Seismic Imaging

William W. Symes

C. I. M. E.
Martina Franca
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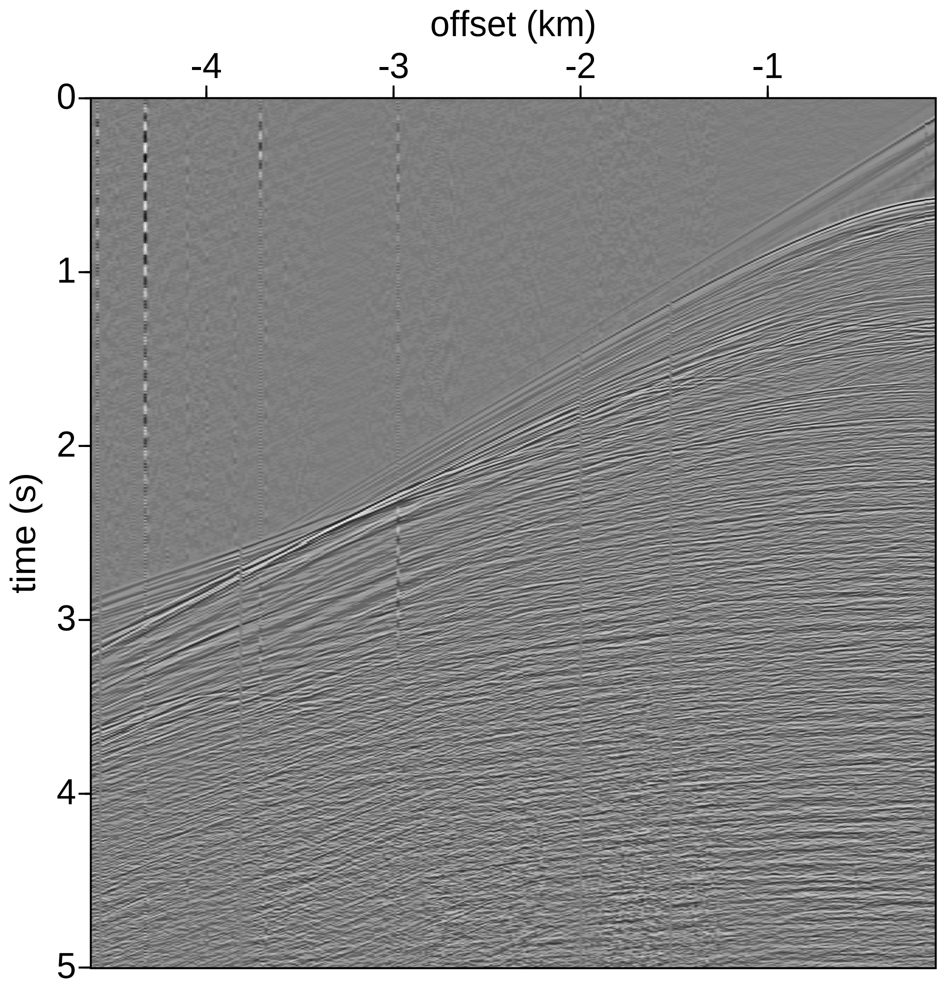
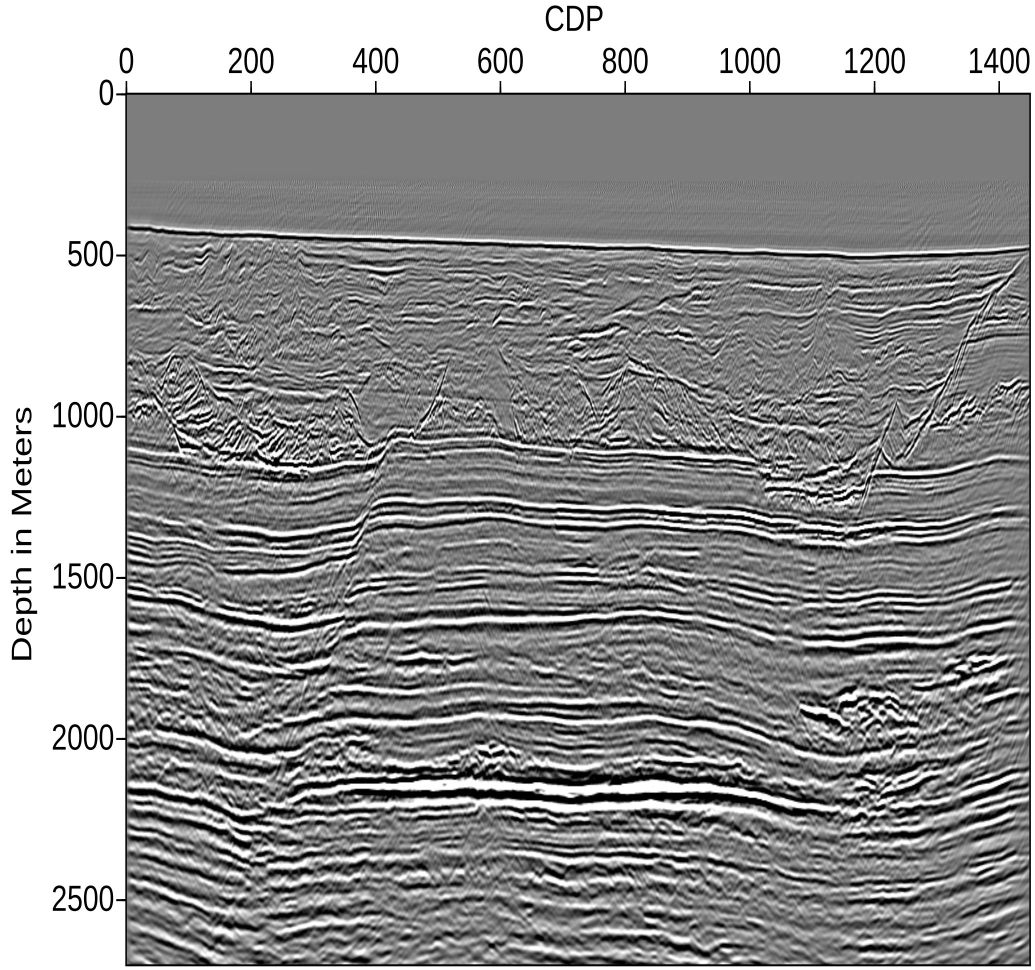
www.trip.caam.rice.edu



