FWI + MVA

William Symes

The Rice Inversion Project

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Agenda

FWI

Extended Modeling

Extended Modeling and WEMVA

Extended Modeling and FWI

Nonlinear WEMVA and LF Control

Summary

$\mathcal{M}$ = model space, $\mathcal{D}$ = data space

$\mathcal{F} : \mathcal{M} \rightarrow \mathcal{D}$ modeling operator = forward map = ...

Full Waveform Inversion problem:

given $d \in \mathcal{D}$, find $m \in \mathcal{M}$ so that

$$\mathcal{F}[m] \simeq d$$

Least squares inversion ("the usual suspect"):

given $d \in D$, find $m \in \mathcal{M}$ to minimize

$$J_{LS}[m] = \|\mathcal{F}[m] - d\|^2 [+ \text{ regularizing terms}]$$

($\|\cdot\|^2$ = mean square)

[Jackson 1972, Bamberger et al 1979, Tarantola & Vallette 1982,...]

Known since 80's:

- tendency to get trapped in "local mins"
- transmission modeling more linear than reflection modeling, so J_{LS} more quadratic
- continuation [low frequency $\rightarrow$ high frequency] helps

[Gauthier et al. 1986, Kolb et al. 1986]

The critical step is the first:

initial model $\Leftrightarrow$ data bandwidth

$\Rightarrow$ tomography, either waveform or travel time

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$\mathcal{M}$ = physical model space

$\bar{\mathcal{M}}$ = *bigger* extended model space

$\bar{\mathcal{F}} : \bar{\mathcal{M}} \rightarrow \mathcal{D}$ extended modeling operator

Extension property:

- $\mathcal{M} \subset \bar{\mathcal{M}}$
- $m \in \mathcal{M} \Rightarrow \bar{\mathcal{F}}[m] = \mathcal{F}[m]$

Acoustics: $m = \kappa/\rho$:

$$\partial_t^2 p - \kappa \nabla^2 p = f$$

$\mathcal{F}[m] = p$ sampled at receiver positions $\mathbf{r}$

f depends on source position $\mathbf{s}$

Extended acoustics via "survey sinking" - $\bar{m}$ is an *operator*,

$$(\bar{m}\nabla^2\bar{p})(\mathbf{x}) = \int dy\, \bar{m}(\mathbf{x},\mathbf{y})\nabla^2\bar{p}(\mathbf{y})$$

$\mathcal{M} \subset \bar{\mathcal{M}}$: multiplication by $m(\mathbf{x}) \sim$ application of

$\bar{m}(\mathbf{x},\mathbf{y}) = m(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$

Physical meaning: action at a positive distance

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Born approximation about *physical* (non-extended) background model

$$\bar{m}(\mathbf{x},\mathbf{y}) = m(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) + \delta\bar{m}(\mathbf{x},\mathbf{y})$$

then $\bar{p} \simeq p + \delta p$,

$$\partial_t^2 \delta p - m\nabla^2 \delta p = \int dy \delta\bar{m}(\mathbf{x},\mathbf{y})\nabla^2 p(\mathbf{y},t)$$

Born modeling (derivative of $\bar{\mathcal{F}}$) = samples of δp at receiver locations $\mathbf{r}$

$G(\mathbf{x}, \mathbf{y}, t)$ = Green's function

$$\delta p(\mathbf{s}, \mathbf{r}, t) = \int d\tau \int dx G(\mathbf{r}, \mathbf{x}, t - \tau)$$

$$\times \int dy \delta \bar{m}(\mathbf{x}, \mathbf{y}) \nabla^2 p(\mathbf{s}, \mathbf{y}, \tau)$$

$$= \int dx \int dy \left[\int d\tau \, G(\mathbf{r}, \mathbf{x}, t-\tau) \nabla^2 p(\mathbf{s}, \mathbf{y}, \tau) \right]$$

$$\times \delta \bar{m}(\mathbf{x}, \mathbf{y})$$

$\Rightarrow$ *adjoint* of Born modeling = imaging operator applied to data residual $\delta d(\mathbf{s}, \mathbf{r}, t)$

$$I(\mathbf{x}, \mathbf{y}) = \int ds \int dr \int dt \delta d(\mathbf{s}, \mathbf{r}, t)$$

$$\times \left[\int d\tau \, G(\mathbf{r}, \mathbf{x}, t-\tau) \nabla^2 p(\mathbf{s}, \mathbf{y}, \tau) \right]$$

