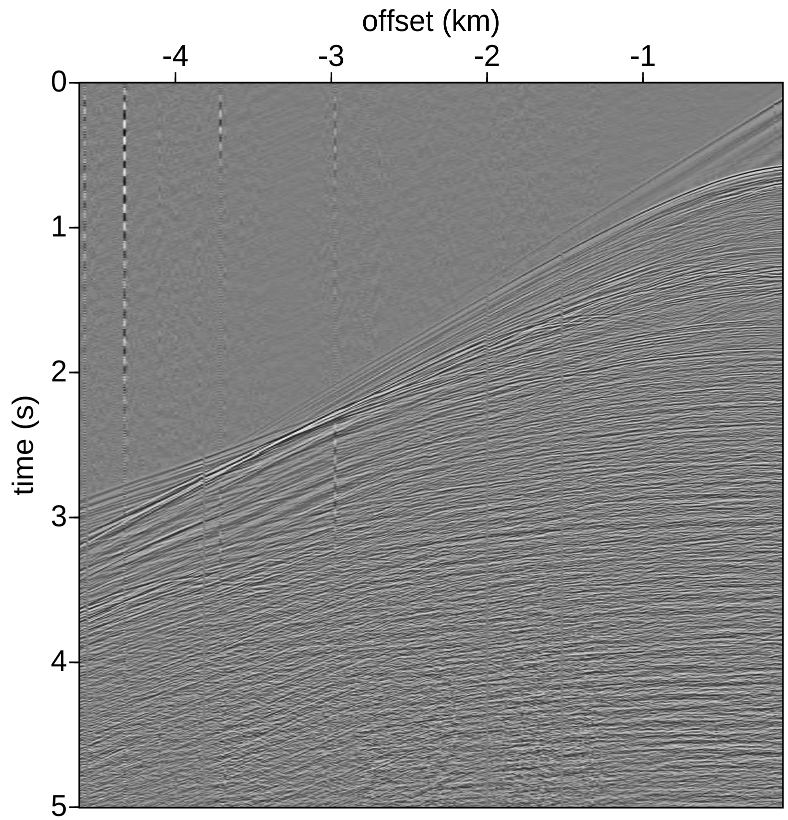
(2) then sum the resulting images, one for each source/receiver pair.

Each image will exhibit elliptical wavefronts.

Principle of Stationary Phase: Sum of oscillatory signals *cancels*, *except* along envelope of wavefronts.

