Statistically stable velocity macro-model estimation

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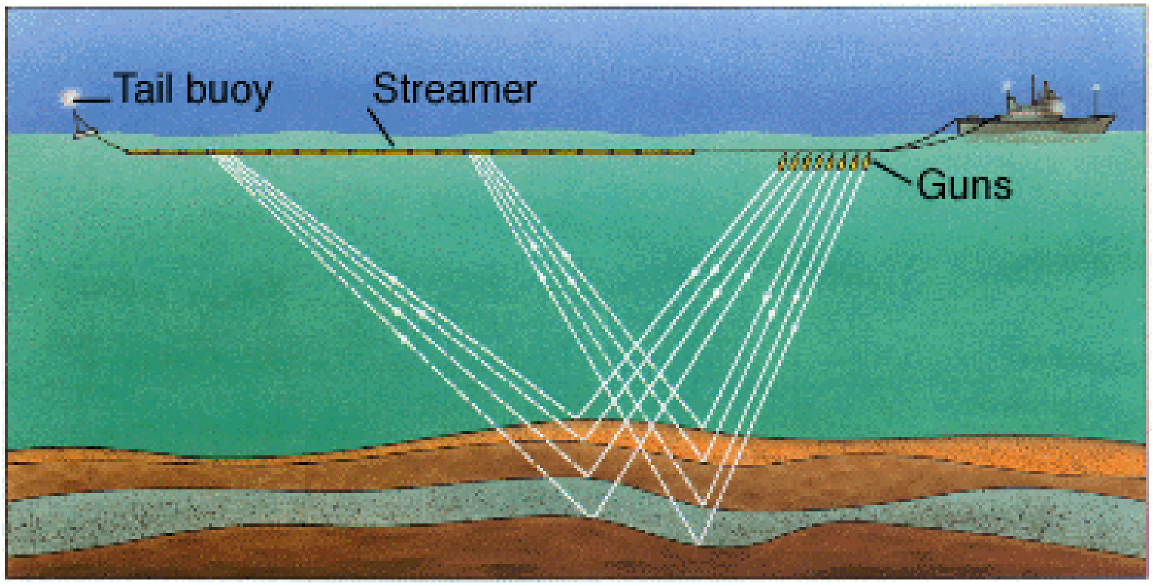


Courtesy: Schlumberger

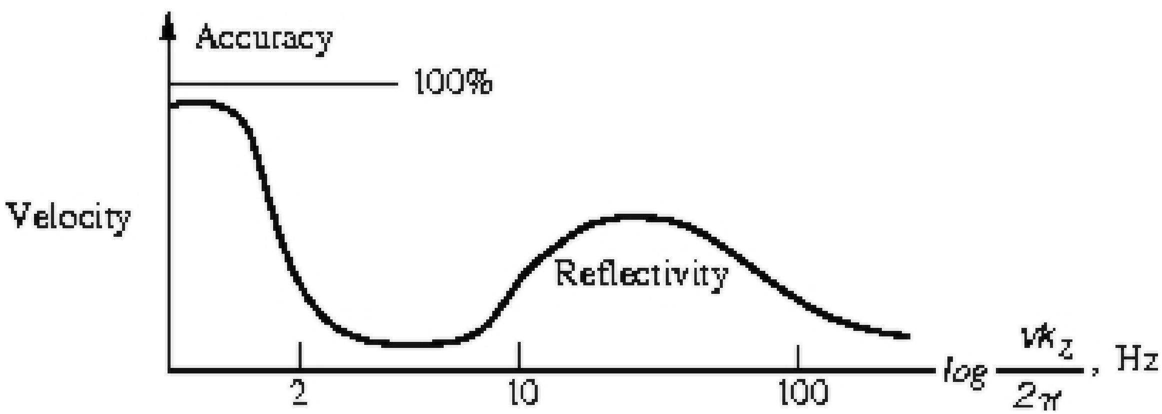
- Seismic waves are reflected where the medium varies discontinuously.
- From the recorded reflections that can be observed in the data, the problem is reconstruct the discontinuities.
- The simplest theory to explain the reflections is the linear (constant density) acoustics model: the significant property of the Earth is the wave speed.

The scale gap

- Seismic imaging methods are typically based on the splitting of the seismic model into a reflecting part (short-scale) and a propagating part (long-scale).
- This scale separation can be established theoretically on the basis of the Born approximation (Lailly, 1983). In practice,
  - Long scale fluctuations (km for sediments) of the velocity are resolved via **velocity analysis**.
  - Short scale variations (10's m) of the velocity (i.e. the reflectivity) are resolved via **migration** or **linearized inversion**.
- The traditional seismic imaging techniques do **not** appear to estimate the intermediate scale wavelengths ($\sim$ 60m - 300m).

What can we get from reflection seismology?

According to Claerbout (IEI, page 47) and Tarantola (1989), seismic data do **not** contain reliable information on the intermediate scales of velocity.



Note: The above conclusion is purely empirical. No theoretical basis has been set forth to back it up.

Proposed work

- We think we can provide a new way to look at this familiar ”fact”.
- However, because the seismic problem is nonlinear - these are components of the velocity, one would expect “energy” or (lack of) ”information” to cascade between scales.
- Try to understand the influence of the medium scale on the resolution of the long (background velocity) and short (image) scales.
- Take this intermediate scale velocity into account and treat it as a random process precisely to model the associated uncertainty (and its consequences).
- **Goal**: Estimate the background velocity by combining ideas on time reversal and imaging in randomly inhomogeneous media set forth by Borcea, Papanicolaou et al., and the velocity estimation methods of differential semblance type.