How do you turn lots of this... (field seismogram from the Gulf of Mexico - thanks: Exxon.)



into this (a fair rendition of subsurface structure)?

Central Point of These Talks:

Estimating the index of refraction (wave velocity) is *the central issue* in seismic imaging.

Combines elements of

- optics, radar, sonar - reflected wave imaging
- tomography - with curved rays

Many unanswered mathematical questions with practical implications!

A mathematical view of reflection seismic imaging, as practiced in the petroleum industry:

- an inverse problem, based on a model of seismic wave propagation

- contemporary practice relies on *partial linearization* and high-frequency asymptotics

- recent progress in understanding capabilities, limitations of methods based on linearization/asymptotics in presence of *strong refraction*: applications of *microlocal analysis* with implications for practice

- limitations of linearization lead to many open problems

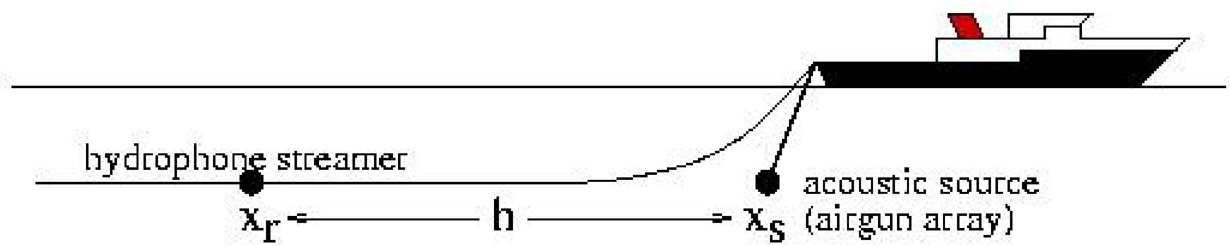
Agenda

1. Seismic inverse problem in the acoustic model: nature of data and model, linearization, reflectors and reflections idealized via *harmonic analysis of singularities*.
2. High frequency asymptotics: why adjoints of modeling operators are imaging operators ("Kirchhoff migration"). Beylkin theory of high frequency asymptotic inversion.
3. Adjoint state imaging with the wave equation: reverse time and reverse depth.
4. Geometric optics, Rakesh's construction, and asymptotic inversion w/ caustics and multipathing.
5. A step beyond linearization: a mathematical framework for velocity analysis, imaging artifacts, and prestack migration après Claerbout.

1. The Acoustic Model and Linearization

Marine reflection seismology apparatus:

- acoustic source (airgun array, explosives,...)
- acoustic receivers (hydrophone streamer, ocean bottom cable,...)
- recording and onboard processing



Land acquisition similar, but acquisition and processing are more complex. Vast bulk (90%+) of data acquired each year is marine.