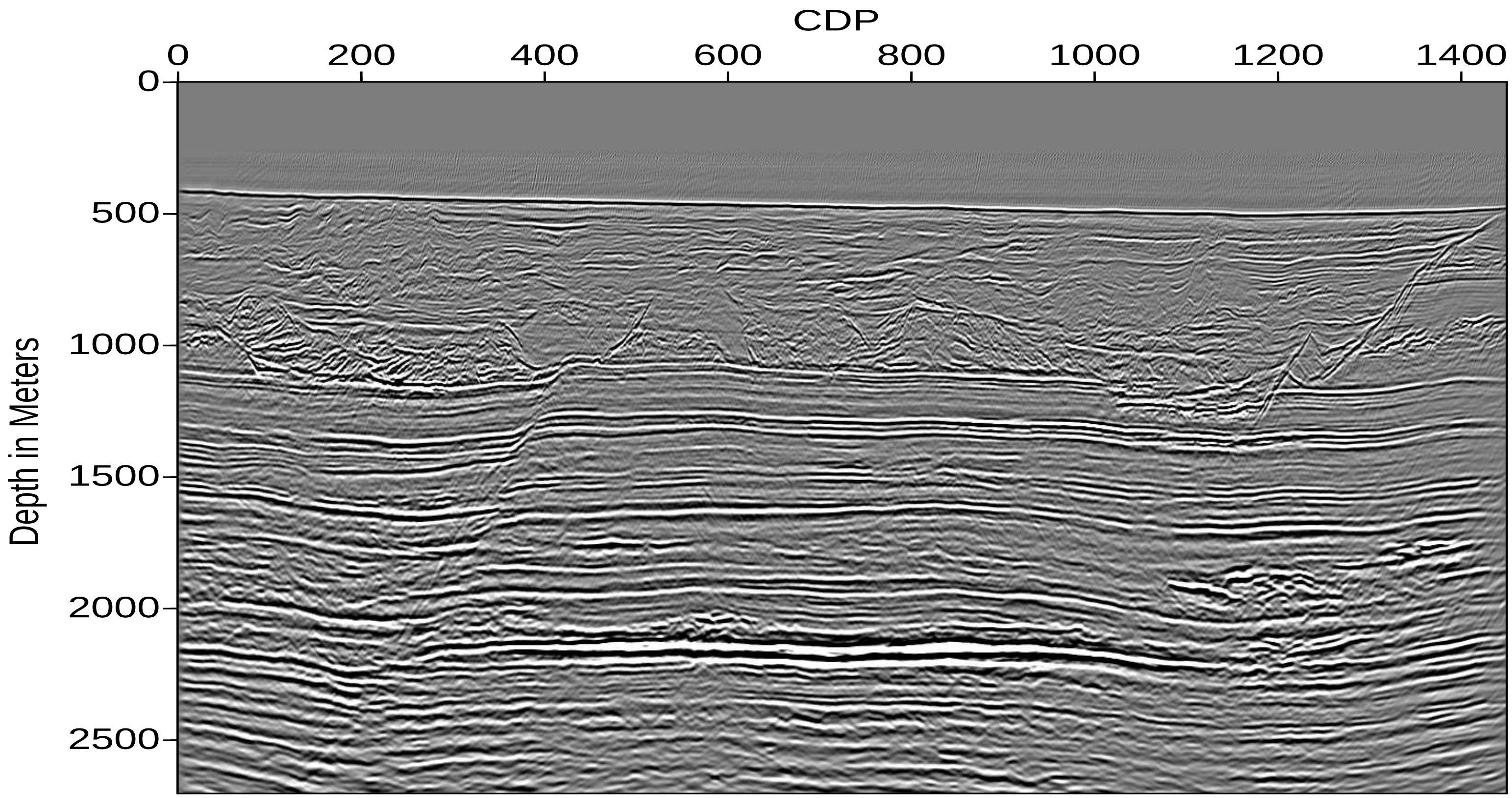
Consequence: after the summation, *only the envelope is visible!!!*.

And that's how we turned lots of this...



(one of 500 data panels from Exxon GofM data = 750 MB.)

into this!



(structural image, Mississippi Canyon, offshore Louisiana)

Two major details

(1) Q. how do you know v? A. You don't!

(2) Spatial variation of velocity and refraction of waves

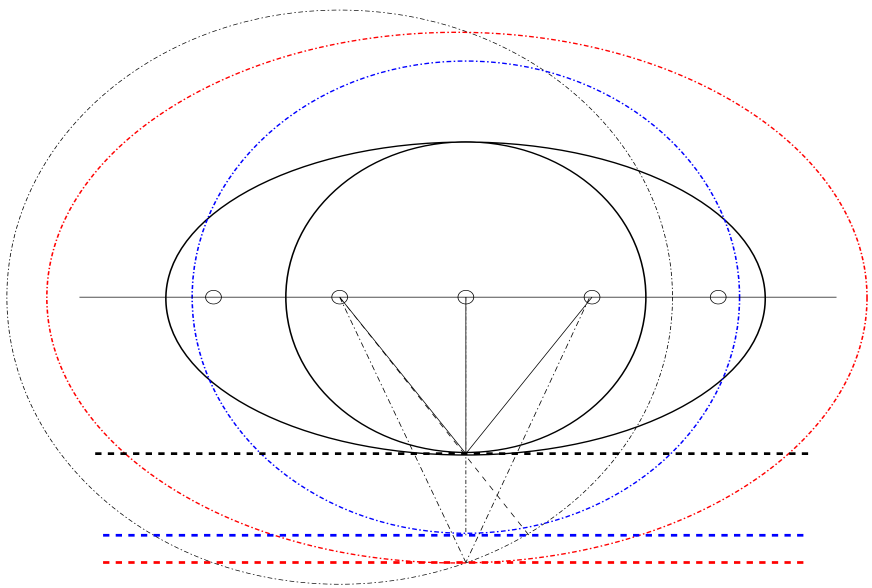
Velocity via redundancy (1/6)

- Typical survey contains source-receiver pairs with a variety of offsets.