Agenda

- Motivation
- The seismic inverse problem
  - The convolutional model
  - Differential semblance optimization
- Time reversal and imaging in random media
  - The point source example
  - Application to seismic imaging
- Proposed work
  - Estimation of the background velocity
  - Cross-correlation tomography
- Conclusions and future work

The forward map

- Constant density linear acoustics model: the mechanical properties of the Earth are represented by the velocity c.
- Linearization: split $c = c_0 + \delta c$ where:
  - c_0 is the smooth background velocity (the macro-model medium)
  - δc is a first-order perturbation which contains the high-frequency content of the wave speed (define the **reflectivity** by $r \sim \delta c / c_0$).
- High-frequency asymptotics.

The reflection data is predicted by the linearized forward map $F[c_0]$

$$(c_0, r) \mapsto F[c_0]r$$

N.B. the forward map $F[c_0]$ is a linear operator acting on the reflectivity r and parametrized by c_0. The dependence on c_0 is (highly) nonlinear!

The seismic inverse problem

- The seismic inverse problem problem can be stated as follows: given observed seismic data d, determine c_0 and r so that $F[c_0]r \simeq d$.
- **Caveat**: as stated, this inverse problem is intractable, e.g. the data fitting formulation via least-squares requires global methods such as simulated annealing.
- **Solution**: decouple the problem into two steps.

1. Assume c_0 known; then try to reconstruct the reflectivity r (this is migration). In this case, the resulting data formulation is a linear least-squares, hence "easy" to solve.
2. Use the redundancy in the data; we have $d = (x_s, y_s, x_r, y_r, t) \in \mathbb{R}^5$ and $r = r(x, y, z) \in \mathbb{R}^3$, so the data can be partitioned into 3-D subsets (called **bins**), and each of these subsets may be used for an independent reconstruction of the reflectivity (basis for velocity analysis, i.e. for reconstructing c_0).

The convolutional model (1/2)

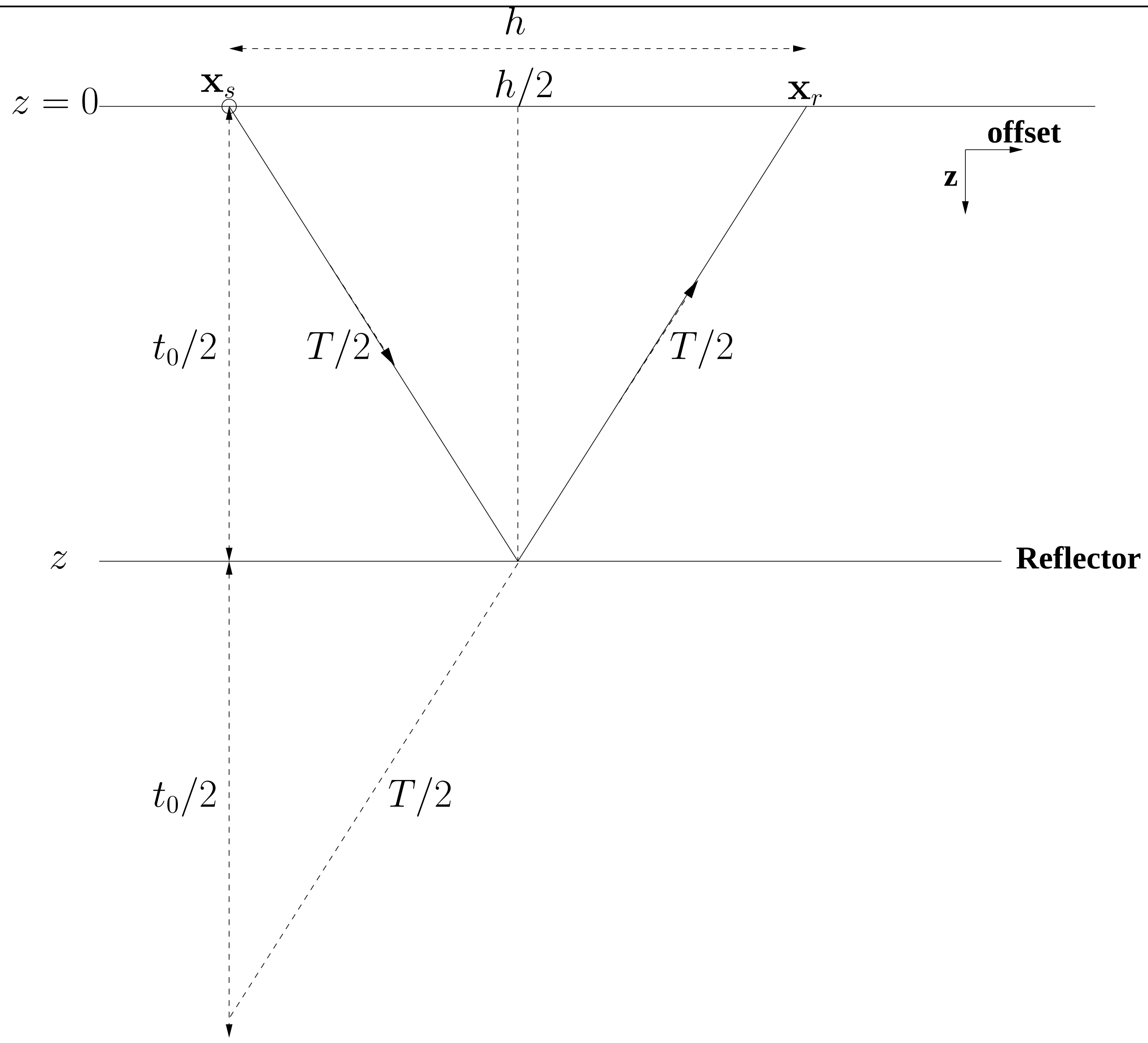
- Assume a laterally homogeneous medium, i.e. $c = c(z)$.
  - Each data bin is parametrized by offset $h = (x_s - x_r, y_s - y_r, 0)$. Therefore, independent reconstructions of the reflectivity r are regarded as offset dependent, i.e. $r \equiv r(z, h)$.
- In practice, the following time-depth conversion is used:

