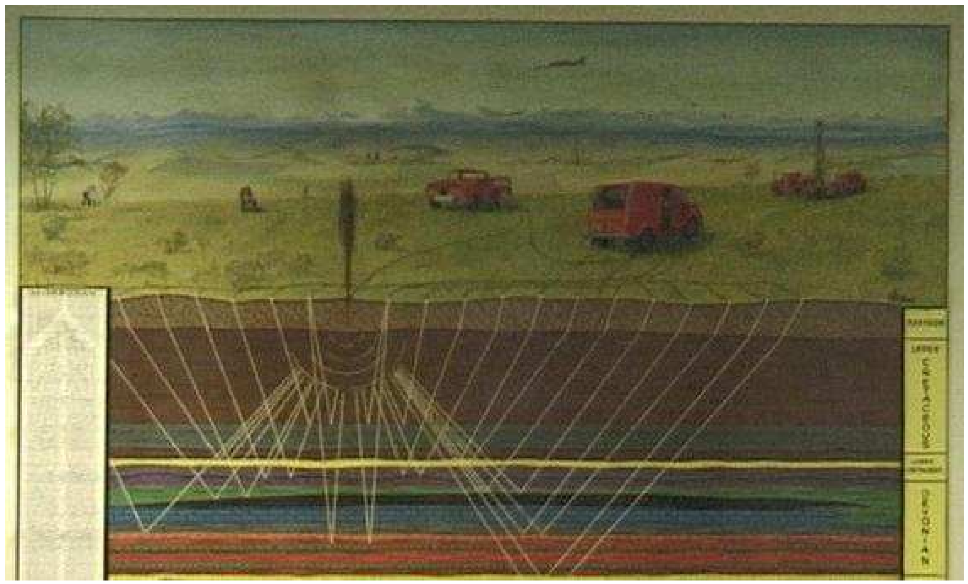
Seeing through rock: the mathematics of Reflection Seismology

William W. Symes

Reflection seismology is...

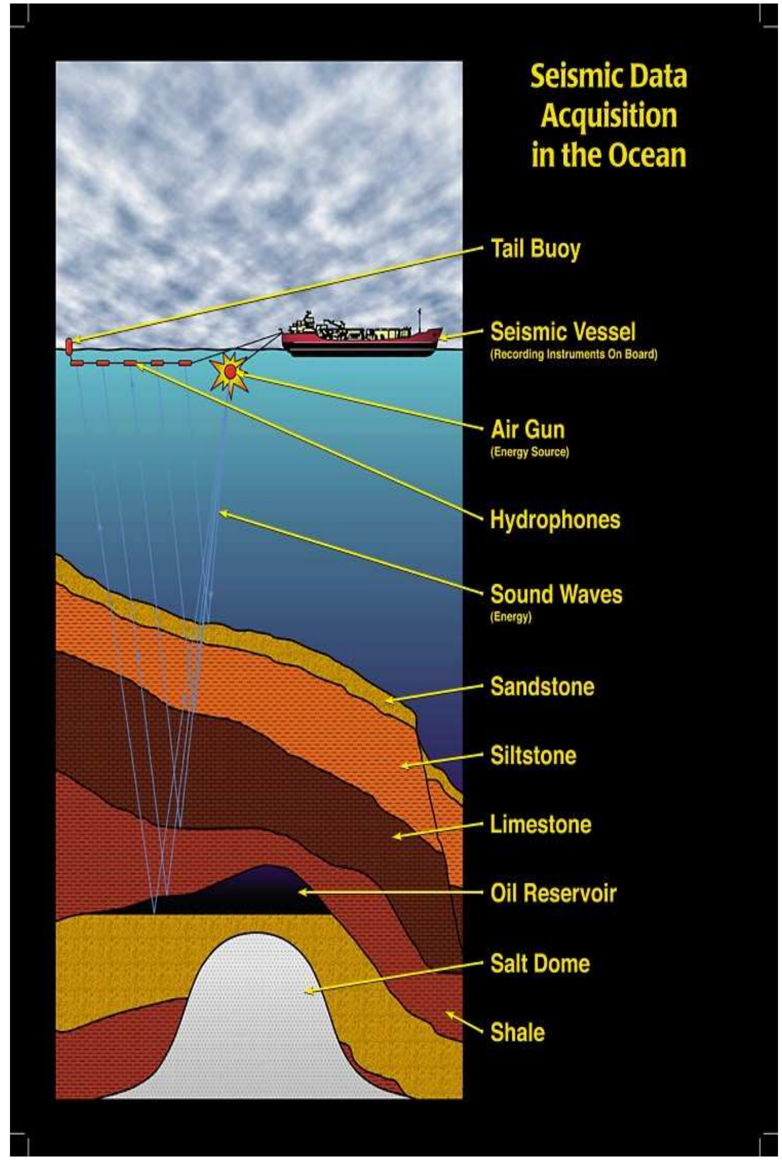
- the main prospecting tool of the oil and gas industry
- also useful in civil/environmental engineering and academic earth science
- a $4 billion / year industry
- a big deal in Houston
- *rich* in mathematical ideas and questions

The basic idea: collect echoes...



and - through various mathematical operations - convert them to maps of the Earth's structure. Infer location of oil and/or gas from structural maps, sometimes from character of echoes also.

Marine Operations
90% of all data
collected worldwide



Data also acquired on land, but it's harder...