- Any one set of ellipses with fixed offset (source-receiver distance) is enough to determine reflecting surfaces by forming envelopes - *common offset* construction

- the envelopes constructed using different offsets will only agree if the velocity is correct!

Velocity via redundancy (2/6)



Black circle, ellipse: zero, nonzero offset, correct velocity - both have dashed black line as possible envelope.

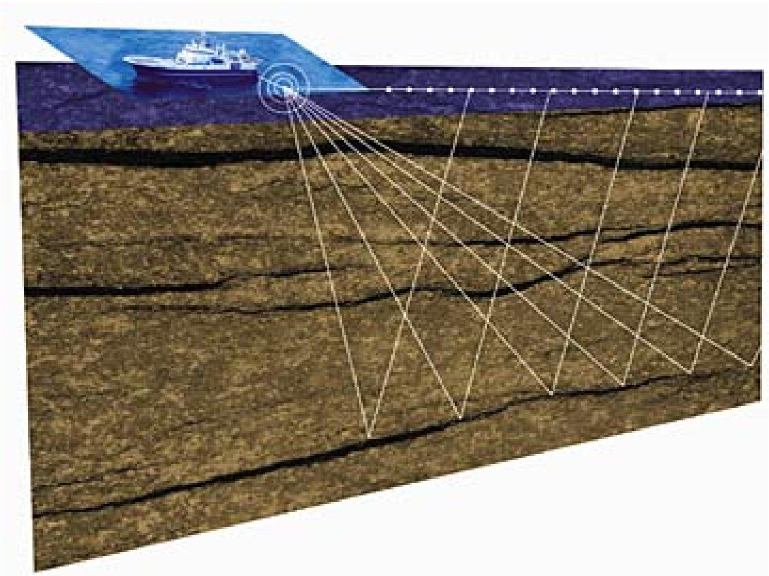
Blue circle, red ellipse: same offsets, velocity 50% high. Envelopes are *different*, so predicted depths of reflecting surface at *same* position will *disagree*.

Velocity via redundancy (3/6)

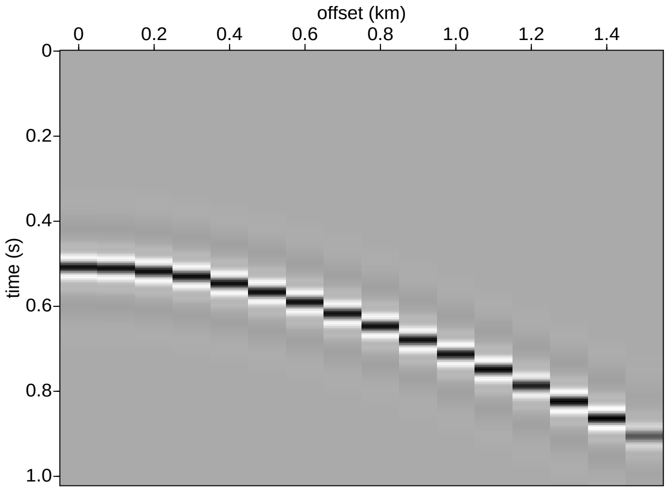
You can see this with the data directly:

- sort the data recordings into piles, each pile ("gather") sharing common offset;
- carry out migration for each common offset gather separately: place data value at time t on ellipse of diameter vt, sum together to get *common offset image* (just like we did for zero offset - a special case!);
- from each image, extract the part of the image that lies below a specific point on the surface - a "virtual well";
- if the velocity is correct, then all "virtual wells" will look same! To compare, put them together, one for each offset, to form a *common image gather*. Look at it!