$$t_0 = 2 \int_0^z \frac{dz}{c_0(z)} \;\Rightarrow\; c_0 = c_0(t_0),\; r = r(t_0, h).$$

  is the vertical (zero-offset) two-way travel time.

- Denote by $T(t_0, h)$ the two-way travel time function corresponding to depth t_0 and offset h and by $T_0(t, h)$ the inverse function, i.e.

$$T\left(T_0(t, h), h\right) = t, \quad T_0\left(T(t_0, h), h\right) = t_0$$


h
$z = 0$
$\mathbf{x}_s$
$h/2$
$\mathbf{x}_r$
offset
z
$t_0/2$
$T/2$
$T/2$
z
Reflector
$t_0/2$
$T/2$

The convolutional model (2/2)

- With these conventions, the forward modeling operator is (Symes, 1999)

$$d(t,h) = f(t) *_t r(T_0(t,h),h) \equiv F_h[c_0]r(t,h),$$

- Ignore convolution (assume perfect source signature deconvolution, i.e. $f \sim \delta$).
- Optimum choice of reflectivity r for each offset h:

$$r(t_0,h) = d(T(t_0,h),h) \equiv G_h[c_0]d(t_0,h)$$

  Here $G_h[c_0]$ is the inverse of $F_h[c_0]$ (obtained by an inverse change of variables). Note that it produces r which depends (artificially) on h!

- **Note**: for more complex models, G_h is an asymptotic inverse to F_h, i.e.

$$G_h[c_0] \equiv F_h^{-1}[c_0] \simeq (F_h^*[c_0]F_h[c_0])^{-1} F_h^*[c_0] \simeq F_h^*[c_0].$$

  The proof involves showing that $F_h^* F_h$ is pseudodifferential. The aggregate operator G performs the so-called migration of the seismic data.

The semblance principle

- **Semblance principle**: if the background model c_0 is "right", then all the $r(h)$'s should be the **same**, or at least similar (there is only one Earth!).

- Given an operator W measuring semblance, the velocity analysis problem can be cast as an optimization problem: given data d, determine c_0 so as to optimize

$$Wr \text{ such that } r = G[c_0]d \ (\text{i.e. } F[c_0]r \simeq d)$$

- Specialization to layered acoustics model: Wr must vanish when r is independent of offset h. Therefore, we take

$$W = \partial/\partial h.$$

Differential semblance optimization

- Formulation via differential semblance (Gockenbach, 1994 and Song, 1994):

$$\min_{c_0} J[c_0] = \frac{1}{2}\|HWG[c_0]d\|^2$$

  Here H is a smoothing pseudodifferential operator designed to keep the spectrum of the functional output comparable to that of the data.

- **Remark**: if c_0 is correct, $WG[c_0]$ annihilates the data.

- Many theoretical results on DSO (Symes, 1999, Stolk & Symes, 2003, Stolk, 2002): e.g. in the layered medium case, all stationary points of the above objective are global minimizers (Symes, 1999).

- Also, there have been many numerical implementations of DSO on real data sets to support these results (Chauris, 2000, Chauris & Noble, 2001).

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Time reversed acoustics

- In time-reversal experiments, a signal emitted by a localized source is recorded by an array of transducers. It is then re-emitted into the medium reversed in time, i.e. the tail of the signal is sent back first.
- Because of the time-reversability of the wave equation, the back-propagated signal retraces its path backwards and refocuses approximately near the source (since the array is limited in size).
- Time reversal has two striking properties in randomly inhomogeneous media:
  - the presence of inhomogeneities in the medium improves the refocusing resolution: this is the super-resolution effect.
  - the refocused signal does not depend on the realization of the random medium: it is self-averaging (i.e. deterministic).