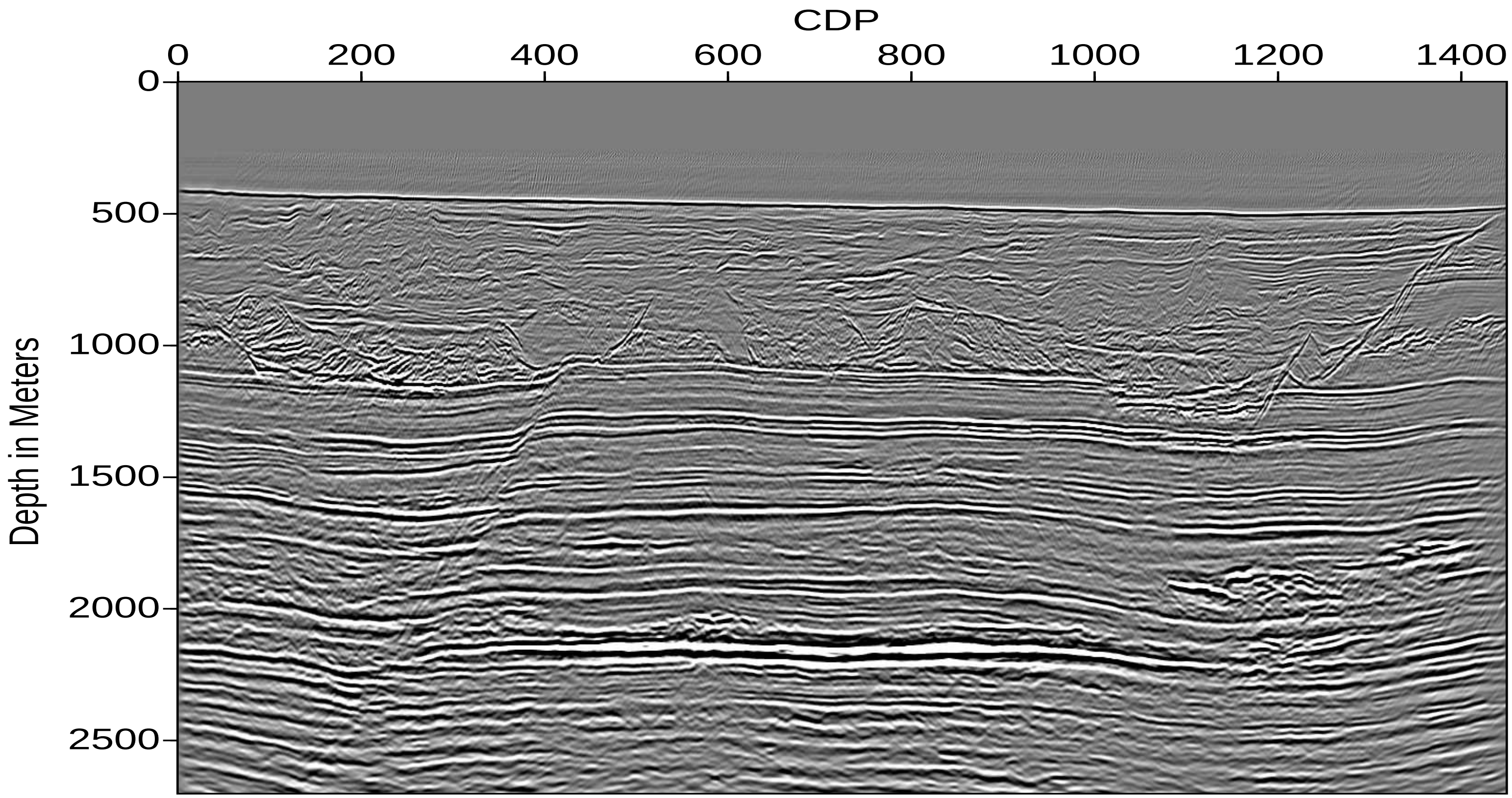
Data, data everywhere

Seismic surveys produce *vast* quantities of data!

- each geo/hydro phone collects perhaps 1500 digitized samples per experiment ("shot").
- each experiment deploys 100's of geo/hydro phones.
- each survey consists of 100's (1980's) to 100,000's (today) of shots.
- upshot: 750 MB = small, old; 150 GB = modest, contemporary
- equivalent: 1 - 300 CD's. Immediate future: 100's of DVD's!

This data gets turned into...

An Image of the Earth's Interior



(structural image, Mississippi Canyon, offshore Louisiana)

How do you do that?

Outline

- Acoustic fundamentals
- Zero offset imaging
- Multioffset imaging
- Velocity via redundancy
- Optimize this!

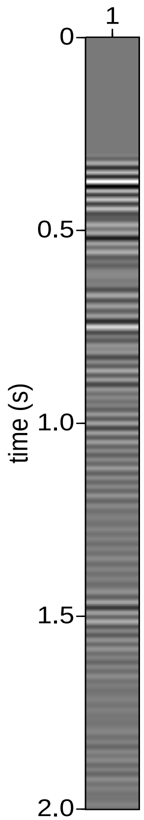
Acoustic fundamentals

Basic facts:

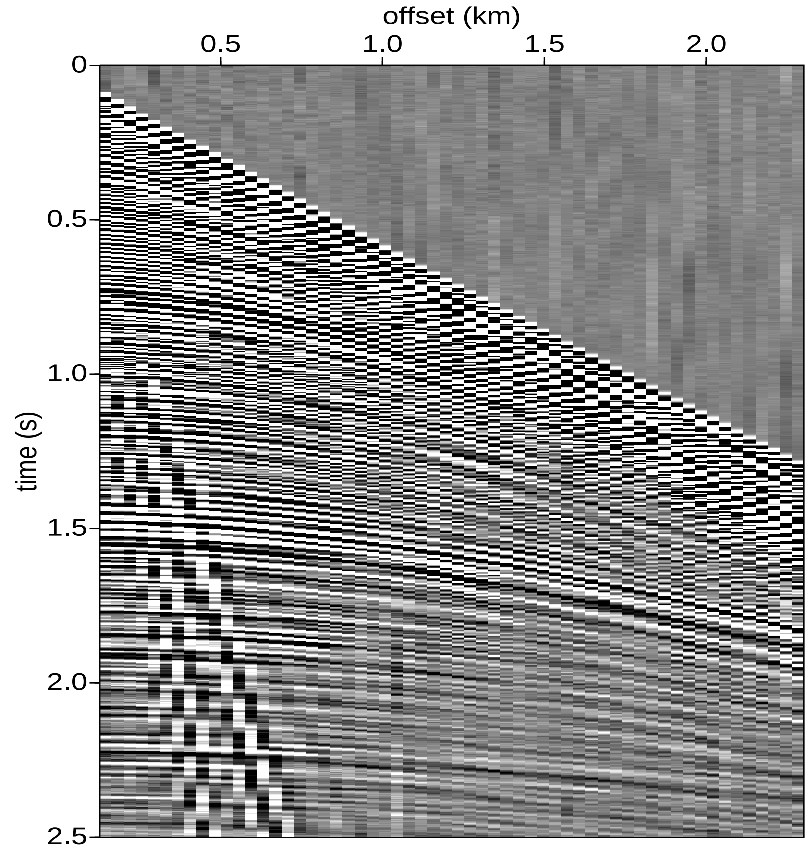
- sound is *pressure* fluctuations about equilibrium - quantify in lb/ft^2 or N/m^2
- sound is *oscillatory* - rapid sequence of positive, negative excursions
  - *A below middle C = 440 cycles/sec ("Hertz", Hz)*
  - typical frequency range of significant energy in seismic exploration: 5 - 70 Hz
- $\Rightarrow$ represent pictorially as grey - dark = higher pressure, light = low pressure, plotted vs. time.

Recording of single hydrophone

North Sea survey (thanks: Shell)



Shot record = recording of
single experiment, 64 hydrophones
North Sea survey (thanks: Shell)