Velocity via redundancy (4/6)

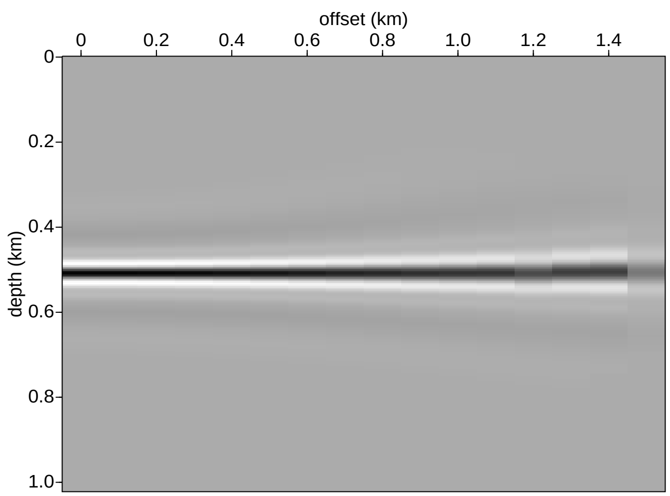


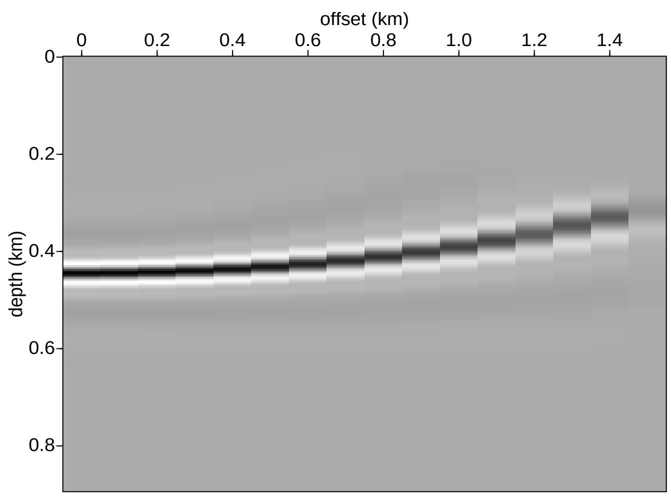
,



Synthetic experiments - data from different offsets

Velocity via redundancy (5/6)