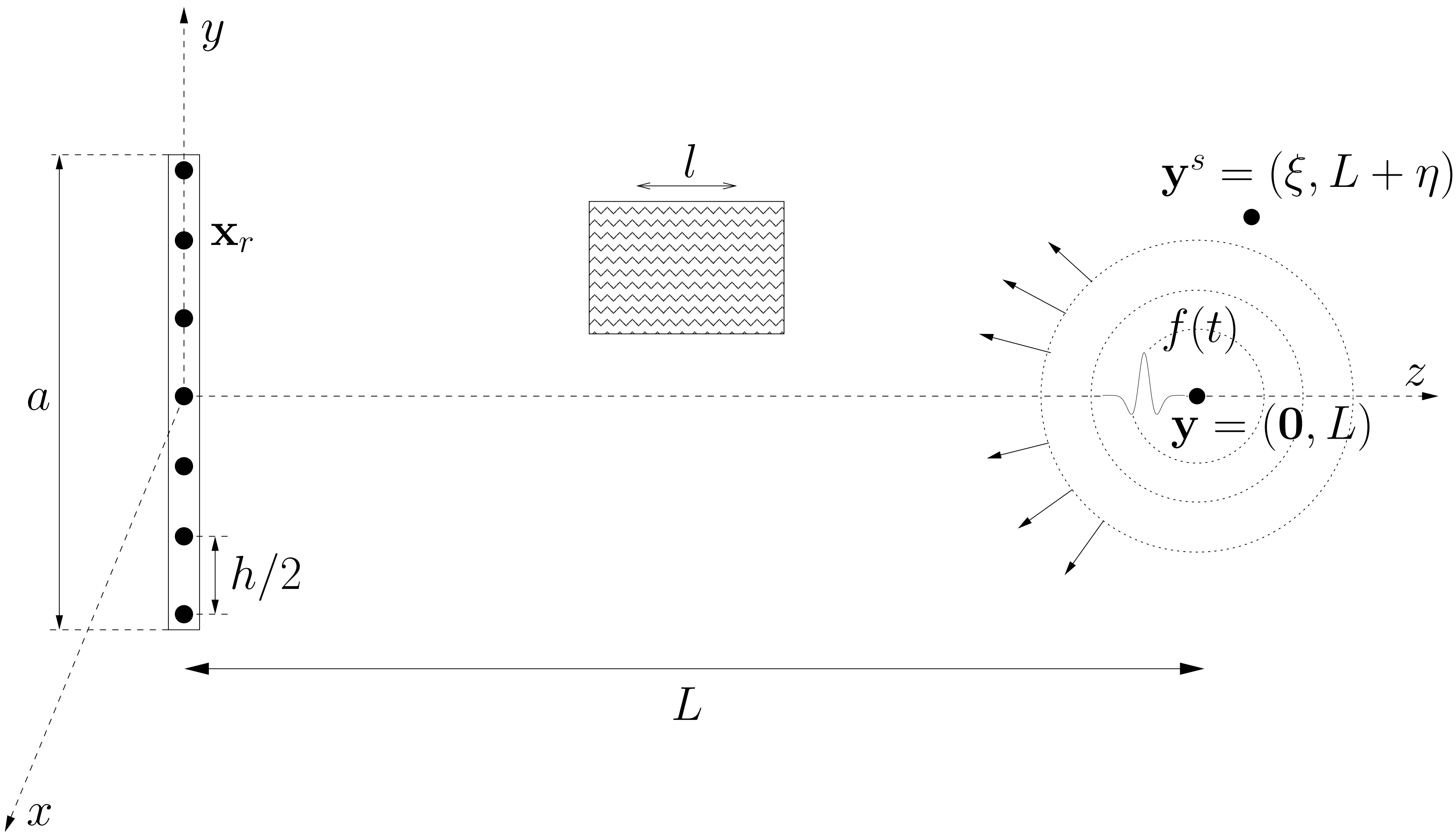
Three scale asymptotics

The **setting** is as follows: single scattering approximation, 3-scale asymptotics:

- "Deterministic" reflectors are structures on wavelength scale λ (corresponding to the short-scale component of velocity).
- Propagation distance L is also the scale of the background velocity "macro-model" (the component which may be estimated via VA).
- The intermediate scale velocity is assumed to randomly fluctuate on the scale l.

Asymptotic assumption: high-frequency regime $\lambda \ll l \ll L$, i.e.

- waves propagate over many correlation lengths so multipathing is significant.
- random fluctuations are slowly varying on the wavelength scale, i.e. the geometrical optics approximation is appropriate.



Setup for “imaging” a point source target (Borcea et al, 2003)

Modeling of the point source example (1/3)

- Wave propagation modeled by (stochastic) acoustic wave equation. Note that the right-hand-side (the source) is $g(\mathbf{x}, t) = f(t)\delta(\mathbf{x} - \mathbf{y})$.

- The data measured at receiver $\mathbf{x}_r$ is given by the time convolution

$$d(\mathbf{x}_r, t) = \left(f(\cdot) *_t G(\mathbf{x}_r, \mathbf{y}, \cdot)\right)(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \widehat{f}(\omega)\widehat{G}(\mathbf{x}_r, \mathbf{y}, \omega)e^{-i\omega t}d\omega.$$

where $\widehat{G}$ solves the Helmholtz equation

$$\Delta\widehat{G}(\mathbf{x}, \mathbf{y}, \omega) + k^2 n^2(\mathbf{x})\widehat{G}(\mathbf{x}, \mathbf{y}, \omega) = -\delta(\mathbf{x} - \mathbf{y}),$$
$$\lim_{r\to\infty} r\left(\partial\widehat{G}/\partial r - ikn\widehat{G}\right) = 0, \quad r = |\mathbf{x} - \mathbf{y}|.$$

Here $k = \omega/c_0$ is the wavenumber, c_0 is the reference speed, $n = c_0/c(\mathbf{x})$ is the random index of refraction, and $c(\mathbf{x})$ is the random propagation speed.

The scattering regime

We assume that the refraction index is randomly fluctuating about the (constant) background velocity on the scale l:

$$n^2(\mathbf{x}) = 1 + \sigma\mu\left(\frac{\mathbf{x}}{l}\right)$$

where

- l is the correlation length, i.e. the scale at which the medium fluctuates.
- $\sigma \ll 1$ (weak fluctuations - waves scattered mostly in the direction of propagation).
- μ is a stationary, isotropic random field with mean $\langle \mu(\mathbf{x}) \rangle = 0$, and covariance

  $$R(\mathbf{x}) = R(|\mathbf{x}|) = \langle \mu(\mathbf{x}' + \mathbf{x})\mu(\mathbf{x}') \rangle \, .$$

  which decays at ∞ so that there are no long range correlations of the fluctuations.