Receiver wavefield (back-propagate receiver traces)

$$R(\mathbf{s},\mathbf{x},\tau) = \int dt \int dr \delta d(\mathbf{s},\mathbf{r},t) G(\mathbf{r},\mathbf{x},t-\tau)$$

Source wavefield

$$S(\mathbf{s},\mathbf{y},\tau) = \nabla^2 p(\mathbf{s},\mathbf{y},\tau)$$

$$I(\mathbf{x},\mathbf{y}) = \int ds \int d\tau R(\mathbf{s},\mathbf{x},\tau) S(\mathbf{s},\mathbf{y},\tau)$$

- propagate receiver field to sunken receiver position $\mathbf{x}$
- propagate source field to sunken source position $\mathbf{y}$
- cross-correlate at zero time lag
- sum over sources

[Claerbout 1985]

- Image formation possible for any background model m, data residual δd: $I = I[m, \delta d]$
- image $I[m, \delta d]$ is actually a *model update*
- updated model $m + \alpha I$ is *physical* if it is concentrated on diagonal $\mathbf{x} = \mathbf{y}$ (zero offset)

Conclusion: background model m consistent with data residual δd

$\Leftrightarrow$ image $I[m, \delta d](\mathbf{x}, \mathbf{y})$ focused on zero offset locus $\mathbf{x} = \mathbf{y}$

$\Rightarrow$ WEMVA

WEMVA via optimization:

- choose a function ϕ on $\bar{\mathcal{M}}$ so that (i) $\phi \geq 0$, (ii) $\phi[\bar{m}] = 0 \Leftrightarrow \bar{m} \in \mathcal{M}$
- minimize $\phi(I[m, \delta d])$ over m

Typical choice: choose operator A on extended model space $\bar{\mathcal{M}}$ so that $\mathcal{M}$ = null space of A, $\phi[\bar{m}] = \|A[\bar{m}]\|^2$

[de Hoop and Stolk 2001, Shen et al. 2003, 2005, Albertin et al 2006,...]

edging towards inversion...

recall that $I[m, \delta d]$ updates $\delta \bar{m}$ - in fact is a Born inversion, with care!

So paraphrase inversion as:

amongst extended models that fit data residual (under Born modeling), find physical one (perhaps by minimizing ϕ)

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Extended FWI problem:

given d, find $\bar{m} \in \bar{\mathcal{M}}$ so $\bar{\mathcal{F}}[\bar{m}] \simeq d$

- extended FWI is too easy - many solutions!
- extension property: a physical model is an extended model

Inversion paraphrase:

amongst extended models which fit data, find a physical one

sounds just like WEMVA!

Difference: full wave field modeling/inversion, rather than Born

WEMVA-like, using ϕ:

minimize $\phi[\bar{m}]$ subject to $\|\bar{\mathcal{F}}[\bar{m}] - d\| \simeq 0$

Contrast with FWI:

minimize $\|\bar{\mathcal{F}}[\bar{m}] - d\|$ subject to $\phi[\bar{m}] \simeq 0$

All in the family:

$$J_\sigma[\bar{m}, d] = \frac{1}{\sigma}\|\bar{\mathcal{F}}[\bar{m}] - d\|^2 + \sigma\phi[\bar{m}]$$

$\sigma \rightarrow \infty \Rightarrow$ FWI

$\sigma \rightarrow 0 \Rightarrow$ "nonlinear WEMVA"

Gockenbach 1995: path of minima $\bar{m}[\sigma]$ can be followed from small σ to large, leads to FWI solution *provided* that small-σ problem can be solved $\langle$fine print$\rangle$

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Natural strategy: optimize $\lim_{\sigma \to 0} J_\sigma$ using a gradient method

BUT for small σ, gradient points mostly in data-consistency direction - can lead to inefficient solves

Better: compute updates *within* the data-consistent extended models

IF VLF band [0,?] Hz were available, acoustic, elastic extended inversion *always* solvable - no ambiguity, can always start frequency continuation

But VLF typically not recorded - what to do?

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But VLF typically not recorded - what to do?

Our solution: make them up!