Data parameters: time t, source location $\mathbf{x}_s$, and receiver location $\mathbf{x}_r$ or *half offset* $\mathbf{h} = \frac{\mathbf{x}_r - \mathbf{x}_s}{2}$, $h = |\mathbf{h}|$.

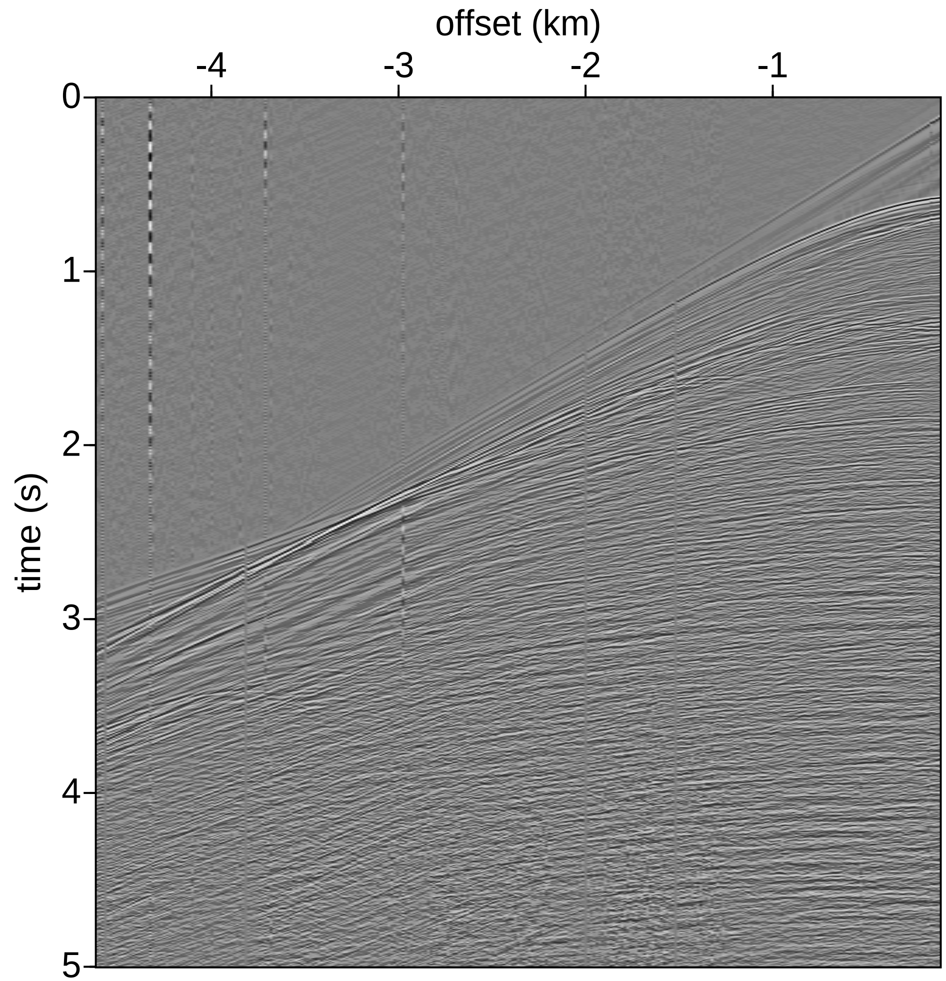
Idealized marine "streamer" geometry: $\mathbf{x}_s$ and $\mathbf{x}_r$ lie roughly on constant depth plane, source-receiver lines are parallel $\rightarrow$ 3 spatial degrees of freedom (eg. $\mathbf{x}_s, h$): *codimension 1*. [Other geometries are interesting, eg. ocean bottom cables, but streamer surveys still prevalent.]