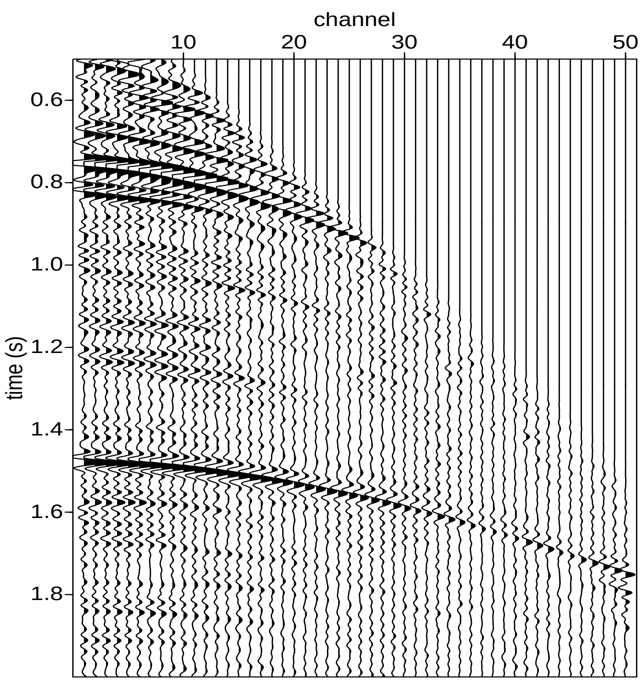


,

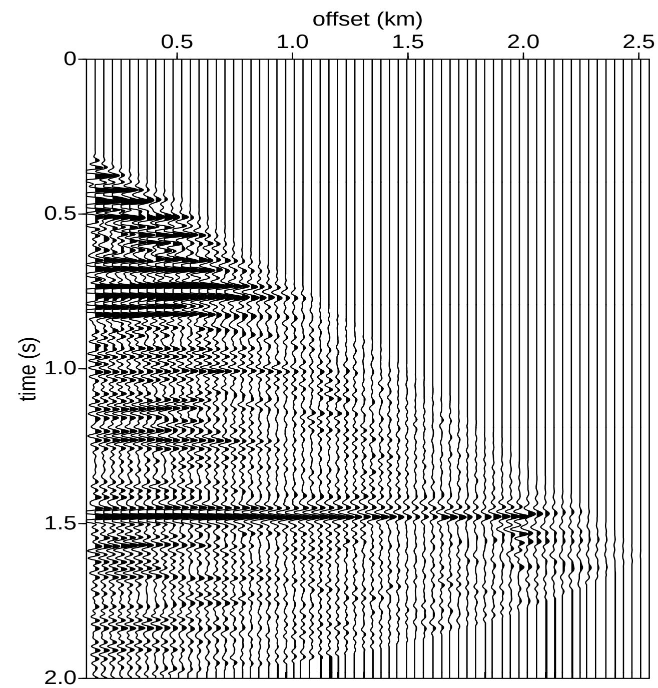
“Virtual wells” displayed next to each other: Left, correct v; Right, 10% low.

“Correct velocity flattens common image gathers” - the practical principle for finding sound velocity from seismic data.

Velocity via redundancy (6/6)



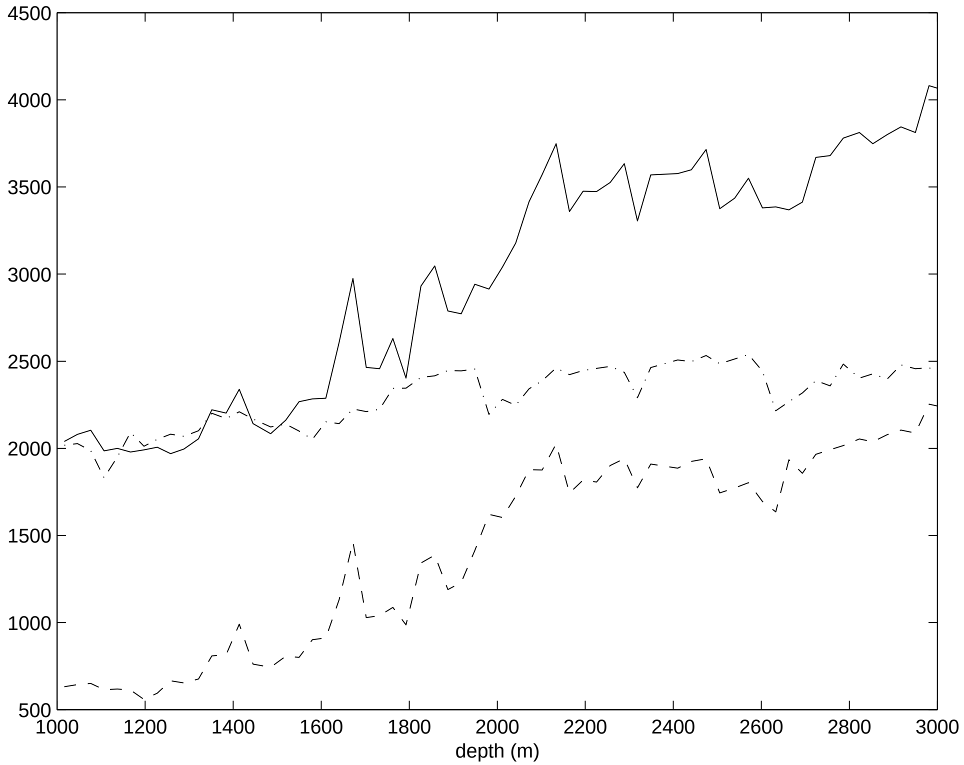
,



,

Works with actual data: Left, data with different offsets, North Sea survey; Right, common image gather created with “ pretty good” velocity.