[non-observed VLF *parametrize* data-consistent extended models]

- choose *low frequency control model* $m_{LF} \in \mathcal{M}$
- choose low frequency source complementary to data passband, build low frequency modeling operator $\mathcal{F}_{LF}$, also extended $\bar{\mathcal{F}}_{LF}$
- create low frequency synthetics $d_{LF} = \mathcal{F}_{LF}[m_{LF}]$
- replace d with full bandwidth data $d_{\rm full} = d + d_{LF}$
- build full bandwidth modeling operator $\bar{\mathcal{F}}_{\rm full} = \bar{\mathcal{F}} + \bar{\mathcal{F}}_{LF}$
- solve full bandwidth extended inversion $\bar{\mathcal{F}}_{\rm full}[\bar{m}] \simeq d_{\rm full}$
- solution $\bar{m}$ depends on m_{LF} and d

LF control model parametrizes data-consistent extended models

[similar: migration macro-model parametrizes extended images]

Nonlinear WEMVA objective:

$$J_{NMVA}[m_{LF}, d] = \phi[\bar{m}[m_{LF}, d]]$$

equivalent to $\lim_{\sigma \rightarrow 0} J_{\sigma}$

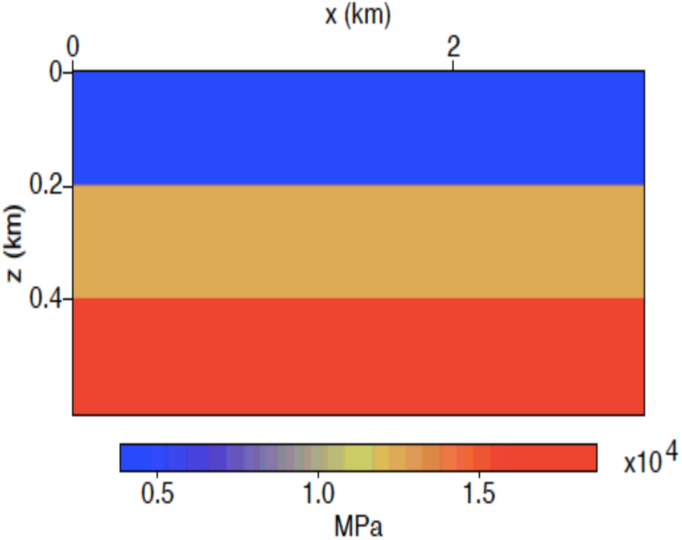
Examples: D. Sun PhD thesis (2012)

Extended modeling by data gathers: *model each gather independently*

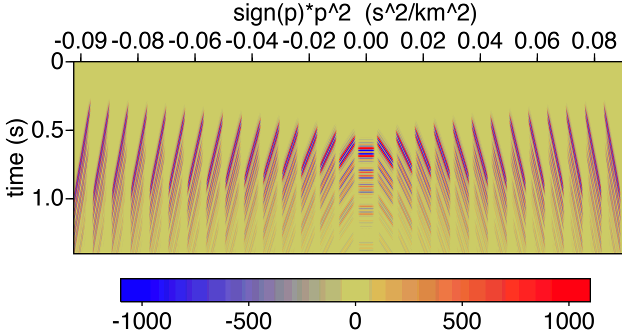
WE solver, economy $\Rightarrow$ source gathers ($\mathbf{s}$)

Extended model space $\bar{\mathcal{M}}$ for acoustics $= \{\bar{m}(\mathbf{x},\mathbf{s})\}$

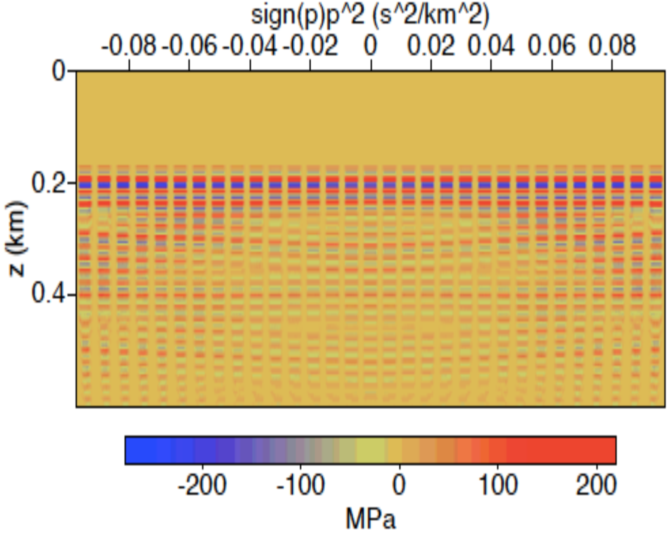
Physical model = independent of $\mathbf{s}$; natural choice of $A = \nabla_{\mathbf{s}}$