How much data? Contemporary surveys may feature

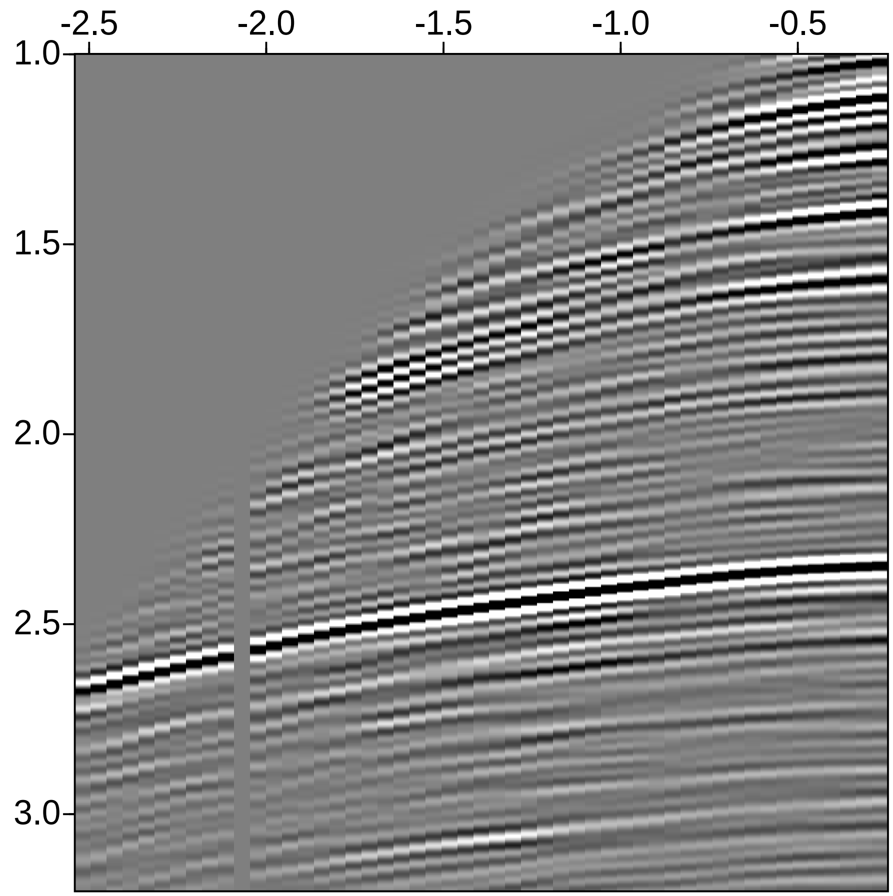
- Simultaneous recording by multiple streamers (up to 12!)
- Many (roughly) parallel ship tracks ("lines"), areal coverage

- single line ("2D") $\sim$ Gbyte; multiple lines ("3D") $\sim$ Tbyte

Main characteristic of data: *wave nature*, presence of **reflections** = amplitude coherence along trajectories in space-time.



Data from one source firing, Gulf of Mexico (thanks: Exxon)



Lightly processed version of data displayed in previous slide - bandpass filtered (in t), truncated ("muted").

Distinguished data subsets: "gathers", "bins", extracted from data after acquisition.

Characterized by common value of an acquisition parameter

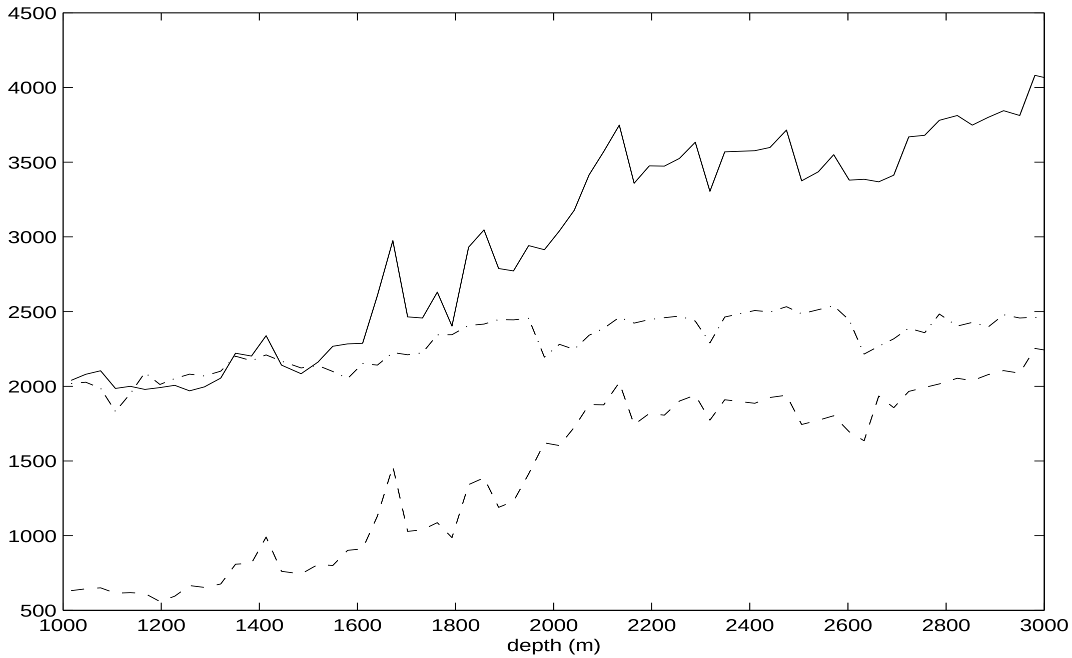
- shot (or common source) gather: traces with same shot location $\mathbf{x}_s$ (previous expls)

- offset (or common offset) gather: traces with same half offset h

- ...