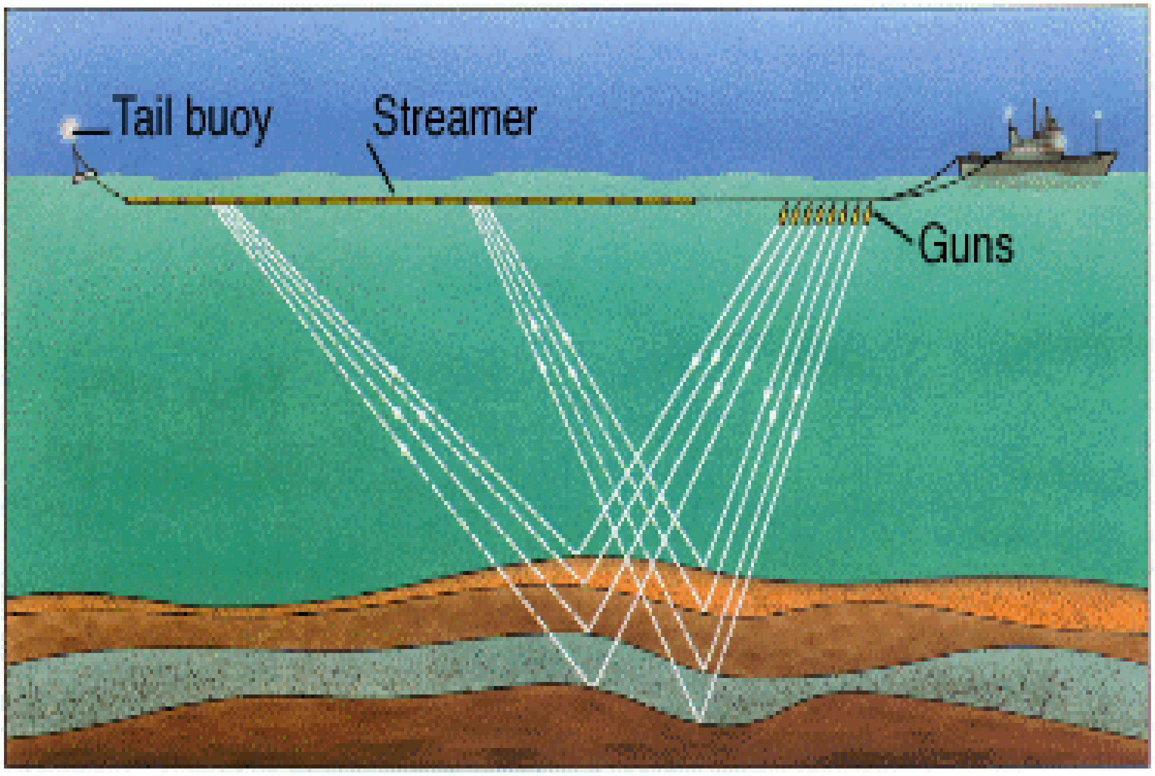


CDP 8501

Acoustic fundamentals

- sound (in rock or in air) is a *wave phenomenon*
- sound travels at a definite speed, characteristic of material
  - *sound speed in air* $\sim$ *330 m/s* $\sim$ *1000 ft/s; so count seconds after lightning flash until thunder arrives, multiply by 1000 to get distance in feet to lightning strike*
- reflection seismographs record *echoes* $\Rightarrow$ sound must travel from *sound source* (dynamite, air gun) to subsurface structure *and back* to geo/hydro phone.
- echoes (or *reflections*) occur when sound reaches a change in mechanical characteristics (eg. a change in sound speed!) - air to marble, sandstone to limestone, water-saturated sandstone to gassy sandstone,...

Acoustic fundamentals

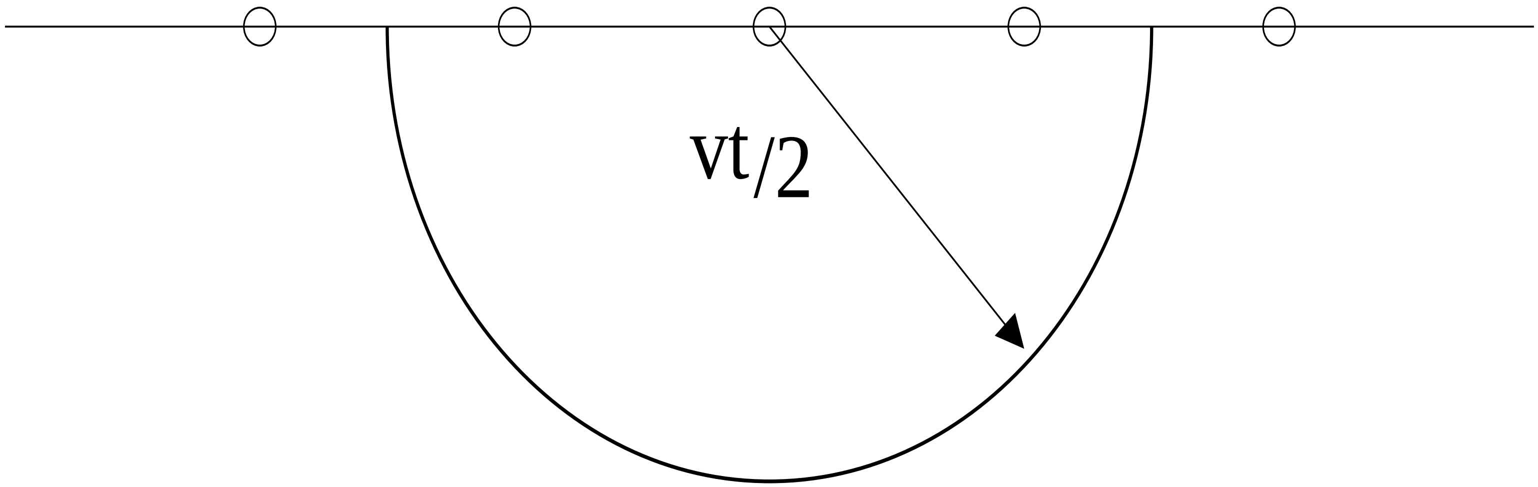


Offset = distance from source to receiver. Picture shows "near" and "far" offsets.

Zero Offset

- source and receiver in same place
- can’t really acquire (dynamite!)
- but can fake it by standard techniques
- easiest to understand how to build image...

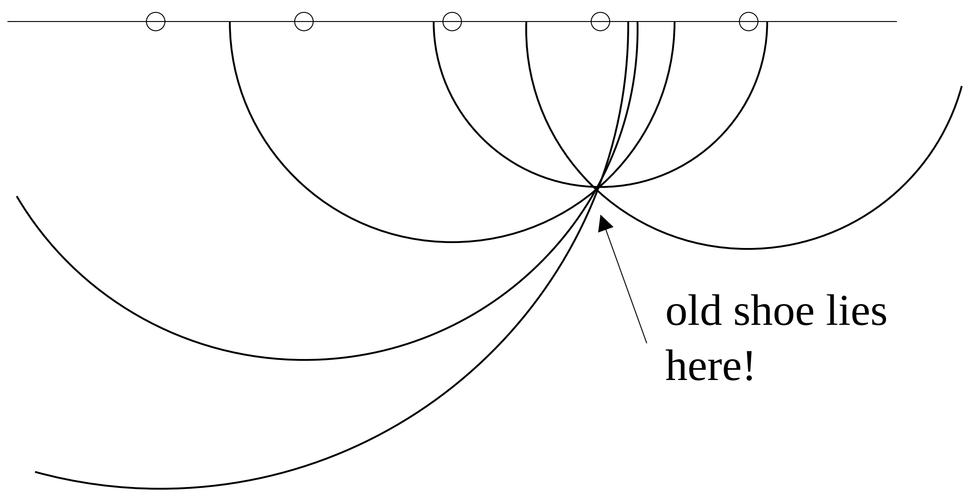
Out and back...



v = sound speed (or *velocity*) - 330 m/s in air, 1500 m/s in water, 2000 - 5000 m/s in sedimentary rock.

If you hear an echo at time t after seismic wave initiation at zero offset, then the source of the echo must lie somewhere on a circle of radius $vt/2$.