Modeling of the point source example (2/3)

- Define the operator F mapping the source g to data d: $Fg = d$.
- Abusing notation, the least-squares solution is

$$g \simeq (F^*F)^{-1}F^*d \simeq F^*d.$$

- To compute the adjoint, start with the definition

$$\langle F^*d, g\rangle = \langle d, Fg\rangle = \ldots$$

- We obtain the so-called **point spread** function

$$F^*d \equiv \Gamma^{TR}(\mathbf{y}^S, t) = \sum_{r=-N}^{N} d(\mathbf{x}_r, -t) *_t G(\mathbf{x}_r, \mathbf{y}^S, t)$$

Note: adjoint = time reversal + backpropagation!

Modeling of an active target (3/3)

- Assuming that the source is real, i.e.

$$f(t) = \overline{f(t)} \quad \Rightarrow \quad \widehat{f}(\omega) = \overline{\widehat{f}(-\omega)},$$

time reversal is **equivalent** to complex conjugation in frequency domain:

$$d(\mathbf{x}_r, -t) = \left(f(\cdot) *_t G(\mathbf{x}_r, \mathbf{y}, \cdot)\right)(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\widehat{f}(\omega)}\, \overline{\widehat{G}(\mathbf{x}_r, \mathbf{y}, \omega)} e^{-i\omega t} d\omega.$$

- Therefore, we obtain:

$$\Gamma^{TR}(\mathbf{y}^S, t) = \sum_{r=-N}^{N} d(\mathbf{x}_r, -t) *_t G(\mathbf{x}_r, \mathbf{y}^S, t)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{f(\omega)} \sum_{r=-N}^{N} \overline{\widehat{G}(\mathbf{x}_r, \mathbf{y}, \omega)} \widehat{G}(\mathbf{x}_r, \mathbf{y}^S, \omega) e^{-i\omega t} d\omega$$

Refocusing resolution

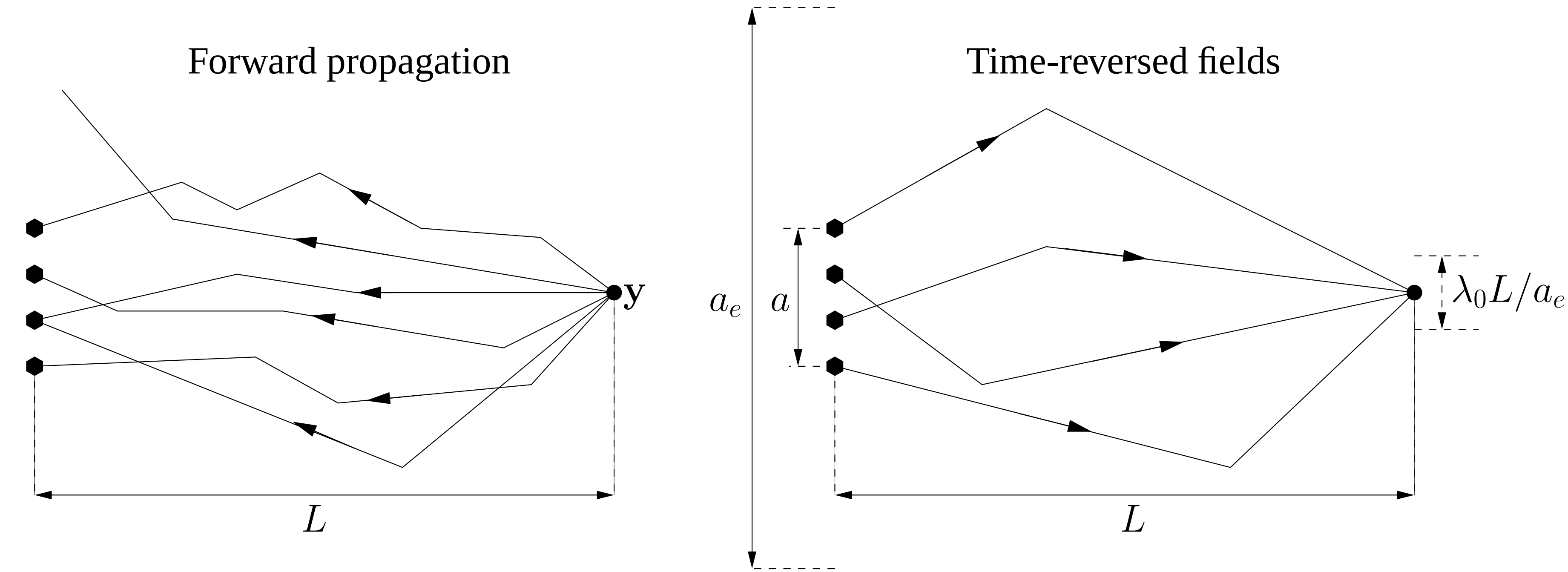
The TR point-spread function is evaluated at the **exact range** $\eta = 0$ and at the **arrival time** $t = 0$:

- In **homogeneous** media, the (deterministic) cross-range resolution can be shown to be $\lambda_0 L/a$. Clearly, the **larger** the physical aperture a of the array, the **better** the resolution.