- presence of wave events ("reflections") = coherent space-time structures - clear from examination of the data.

- what features in the subsurface structure could cause reflections to occur?



Blocked logs from well in North Sea (thanks: Mobil R & D). Solid: p-wave velocity (m/s), dashed: s-wave velocity (m/s), dash-dot: density (kg/m^3). "Blocked" means "averaged" (over 30 m windows). Original sample rate of log tool < 1 m. **Reflectors** $=$ jumps in velocities, density, **velocity trends**.

Known that abrupt (wavelength scale) changes in material mechanics, i.e. reflectors, act as internal boundary, causing reflection of waves.

What is the mechanism through which this occurs?

Seek a simple model which quantitatively explains wave reflection and other known features of the Earth's interior.

The Modeling Task: any model of the reflection seismogram must

- predict wave motion
- produce reflections from reflectors
- accomodate significant variation of wave velocity, material density,...

A *really good* model will also accomodate

- multiple wave modes, speeds
- material anisotropy
- attenuation, frequency dispersion of waves
- complex source, receiver characteristics

Acoustic Model (only compressional waves)

Not *really good*, but good enough for today and basis of most contemporary processing.

Relates $\rho(\mathbf{x})=$ material density, $\lambda(\mathbf{x})$ = bulk modulus, $p(\mathbf{x},t)=$ pressure, $\mathbf{v}(\mathbf{x},t)$ = particle velocity, $\mathbf{f}(\mathbf{x},t)=$ force density (sound source):

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \mathbf{f},$$

$$\frac{\partial p}{\partial t} = -\lambda \nabla \cdot \mathbf{v} \ (+\,\mathrm{i.c.'s}, \mathrm{b.c.'s})$$

(compressional) wave speed $c = \sqrt{\frac{\lambda}{\rho}}$

acoustic field potential $u(\mathbf{x},t) = \int_{-\infty}^{t} ds\, p(\mathbf{x},s)$:

$$p = \frac{\partial u}{\partial t},\ \mathbf{v} = \frac{1}{\rho}\nabla u$$

Equivalent form: second order wave equation for potential

$$\frac{1}{\rho c^2}\frac{\partial^2 u}{\partial t^2} - \nabla \cdot \frac{1}{\rho}\nabla u = \int_{-\infty}^{t} dt\, \nabla \cdot \left(\frac{\mathbf{f}}{\rho}\right) \equiv \frac{f}{\rho}$$

plus initial, boundary conditions.

Weak solution of Dirichlet problem in $\Omega \subset \mathbf{R}^3$ (similar treatment for other b. c.'s):

$$u \in C^1([0,T]; L^2(\Omega)) \cap C^0([0,T]; H^1_0(\Omega))$$

satisfying for any $\phi \in C_0^\infty((0,T) \times \Omega)$,

$$\int_0^T \int_\Omega dt\, dx \left\{ \frac{1}{\rho c^2} \frac{\partial u}{\partial t} \frac{\partial \phi}{\partial t} - \frac{1}{\rho} \nabla u \cdot \nabla \phi + \frac{1}{\rho} f \phi \right\} = 0$$

Theorem (Lions, 1972) Suppose that $\log \rho, \log c \in L^\infty(\Omega), f \in L^2(\Omega \times \mathbf{R})$. Then weak solutions of Dirichlet problem exist; initial data

$$u(\cdot, 0) \in H^1_0(\Omega),\ \frac{\partial u}{\partial t}(\cdot, 0) \in L^2(\Omega)$$

uniquely determine them.

Further idealizations: (i) density is constant, (ii) source force density is *isotropic point radiator with known time dependence* ("source pulse" $w(t)$)

$$f(\mathbf{x},t;\mathbf{x}_s) = w(t)\delta(\mathbf{x}-\mathbf{x}_s)$$

$\Rightarrow$ acoustic potential, pressure depends on $\mathbf{x}_s$ also.