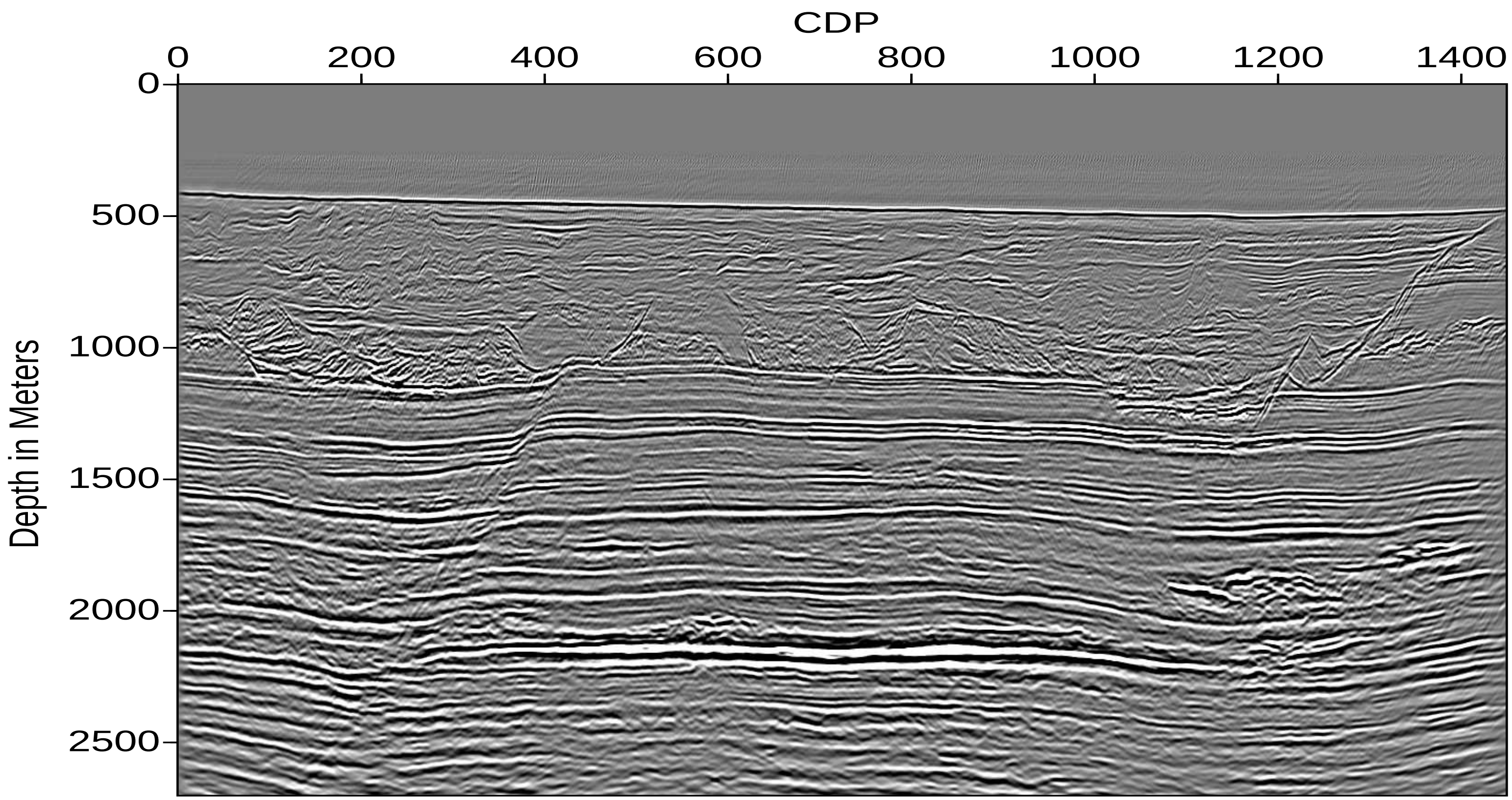
Finding and old shoe...



Time the echo from the old shoe... “hello!” from each source/receiver point, listen at same place, record arrival time of echo.

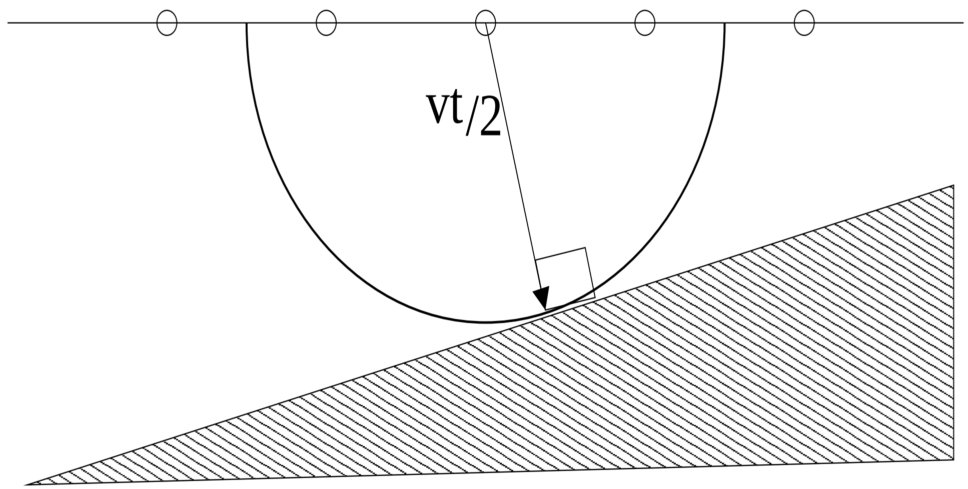
Shoe must lie on circles of various radii, computed as in previous slide - therefore on their intersection!

However...



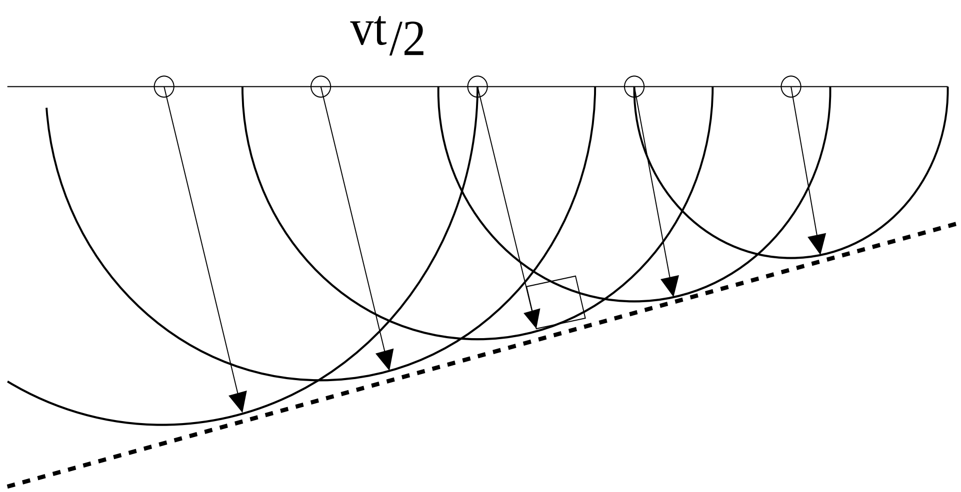
Earth structure does not resemble an old shoe - *extended*, *distributed*

Physics of sound reflection - Zero Offset



Reflection occurs at the place and time at which the wavefront is *tangent* to the reflecting surface - the *specular* (mirrorlike) reflection principle. Radius from source/receiver point is *normal* (perpindicular) to surface.

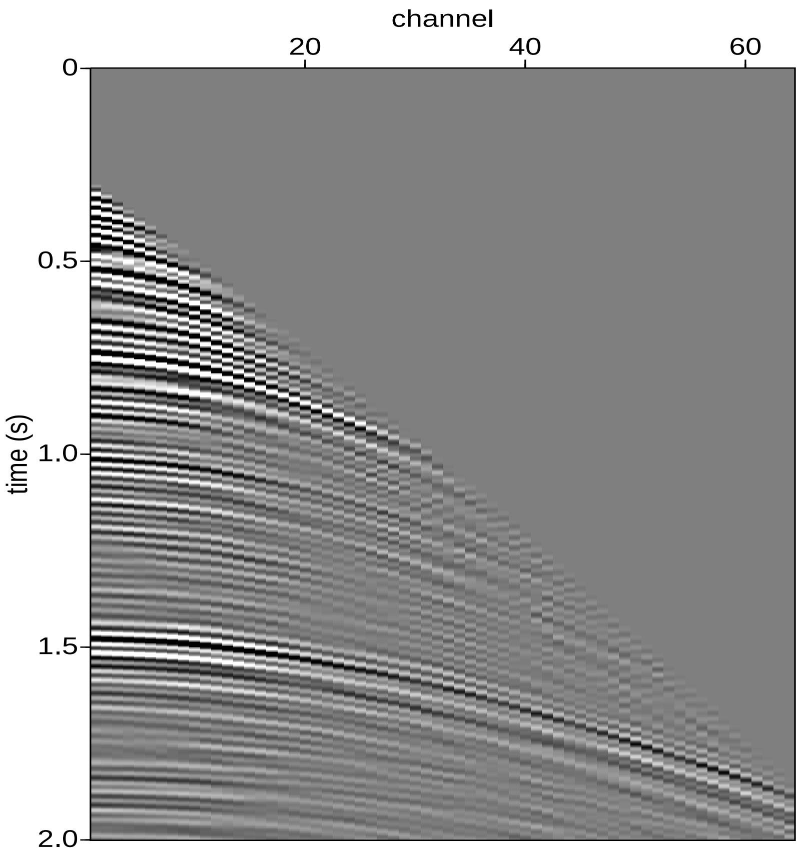
Locating a reflecting surface - zero offset



Reflecting surface must be tangent to each wavefront circle - *envelope* of circles.