Velocity Variation and Refraction (1/5)



Variability of sound velocity in the Earth: well log (direct, local measurement in borehole) from North Sea. Top curve is sound velocity - varies by factor of 2 over 1 km $\rightarrow$ 3 km depth range.

Velocity Variation and Refraction (2/5)

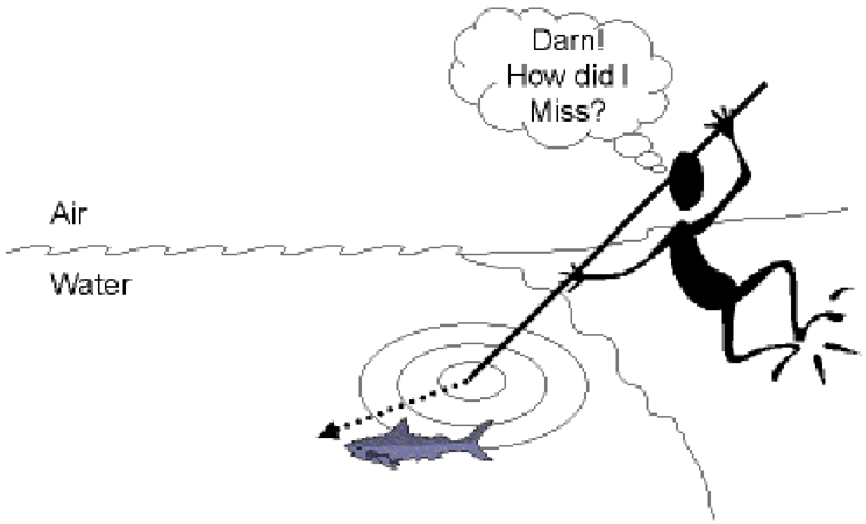
Two major consequences of velocity variability:

- Velocity must be described by many numbers rather than one;

- When sound encounters change in velocity, wavefronts *bend* or *refract* - equal time curves no longer circular or elliptical.

- The same thing happens to light - a familiar phenomenon!

Velocity Variation and Refraction (3/5)



.Figure 3. Ray-bending causes the spear fisherman to miss the fish

For the fisherman, the bending of the light waves at the air/water interface causes the fish to appear offset from where it really is. Similarly, for the dry hole driller, the bending of the seismic energy at the fast/slow layer interface causes the image of the geologic structure to appear offset from the actual structure (Figure 4).

thanks: Landmark Graphics

Velocity Variation and Refraction (4/5)

January/February/March 2000 UserNet 19

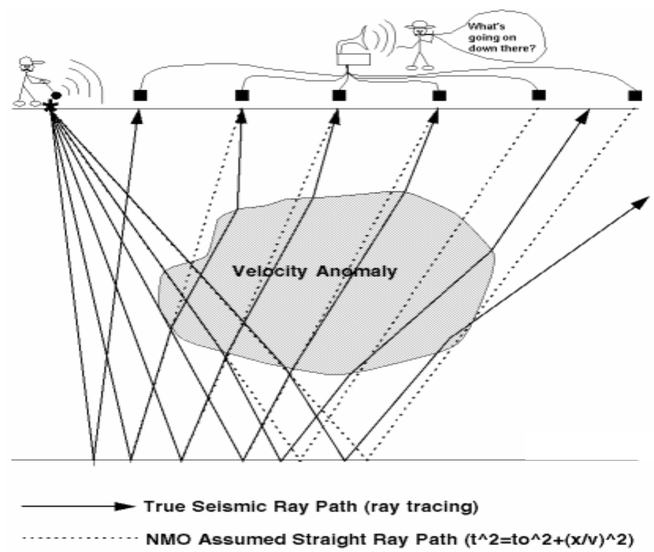


Figure 3. Ray tracing vs. conventional processing. $\left[t^2 = t_0^2 + \left(\frac{x}{v}\right)^2\right]$

This approach uses model-based ray tracing through a fixed overburden velocity field, while varying the interval velocity of the analysis layer. This process provides greater resolution of lateral velocity variation than conventional Dix inversion of stacking velocities. A coherency inversion combined with a map-migration workflow may be considered a "second order" depth conversion.