Forward map $S =$ time history of pressure for each $\mathbf{x}_s$ at receiver locations $\mathbf{x}_r$ (predicted seismic data), depends on velocity field $c(\mathbf{x})$:

$$S[c] = \{p(\mathbf{x}_r,t;\mathbf{x}_s)\}$$

Reflection seismic inverse problem: given *observed seismic data* S^{obs}, find c so that

$$S[c] \simeq S^{\text{obs}}$$

This inverse problem is

- large scale - up to Tbytes, Pflops
- nonlinear

Almost all useful technology to date relies on partial linearization: write $c = v(1 + r)$ and treat r as relative first order perturbation about v, resulting in perturbation of presure field $\delta p = \frac{\partial \delta u}{\partial t} = 0, t \leq 0$, where

$$\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\delta u = \frac{2r}{v^2}\frac{\partial^2 u}{\partial t^2}$$

Define **linearized forward map** F by

$$F[v]r = \{\delta p(\mathbf{x}_r, t; \mathbf{x}_s)\}$$

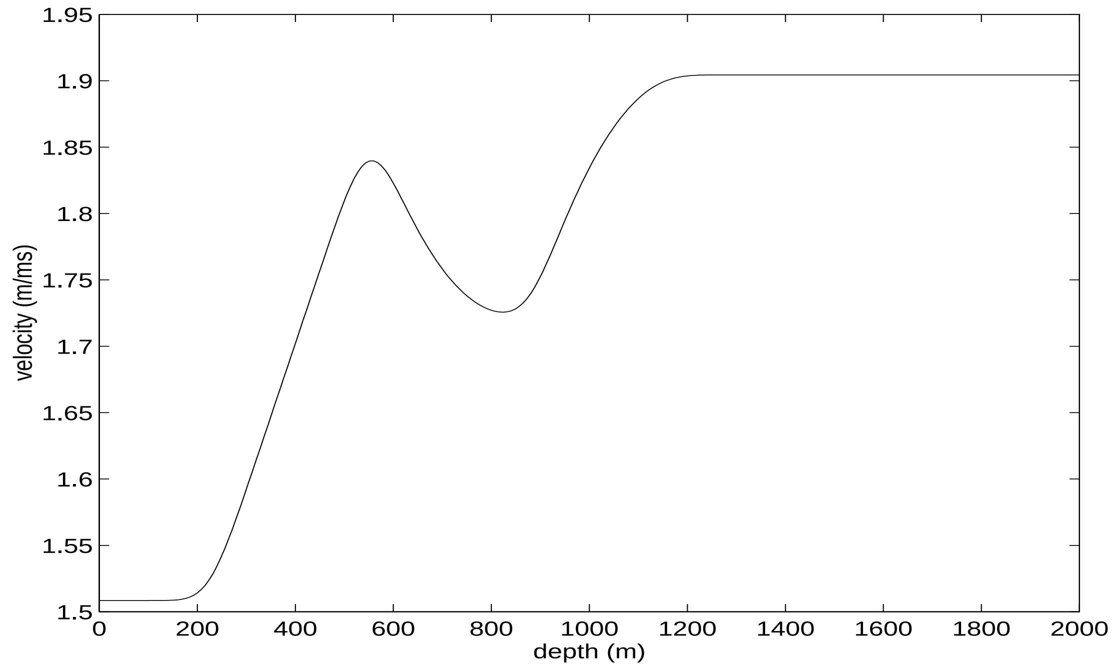
Analysis of $F[v]$ is the main content of contemporary reflection seismic theory.

Critical question: If there is any justice $F[v]r =$ derivative $DS[v][vr]$ of S - but in what sense? Physical intuition, numerical simulation, and not nearly enough mathematics: linearization error