- Amazingly, in inhomogeneous media, the cross-range resolution is $\lambda_0 L/a_e$, where $a_e \gg a$ is the effective aperture of the array.

N.B. A certain number of approximations and calculations have to be made to obtain, in each case, explicit formula that yield these resolution estimates.

Super-resolution in inhomogeneous media



Intuitive explanation: because of multipathing, waves that move away from the array get scattered onto it by the inhomogeneities $\Rightarrow$ the refocusing is much tighter than in homogeneous media ($\sim \lambda_0 L/a_e$), a_e is the effective aperture of the array.

Self-averaging property

The super-resolution phenomenon happens for **essentially** every realization of the random medium! The time-reversed backpropagated field is self-averaging (i.e. deterministic). In the limit $l/L \to 0$, we have:

$$\left\langle \left(\Gamma^{TR}(\mathbf{y}^S, t)\right)^2 \right\rangle \approx \left\langle \Gamma^{TR}(\mathbf{y}^S, t) \right\rangle^2$$

Thus (thereby using Chebyshev inequality)

$$P\left\{ \left| \Gamma^{TR}(\mathbf{y}^S, t) - \left\langle \Gamma^{TR}(\mathbf{y}^S, t) \right\rangle \right| > \delta \right\} \le \frac{\left\langle \left(\Gamma^{TR}(\mathbf{y}^S, t) - \left\langle \Gamma^{TR}(\mathbf{y}^S, t) \right\rangle \right)^2 \right\rangle}{\delta^2} \approx 0$$

That is:

$$\Gamma^{TR}(\mathbf{y}^S, t) \approx \left\langle \Gamma^{TR}(\mathbf{y}^S, t) \right\rangle$$

The refocused field is statistically stable, i.e. it does **not** depend on the particular realization of the random medium.

The moment formula

- Because of the self-averaging property, the product of random Green's functions in the point spread functional may be replaced by its expectation.

- The stochastic analysis yields the so-called moment formula:

$$\left\langle \overline{\widehat{G}(\mathbf{x}_r, \mathbf{y}, \omega)} \widehat{G}(\mathbf{x}_r, \mathbf{y}^S, \omega) \right\rangle \approx \overline{\widehat{G}_0(\mathbf{x}_r, \mathbf{y}, \omega)} \widehat{G}_0(\mathbf{x}_r, \mathbf{y}^S, \omega) e^{-\frac{k^2 a_e^2 \xi^2}{2L^2}}$$

  Note that all of the statistics of the medium are confined to a single parameter, the effective aperture a_e (the super-resolution arises from the Gaussian factor).

- Key to self-averaging is the near cancellation of the random phases. Heuristically, $\widehat{G} \simeq A e^{i(kr+\phi)}$ and since the TR functional contains $\overline{\widehat{G}}\widehat{G}$ (nearby paths), the random phases ϕ almost cancel.

Application to seismic imaging

- Setting: three scale asymptotics $\lambda \ll l \ll L$.
- With the Born approximation, the scattered field measured at a receiver $\mathbf{x}_r$ is

$$d(\mathbf{x}_s, \mathbf{x}_r, t) = \int_{-\infty}^{\infty} \frac{k^2 \widehat{f}(\omega)}{2\pi} \left[\int r(\mathbf{y}) \widehat{G}(\mathbf{x}_s, \mathbf{y}, \omega) \widehat{G}(\mathbf{x}_r, \mathbf{y}, \omega) d\mathbf{y} \right] e^{-i\omega t} d\omega$$

  Note that the above Green's functions are random (they contain both long scale and medium scale components of the velocity).

- **Contrast with TR**: the fluctuations in the medium are not known, so migration is done fictitiously, in the background medium.
- We obtain terms such as $\widehat{G}\overline{\widehat{G}_0}$, i.e. there remain random phases in migration operators corresponding to long random paths from the source to the reflector and back to the receiver: lack of statistical stability.

Local data covariances

- To achieve statistical stability, we must cancel random phases in the data d.