Another issue

Do you really want to find the times of all the echoes?



Direct construction of images - *ZO Migration*

Concept:

(1) for each source/receiver point each sample time t (usually multiples of 2 ms or 4 ms), *place* the sample value at all possible image points at distance $vt/2$ (circle).

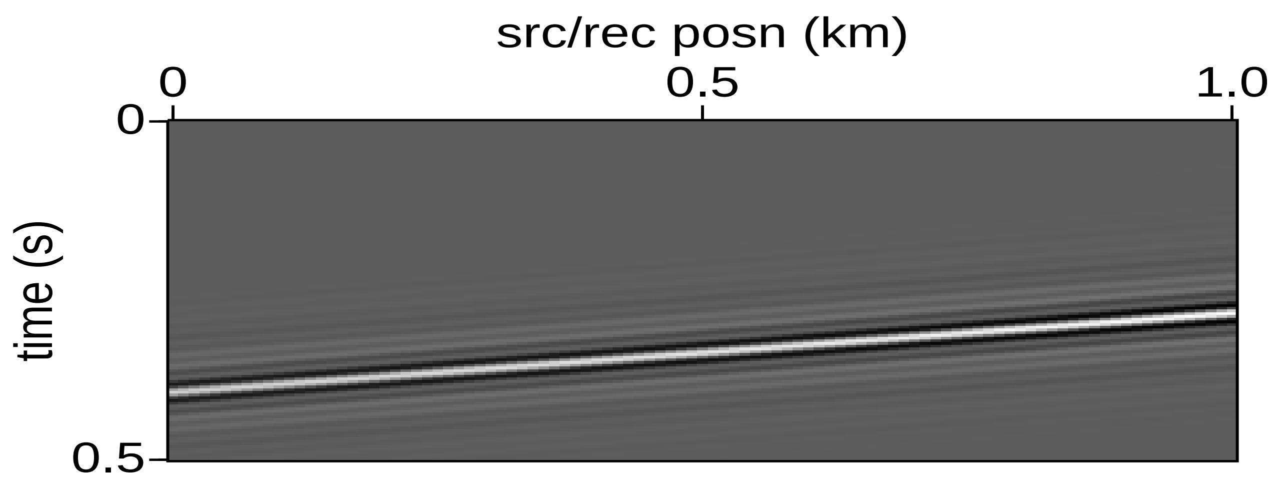
(2) then sum the resulting images, one for each source/receiver point.

Each image will exhibit circular wavefront.

Principle of Stationary Phase: Sum of oscillatory signals *cancels*, *except* along envelope of wavefronts.

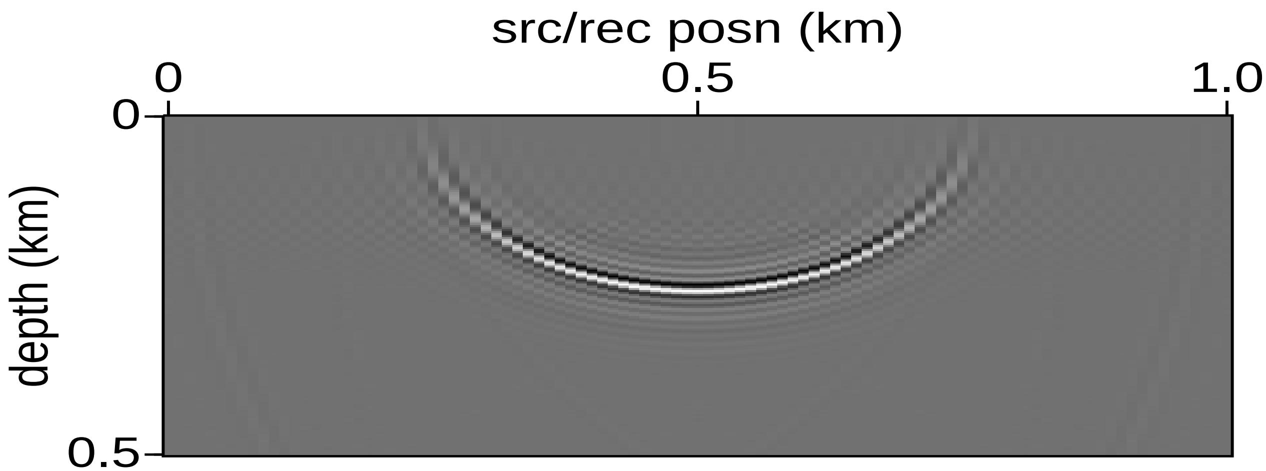
Consequence: after the summation, *only the envelope is visible!!!*.

ZO Migration Example (1/4)



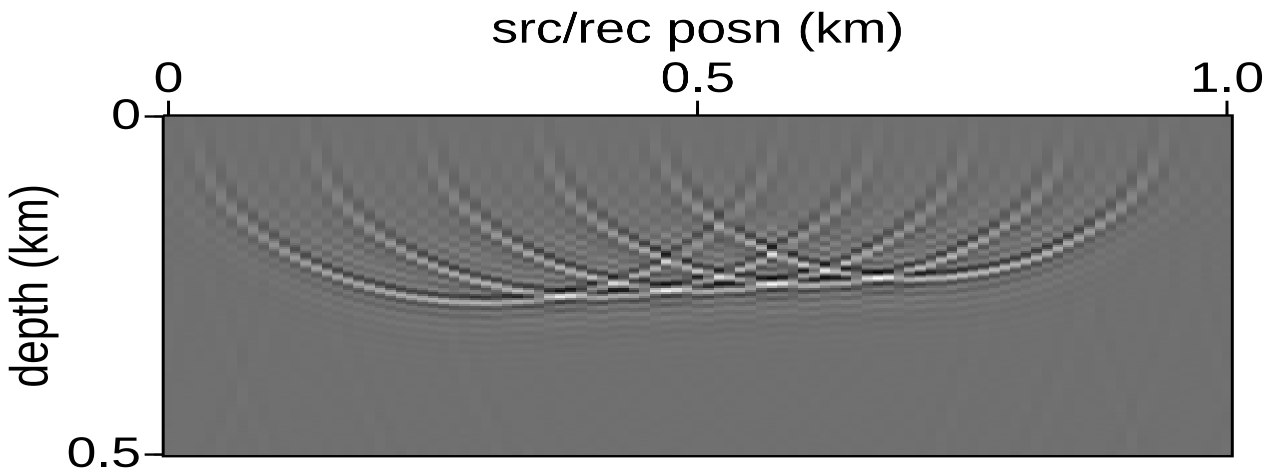
Zero offset simulated data for reflecting surface at depth $z = 0.3$ km at left edge ($x = 0$ km), sloping upwards to the right at 5 deg. 101 source/receiver points, spaced 10 m apart, from left to right.