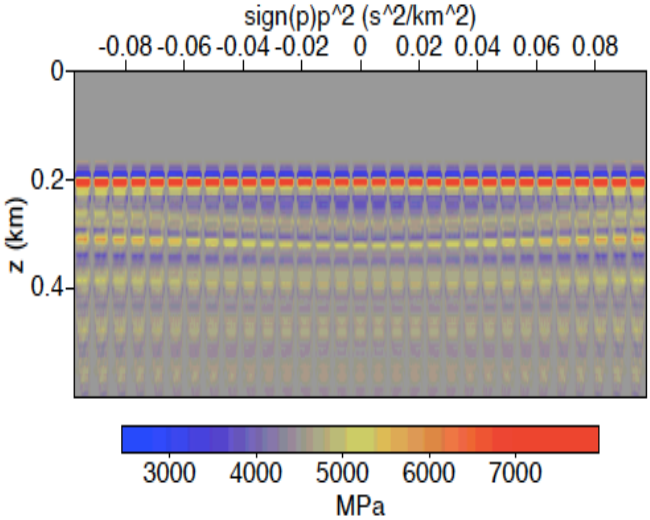
Three layer bulk modulus model. Top surface pressure free, other boundaries absorbing



Plane wave data - source param $\mathbf{s}$ = slowness - note free surface multiples



Prestack RTM $=$ extended model gradient at homogeneous initial model



Extended model inversion at homogenous initial model - ϕ is mean-square of slowness derivative [also: look ma no multiples]

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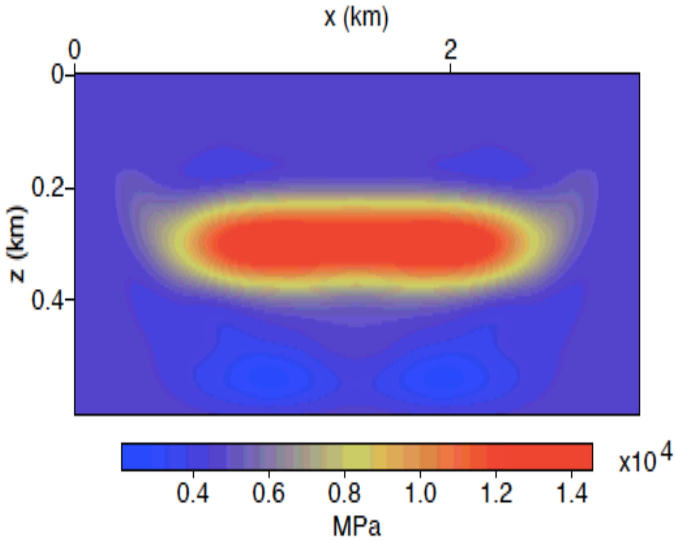
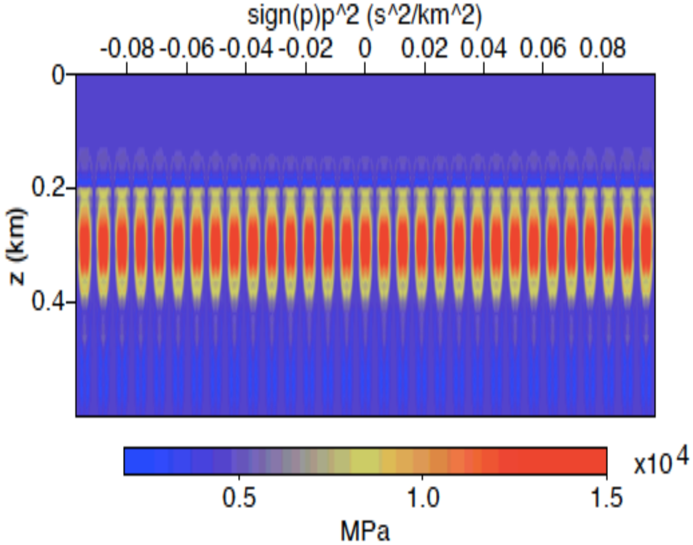
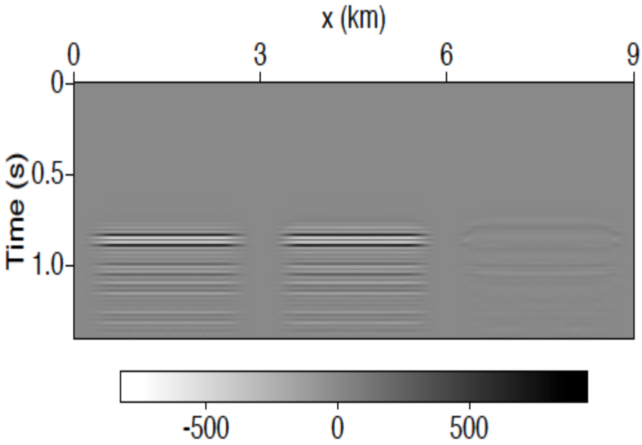