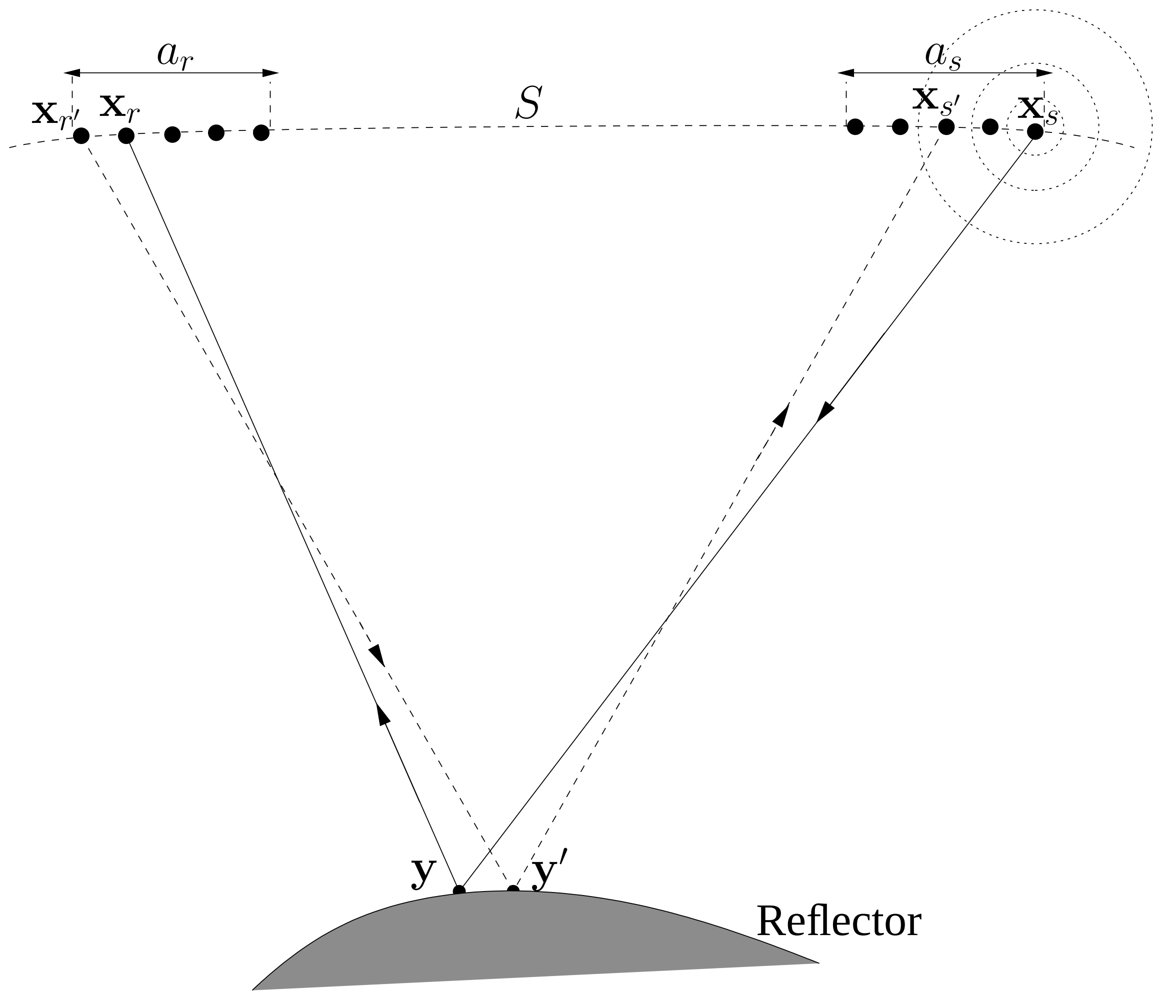
- **Idea** (Borcea et al., 2003): divide the data set into smaller parts and construct local data covariances by cross-correlating nearby traces $d(\mathbf{x}_s, \mathbf{x}_r, t)$ and $d(\mathbf{x}_{s'}, \mathbf{x}_{r'}, t)$, i.e.

$$d(\mathbf{x}_s, \mathbf{x}_r, t) \star d(\mathbf{x}_{s'}, \mathbf{x}_{r'}, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\widehat{d}(\mathbf{x}_s, \mathbf{x}_r, \omega)}\widehat{d}(\mathbf{x}_{s'}, \mathbf{x}_{r'}, \omega)e^{i\omega t}d\omega$$

  Note that we obtain the terms:

$$\overline{\widehat{G}(\mathbf{x}_s, \mathbf{y}, \omega)}\widehat{G}(\mathbf{x}_{s'}, \mathbf{y}', \omega) \quad \text{and} \quad \overline{\widehat{G}(\mathbf{x}_r, \mathbf{y}, \omega)}\widehat{G}(\mathbf{x}_{r'}, \mathbf{y}', \omega)$$

- In essence, this approach can be viewed as a pre-processing step in which, starting with the randomly fluctuating data $d(\mathbf{x}_s, \mathbf{x}_r, t)$, we obtain a reduced, self-averaging data set.



A data pre-processing step

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Cross-correlation tomography (1/8)

- We have seen how to construct self-averaging data sets. So what good is it for velocity analysis?

- The ideas introduced above have been applied by Borcea and colleagues to the problem of "imaging" targets embedded in random media.

- The proposed work addresses (in a first stage) the issue of estimating the background velocity; we will see that this entails establishing new differential semblance principles.

- We first show that cross-correlation of seismic traces do contain velocity information. The question is: how to get it?

Cross-correlation tomography (2/8)

- Suppose there are **no** fluctuations,
- Assume layered Earth model, i.e. $c_0 = c_0(t_0)$.
- Assume there is a single reflector at “depth” t_0.
- Use the **hyperbolic moveout approximation**: for “small” offsets h,

$$T(t_0, h) = \sqrt{t_0^2 + \frac{h^2}{c_{\text{rms}}^2(t_0)}}, \quad c_{\text{rms}}(t_0) = \sqrt{\frac{1}{t_0} \int_0^{t_0} c_0^2}$$

- Define two key quantities:

$$p(t, h) \equiv \frac{\partial T}{\partial h}(T_0(t, h), h) = \frac{h}{t c_{\text{rms}}^2 (T_0(t, h))} \quad \text{(the ray slowness)}$$

$$s(t, h) \equiv \frac{\partial T_0}{\partial t}(t, h) \quad \text{(the stretch factor)}$$

Cross-correlation tomography (3/8)

Because there is only one reflector, the trace has only one event at t_0 so

$$d(t,h) = f\left(t - T(t_0,h)\right).$$

Therefore:

$$\left(d(\cdot,h) \star d(\cdot,h')\right)(t) = f \star f\left(t + T(t_0,h) - T(t_0,h')\right)$$

A first-order Taylor approximation yields

$$\begin{aligned}\left(d(\cdot,h) \star d(\cdot,h')\right)(t) &\simeq \left(f \star f\right)'(t)\frac{\partial T}{\partial h}(t_0,h)(h-h') \\ &= \left(f \star f\right)'(t)\frac{h}{tc_{\text{rms}}^2(t_0)}(h-h')\end{aligned}$$

i.e. the cross-correlation $d \star d'$ contains arrival time slowness, hence background velocity information!