ZO Migration Example (2/4)



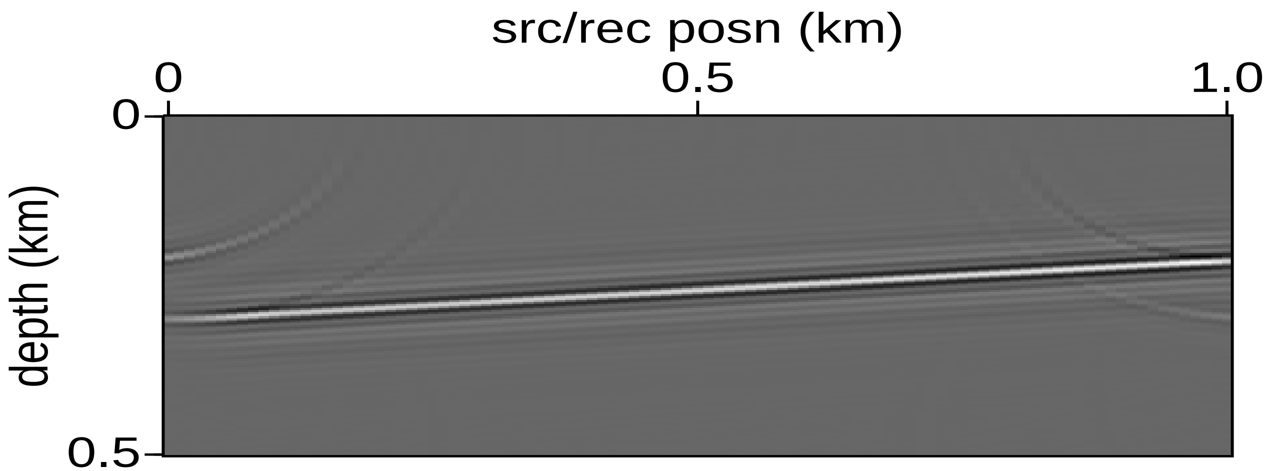
(1) for each source/receiver point each sample time t (usually multiples of 2 ms or 4 ms), *place* the sample value at all possible image points at distance $vt/2$ (circle).

ZO Migration Example (3/4)



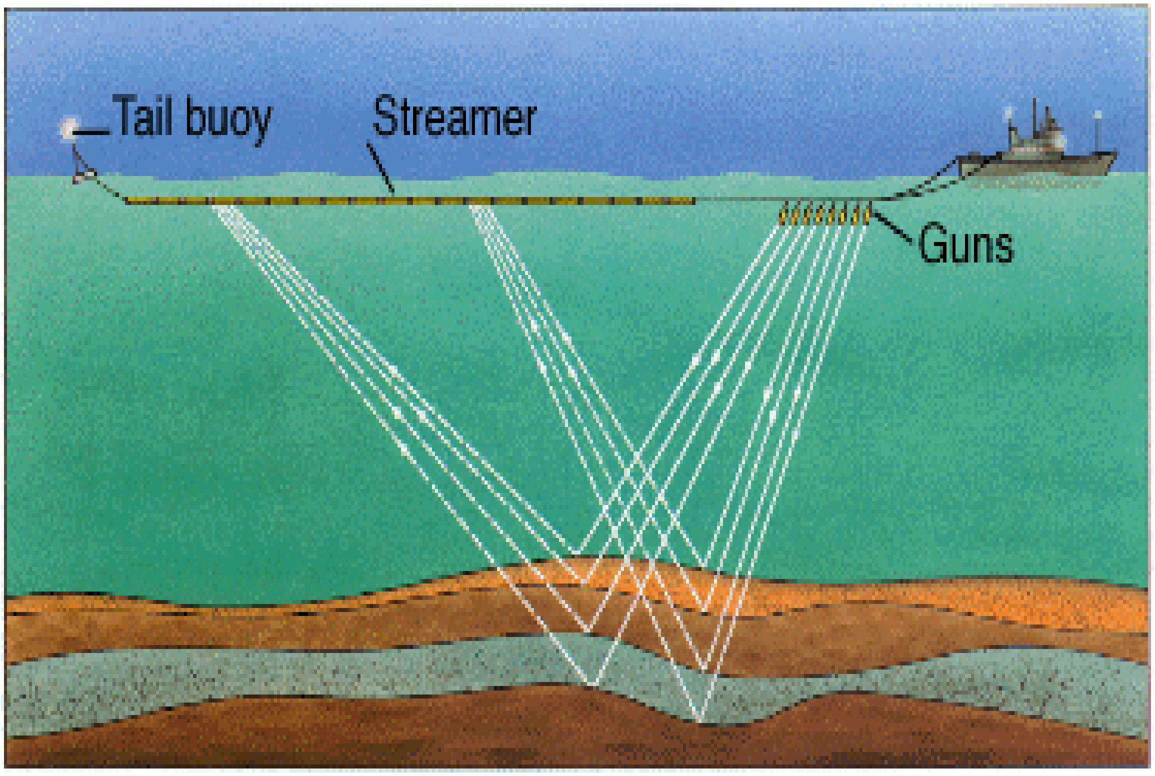
(2) then sum the resulting images, one for each source/receiver point (this figure shows 5 of the 101 source/receiver images added up, at 0.3, 0.4, 0.5, 0.6, 0.7 km from left edge).

ZO Migration Example (4/4)



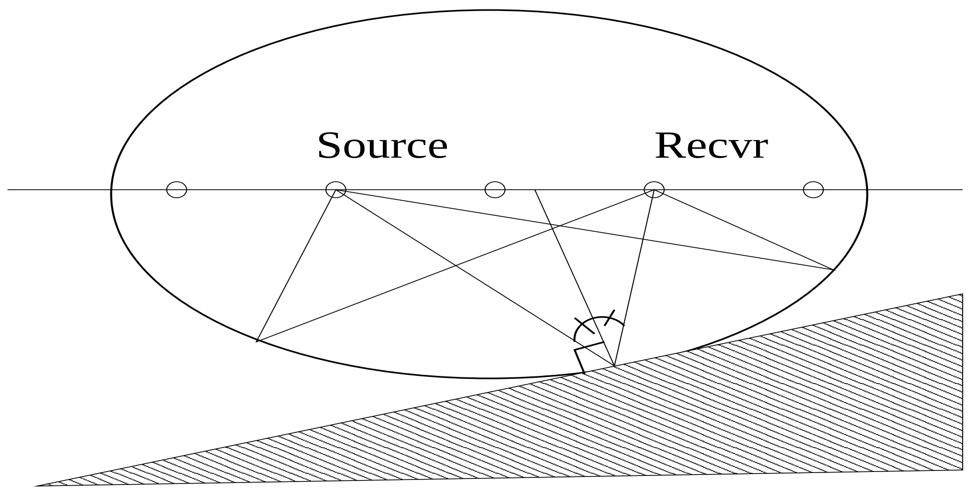
(2) then sum the resulting images, one for each source/receiver point (this figure shows all 101 source/receiver images added up - the wavefront envelope is recovered, sitting exactly on the reflecting surface!)

Multioffset Migration



Multiple Offsets = data that's actually measured. Can either *fake* ZO data from multioffset data (1970's technology) or directly use MO data - routine today.

Physics of sound reflection - Nonzero Offset



Reflection occurs at the place and time t at which ellipse with focii at source, recvr, diam = vt is *tangent* to the reflecting surface - the *specular* (mirrorlike) reflection principle. Bisector of lines from source and receiver points is *normal* (perpindicular) to surface (*Snell's law*) $\Rightarrow$ surface is *envelope* of ellipses.

Direct construction of images - *MO Migration*

Concept:

(1) for each source/receiver point each sample time t (usually multiples of 2 ms or 4 ms), *place* the sample value at all possible image points on ellipse of diameter vt.