LF control model - 3 steps of LBFGS applied to J_{NMVA}



Extended model inversion from LF model data, step 3

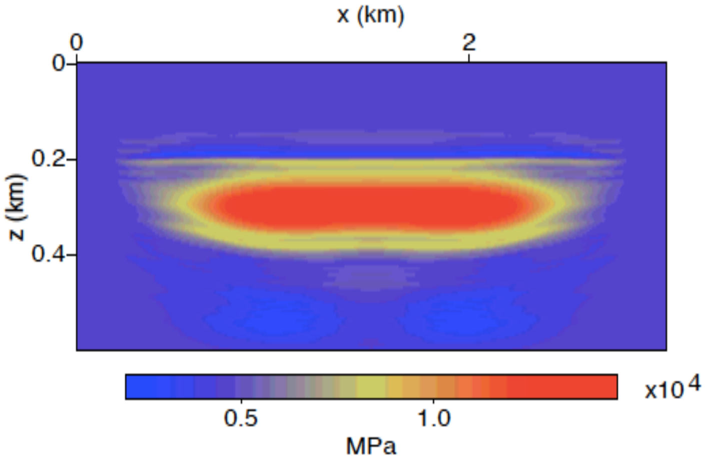


Data residual, slowness = 0 panel: left, target data; middle, resimulated data from extended model inversion; right, residual (11.4% RMS)

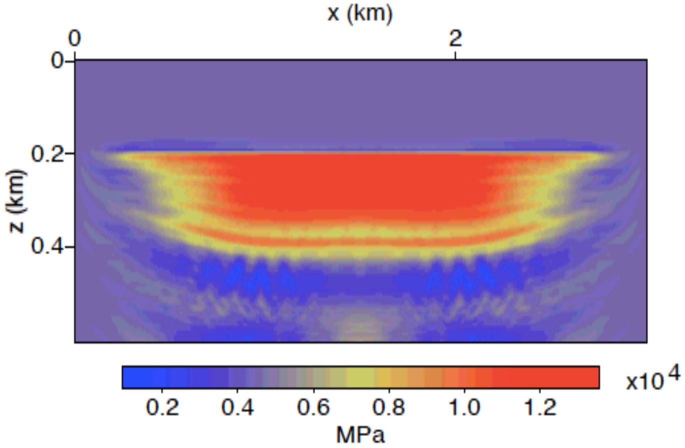
WEMVA $\rightarrow$ FWI by continuation: $\sigma = 0$ to $\sigma = \infty$ in one step!

Use LS fit of (physical model $\rightarrow$ extended model) to produce optimal initial model for LS inversion

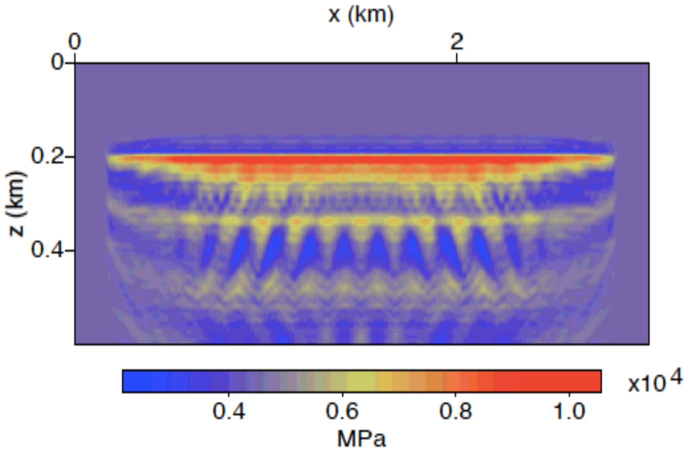
Follow by standard FWI



Initial model for FWI, obtained as best least-squares fit to NMVA extended model inversion