Cross-correlation tomography (4/8)

- Recall that the (simplified) convolutional model writes:

$$d(t,h) = r(T_0(t,h)).$$

- **Idea**: To obtain the background velocity, construct an operator which when applied to the data with the correct background medium yields a vanishing outcome.

- Denote by c_0^* the correct background velocity, with corresponding traveltime $T^*(t_0,h)$ and inverse traveltime $T_0^*(t,h)$.

- Assume model-consistent data (i.e. noise-free): $d(t,h) = r^*(T_0^*(t,h))$.

Cross-correlation tomography (5/8)

- A short calculation shows that: $\dfrac{\partial T_0}{\partial h}(t,h) = -s(t,h)p(t,h)$.
- We will also need:

$$\frac{\partial d}{\partial t}(t,h) = \frac{\partial T_0^*}{\partial t}(t,h)\frac{\partial r^*}{\partial t_0}(T_0^*(t,h)) \;\Rightarrow\; \frac{\partial r^*}{\partial t_0}(T_0^*(t,h)) = \left[\frac{\partial T_0^*}{\partial t}(t,h)\right]^{-1}\frac{\partial d}{\partial t}(t,h)$$

and $\partial d/\partial h$ is computed in a similar way.

- Choose a trial velocity c_0, compute corresponding T_0, and define the weighted cross-correlations:

$$C_t(t,h,h') = \int \left[d(t+\tau,h)\frac{\partial T_0}{\partial \tau}(\tau,h)\int_{-\infty}^{\tau} d(\cdot,h')\right] d\tau$$

$$C_h(t,h,h') = \int \left[d(t+\tau,h)\frac{\partial T_0}{\partial h}(\tau,h)\int_{-\infty}^{\tau} d(\cdot,h')\right] d\tau$$

Cross-correlation tomography (6/8)

Define the functional: $I(t,h) = \left[\frac{\partial C_t}{\partial h'} + \frac{\partial C_h}{\partial t}\right](t,h,h'=h).$

Then it can be shown that:

$$I(t,h) = \int d(t+\tau,h)\left\{\left[\frac{\partial T_0^*}{\partial h}\frac{\partial T_0}{\partial \tau} - \frac{\partial T_0^*}{\partial \tau}\frac{\partial T_0}{\partial h}\right](\tau,h)\left[\frac{\partial T_0^*}{\partial \tau}(\tau,h)\right]^{-1}\right\}d(\tau,h)d\tau$$

Note that $I(t,h)$ vanishes when $T_0^* = T_0$, i.e. when $c_0 = c_0^*$. Using the quantities defined above, we can rewrite $I(t,h)$ as

$$I(t,h) = \int_{-\infty}^{\infty} d(t+\tau,h)s(\tau,h)\left[p(\tau,h) - p^*(\tau,h)\right]d(\tau,h)d\tau$$

This functional measures the mismatch of event slowness, weighted by data auto-correlation and stretch factor.

Cross-correlation tomography (7/8)

- With the hyperbolic moveout approximation, we obtain:

$$I(t,h) = \int_{-\infty}^{\infty} d(t+\tau,h)s(\tau,h)\frac{h}{\tau}\left[c_{\text{rms}}^{-2} - c_{\text{rms}}^{*,-2}\right](T_0(\tau,h))\,d(\tau,h)d\tau$$

- **Velocity analysis algorithm**:

$$\min_{c_0} J = \tfrac{1}{2}\|I(t,h)\|^2$$

Use gradient-based optimization methods (assuming J is smooth in c_0).

- This approach clearly is a variant of differential semblance optimization. It is also a waveform variant of **stereotomography** (Sword, 1986, Biondi, 1990, Billette and Lambaré, 1998).

Cross-correlation tomography (8/8)

Conjectures:

- Objective just defined has global minimums, as has been proved for other DSO variants (e.g. the layered medium case as shown above).
- When intermediate scale random fluctuations are allowed, the cross-correlations with slowly-varying weights are statistically stable, as is the case without weights.
- The gradient of J is also statistically stable.
- Stationary points of J with cross-correlation weights computed from long-scale velocity component are optimal estimators of background velocity.

Ultimately: Velocity analysis is essentially stable against random fluctuations on the medium scale l!

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Conclusions

- The proposed work represents the first attempt at estimating the velocity macro-model in this three-scale asymptotics regime. i.e. when uncertainty at the middle scales is modeled by a random field.

- As such, it also represents an innovative way of combining two very different theories: the traditional (deterministic) approach to the problem coupled to the tools and ideas used to study time reversal in random media.

- In essence, it is an attempt to build a theoretical basis for assessing the influence of the intermediate scale of velocity on the estimation of the background velocity.

Future work

Immediate:

- Work through the set of conjectures set forth. In particular, the first step is to verify that the weighted cross-correlations are self-averaging.
- Implementation in SVL and TSOpt for numerical investigation on real data sets.

Possible extension:

- Extension to more complex models.
- Investigation of the applicability of the imaging results obtained by Borcea, Papanicolaou and colleagues to migration.

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Key references

Geophysics

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