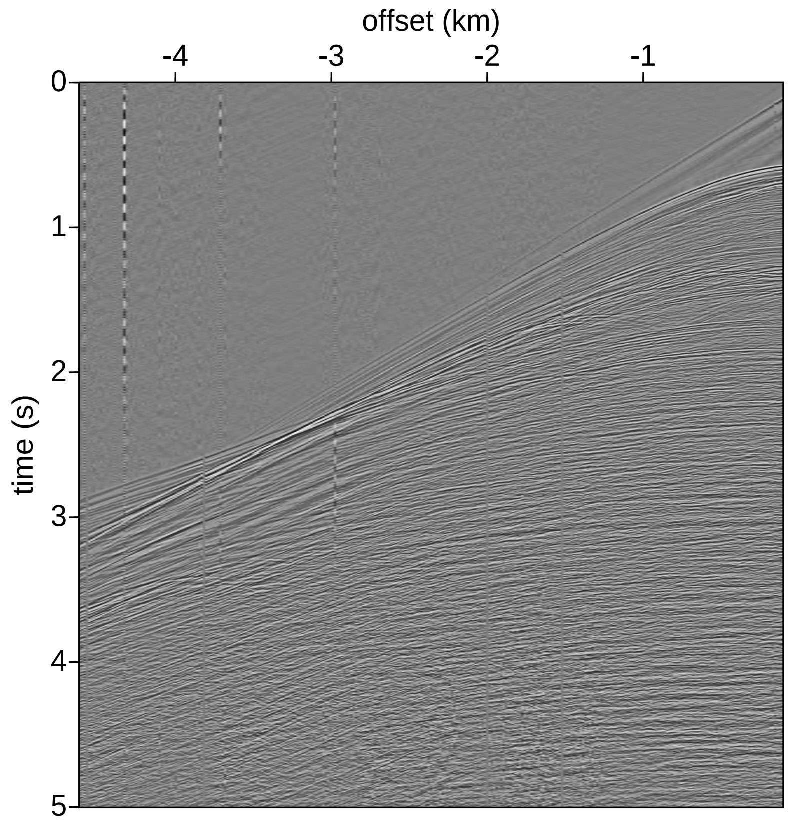
(2) then sum the resulting images, one for each source/receiver pair.

Each image will exhibit elliptical wavefronts.

Principle of Stationary Phase: Sum of oscillatory signals *cancels*, *except* along envelope of wavefronts.

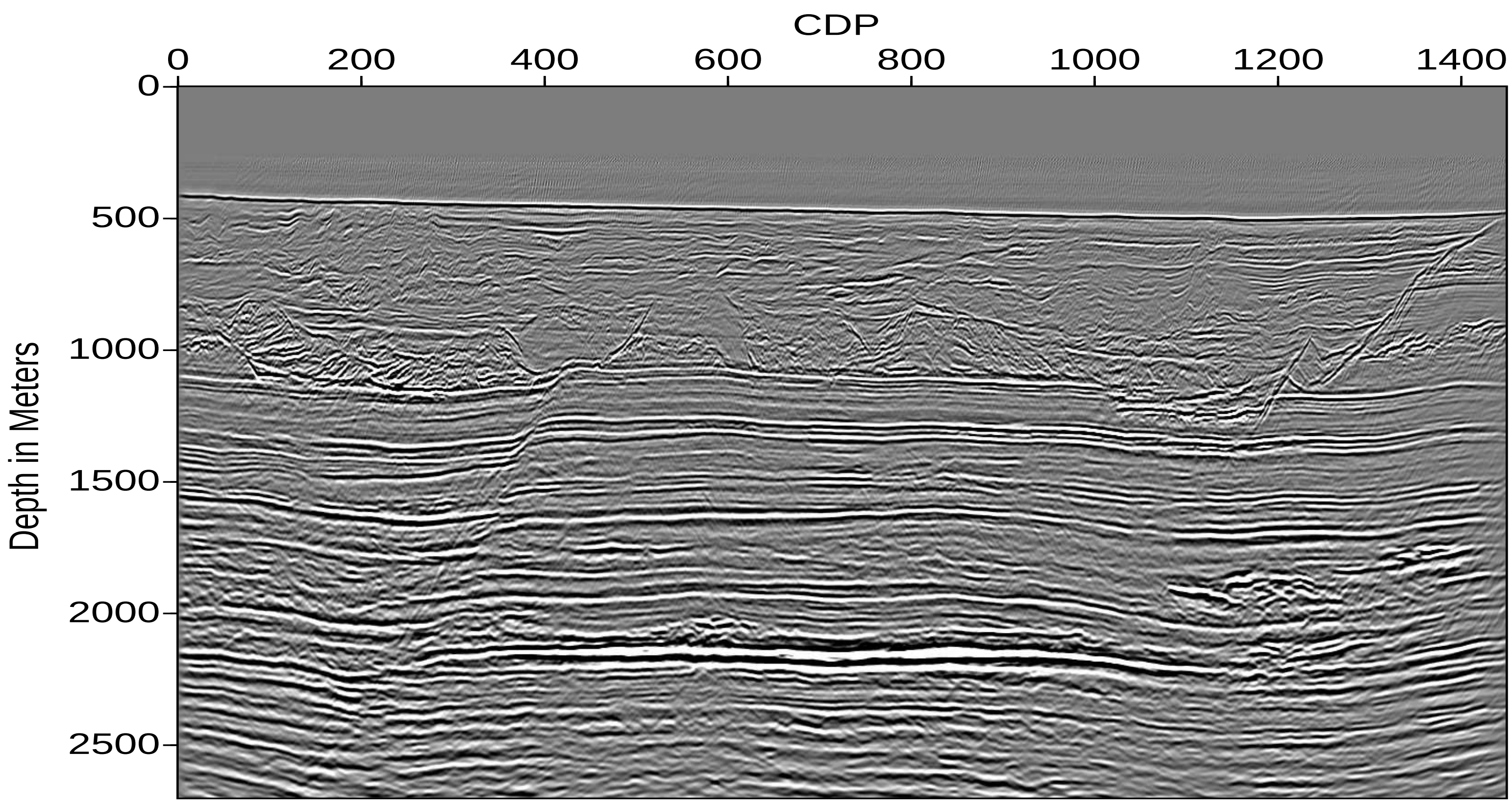
Consequence: after the summation, *only the envelope is visible!!!*.

And that's how we turned lots of this...



(one of 500 data panels from Exxon GofM data = 750 MB.)

into this!



(structural image, Mississippi Canyon, offshore Louisiana)

Two major details

(1) Q. how do you know v? A. You don't!

(2) Spatial variation of velocity and refraction of waves (some other time...)

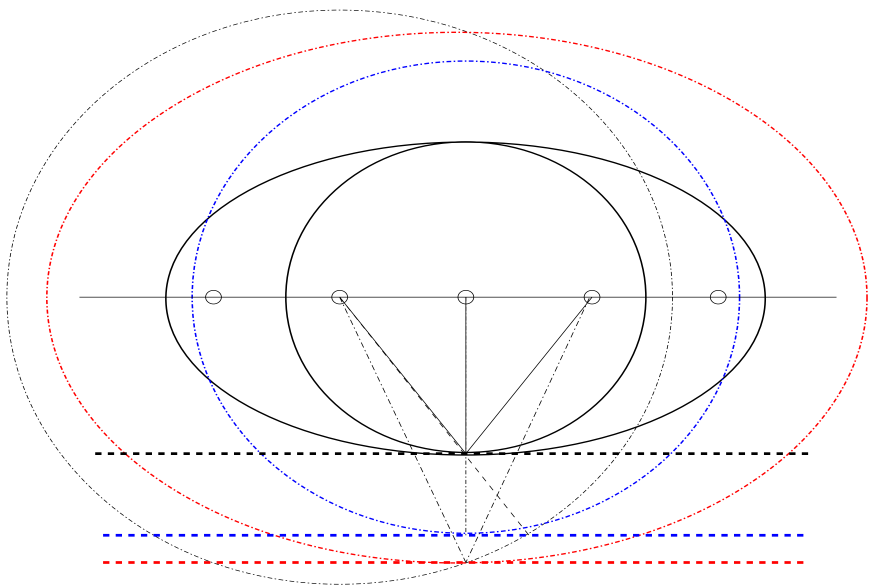
Velocity via redundancy (1/6)

- Typical survey contains source-receiver pairs with a variety of offsets.

- Any one set of ellipses with fixed offset (source-receiver distance) is enough to determine reflecting surfaces by forming envelopes - *common offset* construction

- the envelopes constructed using different offsets will only agree if the velocity is correct!