FWI from NMVA-derived initial model- 60 iterations of LBFGS, 3 frequency bands, 14% RMS residual



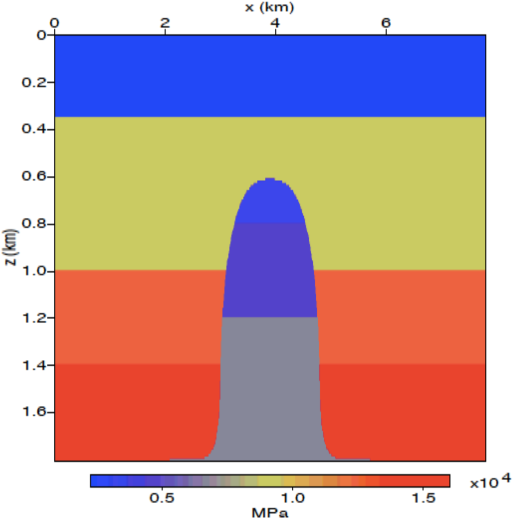
FWI from homogeneous initial model - 60 iterations of LBFGS, 3 frequency bands, 27% RMS residual

Incidental observation - multiple scrubbing effect

Even with incorrect LF control model (e.g. homogeneous), extended model inversion appears to suppress multiple energy

Not limited to layered models...

Theoretical explanation?



Laterally heterogeneous model with dome structure

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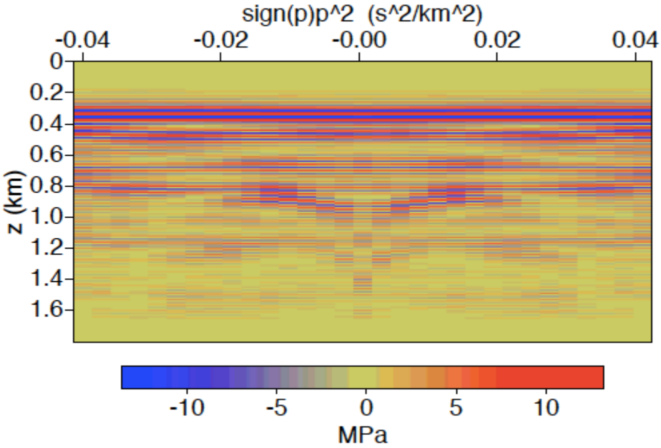
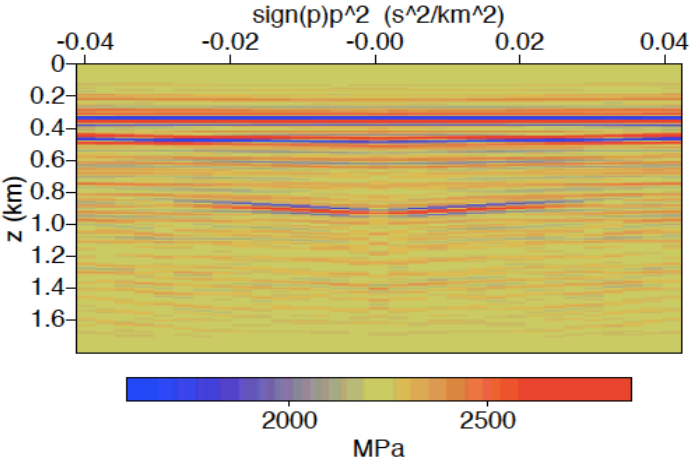


Gather at $x = 1.5$ km from pre stack RTM = extended model gradient, homogeneous background



Gather at $x = 1.5$ km from extended model inversion, homogeneous background

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- via extended modeling, see nonlinear variant of WEMVA, FWI as *end members* of J_σ family
- getting started: LF control model parametrizes data-consistent extended models, analogue of migration macro-model
- Examples suggest *multiple suppression* property of extended inversion

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