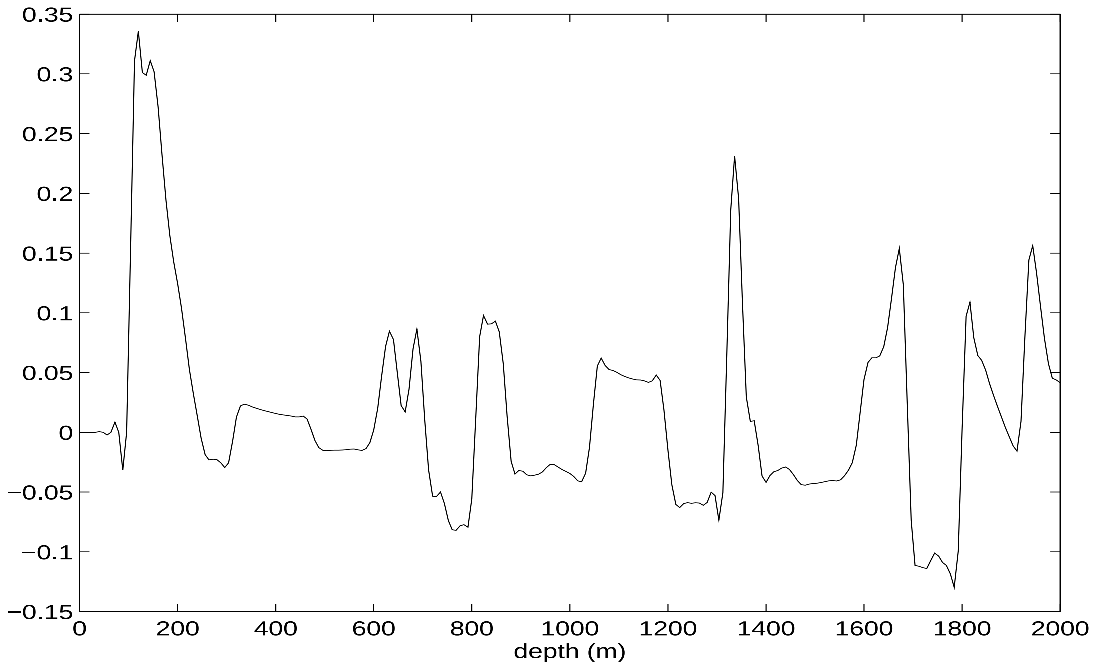
$$S[v(1+r)] - (S[v] + F[v]r)$$

- *small* when v smooth, r rough or oscillatory on wavelength scale - well-separated scales
- *large* when v not smooth and/or r not oscillatory - poorly separated scales

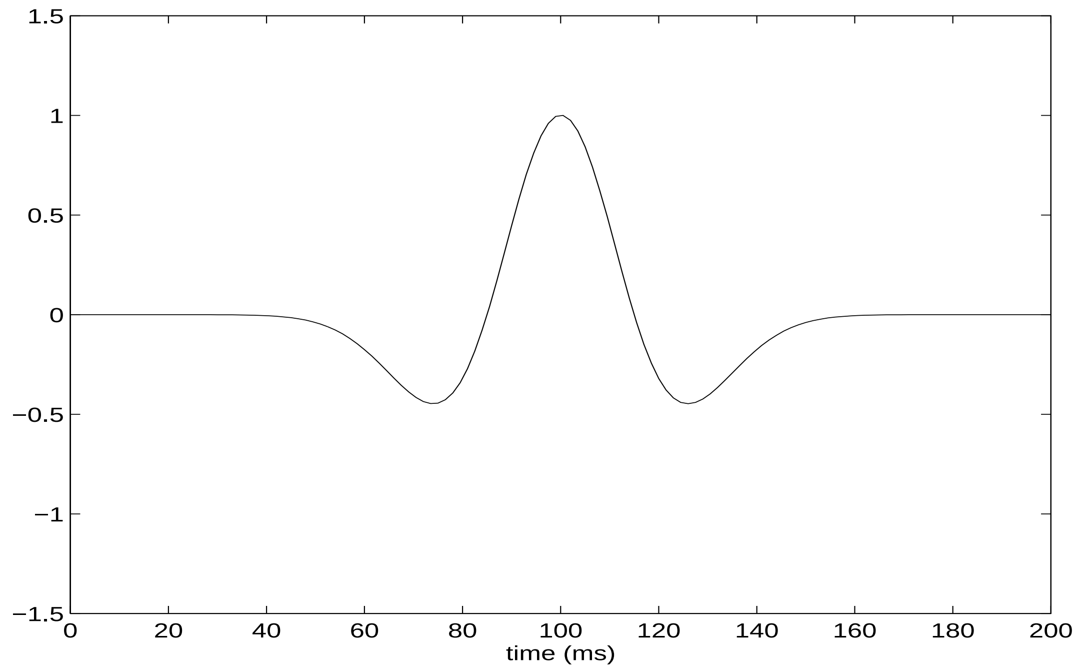
2D finite difference simulation: shot gathers with typical marine seismic geometry. Smooth $v(z)$, oscillatory $r(z)$ ("layered medium") extracted from "Marmousi" synthetic data set (Grau & Versteeg, 1994)