Technical Newsletter for Landmark Users

thanks: Landmark Graphics

Velocity Variation and Refraction (5/5)

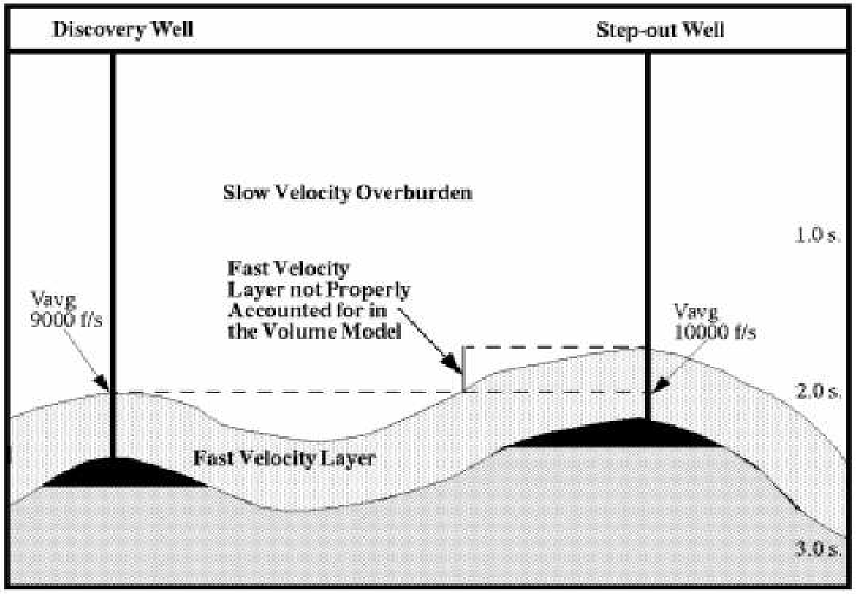


Figure 2. Common Problem with Single Function Depth Conversions.

This happened because the depth conversion used an average velocity of 9000 ft/s down to 2.0 seconds, while in reality, due to the structure, this velocity is really 10000 feet per second. It follows that the geologists' simplified velocity model led to the 1000-foot depth conversion error at 2.0 seconds.

Second, a proper depth conversion is critical to determining the correct lateral position of a structure. This type of depth conversion problem is as old as prehistory, when the first spear fisherman tried to catch a fish (Figure 3).

thanks: Landmark Graphics

Summary

- Seismic imaging concepts grounded in plane geometry, distance = velocity $\times$ time.
- Reflecting surfaces form *envelope* of equal-time curves (*Snell's Law*).
- *Principle of Stationary Phase* permits *imaging* of the reflecting surface directly by data manipulations, without geometrical construction of the envelopes.
- *Redundancy* of seismic survey data enables estimation of velocity directly from the data, via *flattening of common image gathers*.
- *Refraction* complicates all of the above steps.

Frontiers

- Estimating seismic velocity (many numbers, representing sound speed at different places in the Earth) reliably and (semi) automatically is still infeasible.

- Formation of images, given correct velocity, is still not always successful when large velocity changes cause large refraction.

- Many other currently unresolved difficulties will keep mathematicians, geophysicists, and software engineers busy for many years to come!

Credit where credit is due

Photos, drawings in this talk provided by:

- Landmark Graphics, Schlumberger, Western Geco, Veritas DGC, Seismic Consultants, and WWS

Seismic data provided by:

- Exxon Production Research Co., Shell International Research

Software used to produce data-derived graphics:

- Seismic Unix (Colorado School of Mines), TRIP (Rice University)

Where to find these slides...

`http://www.trip.caam.rice.edu`

> Downloadable Materials