Velocity via redundancy (2/6)



Black circle, ellipse: zero, nonzero offset, correct velocity - both have dashed black line as possible envelope.

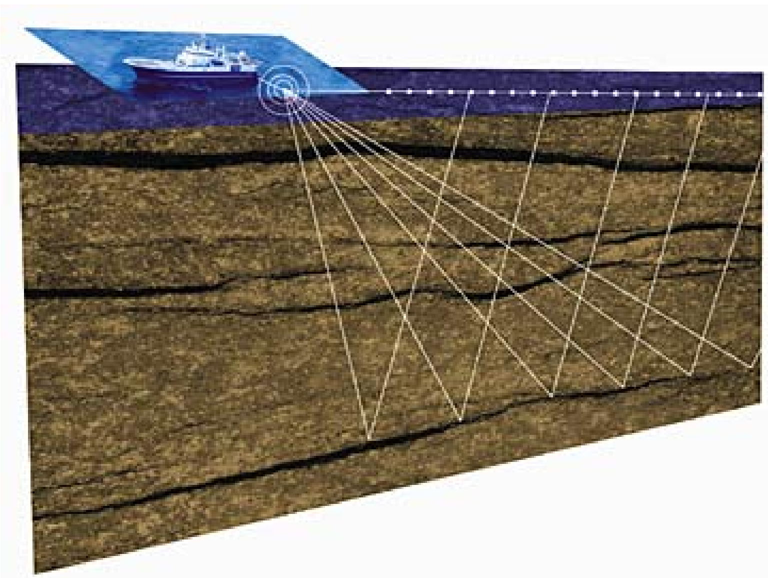
Blue circle, red ellipse: same offsets, velocity 50% high. Envelopes are *different*, so predicted depths of reflecting surface at *same* position will *disagree*.

Velocity via redundancy (3/6)

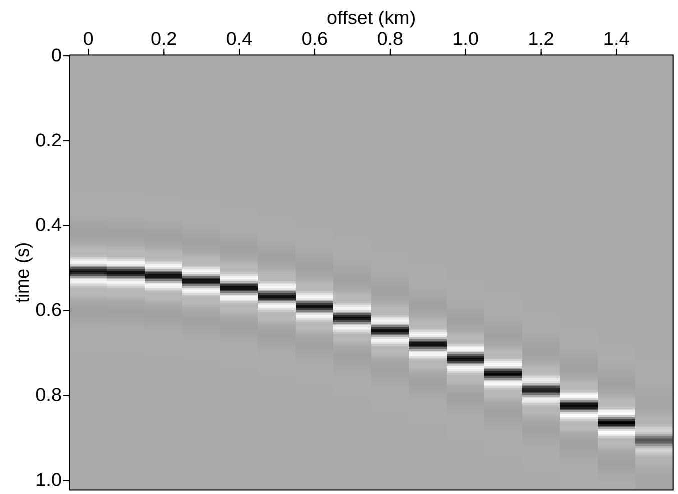
You can see this with the data directly:

- sort the data recordings into piles, each pile ("gather") sharing common offset;
- carry out migration for each common offset gather separately: place data value at time t on ellipse of diameter vt, sum together to get *common offset image* (just like we did for zero offset - a special case!);
- from each image, extract the part of the image that lies below a specific point on the surface - a "virtual well";
- if the velocity is correct, then all "virtual wells" will look same! To compare, put them together, one for each offset, to form a *common image gather*. Look at it!

Velocity via redundancy (4/6)

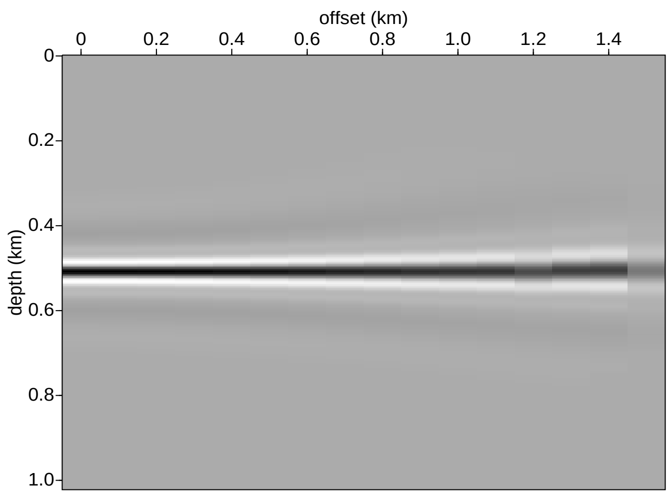


,

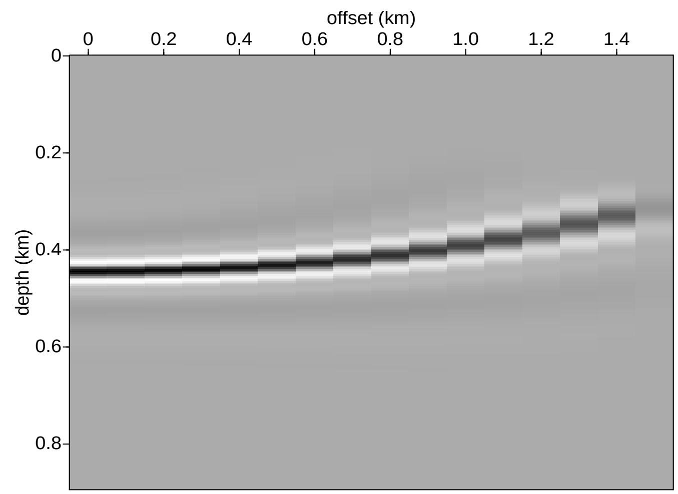


Synthetic experiments - data from different offsets

Velocity via redundancy (5/6)



,



“Virtual wells” displayed next to each other: Left, correct v; Right, 10% low.

“Correct velocity flattens common image gathers” - the practical principle for finding sound velocity from seismic data.

Velocity via redundancy (6/6)

Works with actual data!

Optimize This!

If you can measure "flatness" somehow, then you can turn image-gather-flattening task into optimization problem: *minimize a measure of un-flatness of image gathers*

An obvious possibility, après calculus: measure un-flatness by mean square of *derivative in offset of image gathers* $\equiv$ the *differential semblance* objective (function of assumed velocity, data).

Stolk & WWS *Inverse Problems* 03: differential semblance is *smooth in velocity and data jointly* and *is essentially the only quadratic form in the image gather with this property - so the only one for which you can use Newton's method!* [theory of oscillatory integral operators]

Generally: provided a strong enough dichotomy between length scale in velocity model and wavelengths in data, can show local uniqueness of global minimizer. In some special cases, *all local minimizers are global!* [multiscale asymptotics]

Some Obvious Questions

- what does all of this have to do with physics?
- waves obey the wave equation (acoustic, elastic, whatever) - where is the wave equation?
- what the heck is an image, really? what does it have to do with the mechanics of rocks? why is it separate, apparently, from “the velocity”?
- I thought you did inverse problems - is there an inverse problem somewhere in here?
- why not “image” the earth’s structure by modeling the data and fitting the model predictions to the data, say by least squares?

Where to Find Some Answers

- CAAM Colloquium 13 September
- CIME Short Course, `www.trip.caam.rice.edu` > Downloadable Materials
- *Mathematical Foundations of Reflection Seismology*, ditto

Credit where credit is due

Photos, drawings: Landmark Graphics, Schlumberger, Western Geco, Veritas DGC, Seismic Consultants, and WWS

Seismic data: Exxon Production Research Co., Shell International Research

Software used to produce data-derived graphics: Seismic Unix (Colorado School of Mines), The Rice Inversion Project (Rice University)

Money: sponsors of The Rice Inversion Project, NSF, DoE