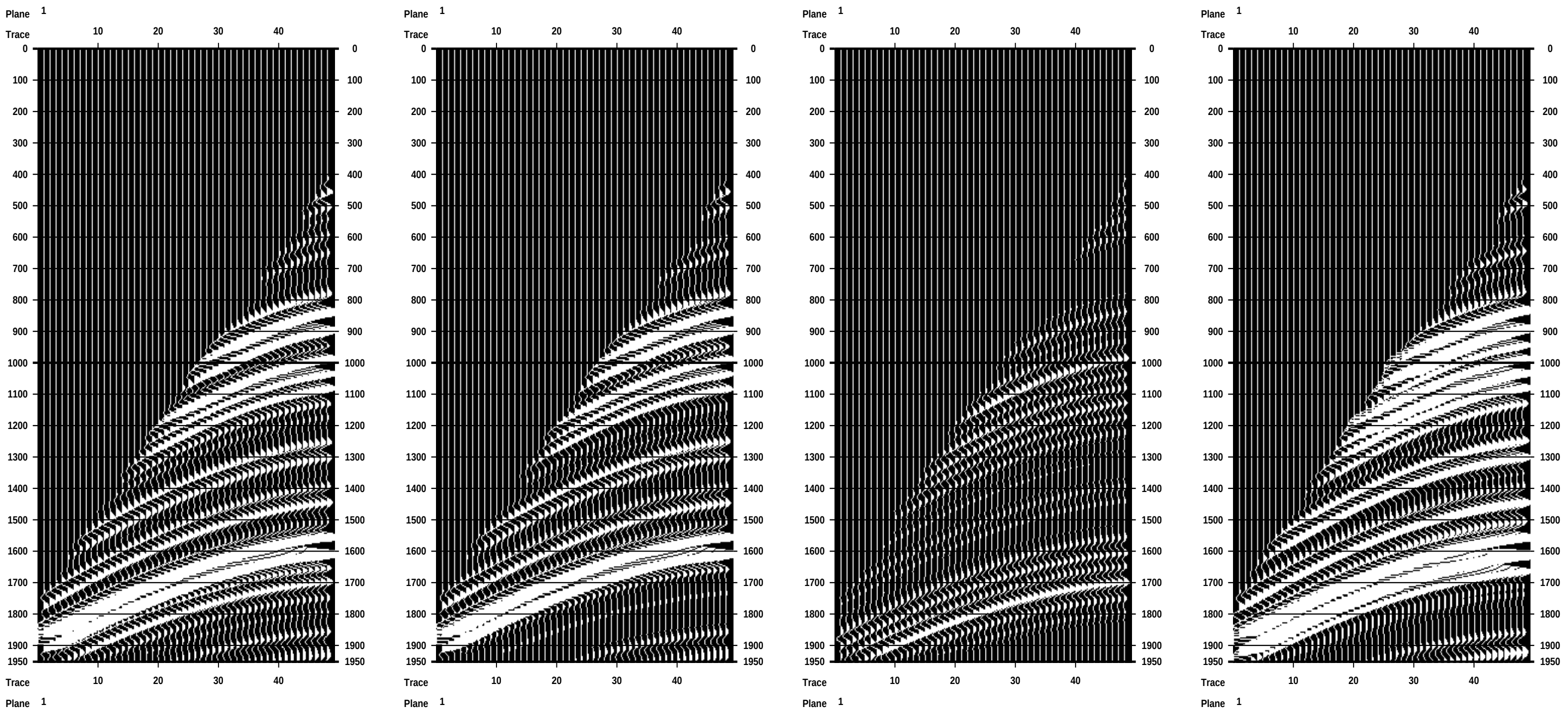
Velocity $v(x_1)$ (function of depth only) used in numerical linearization study.



Reflectivity function $r(x_1)$ (function of depth only) used in numerical linearization study.



Source pulse $w(t)$ used in numerical linearization study.



(a) $S[v(1+r)]$, (b) $S[v]+F[v]r$, (c) $S[v(1+r)]-S[v]-F[v]r$,
(d) $S[v(1+r)+0.02v]-S[v(1+r)]+F[v(1+r)](0.02v)$

Implications:

- Some geologies have well-separated scales - cf. sonic logs - linearization-based methods work well there. Other geologies do not - expect trouble!
- v smooth, r oscillatory $\Rightarrow F[v]r$ approximates **primary reflection** $=$ result of wave interacting with material heterogeneity only once (single scattering); error consists of **multiple reflections**, which are "not too large" if r is "not too big", and sometimes can be suppressed (lecture 4).
- v nonsmooth, r smooth $\Rightarrow$ error consists of *time shifts* in waves which are very large perturbations as waves are oscillatory.

No mathematical results are known which justify/explain these observations in any rigorous way.

Partially linearized inverse problem = **velocity analysis problem**: given S^{obs} find v, r so that

$$S[v] + F[v]r \simeq S^{\mathrm{obs}}$$

Linear subproblem = **imaging problem**: given S^{obs} and v, find r so that

$$F[v]r \simeq S^{\mathrm{obs}} - S[v]$$

Last 20 years:

- much progress on imaging problem
- much less on velocity analysis problem.