Mathematics of Seismic Imaging
Part II

William W. Symes

PIMS, June 2005

Review: Normal Operators and imaging

If $d = F[v]r$, then

$$F[v]^*d = F[v]^*F[v]r$$

Recall: In the layered case, $F[v]^*F[v]$ is an operator which preserves wave front sets.

Whenever $F[v]^*F[v]$ *preserves wave front sets,* $F[v]^*$ *is an imaging operator.*

Review: Generalized Radon Representation

Assume (1) r (oscillatory) supported in simple geometric optics domain for v (smooth), (2) no forward scattering. Then

$$F[v]r(\mathbf{x}_r, t; \mathbf{x}_s) \simeq$$

$$\int dx \, \frac{2r(\mathbf{x})}{v^2(\mathbf{x})} a(\mathbf{x}, \mathbf{x}_r) a(\mathbf{x}, \mathbf{x}_s) \delta''(t - \tau(\mathbf{x}, \mathbf{x}_r) - \tau(\mathbf{x}, \mathbf{x}_s))$$

Similar representation of adjoint follows:

$$F[v]^* d(\mathbf{x}) = \int \int \int \, dx_r \, dx_s \, dt \, a(\mathbf{x}, \mathbf{x}_r) a(\mathbf{x}, \mathbf{x}_s) \delta''(t - \tau(\mathbf{x}; \mathbf{x}_s) - \tau(\mathbf{x}; \mathbf{x}_r)) d(\mathbf{x}_r, t; \mathbf{x}_s)$$

Beylkin, *J. Math. Phys.* 1985

For r supported in simple geometric optics domain,

- $WF(F[v]^*F[v]r) \subset WF(r)$
- if $d = \mathcal{F}[v] + F[v]r$ (data consistent with linearized model), then $F[v]^*(d - \mathcal{F}[v])$ is an image of r
- an operator $F[v]^\dagger$ exists for which $F[v]^\dagger(d - \mathcal{F}[v]) - r$ is *smoother* than r, under some constraints on r - an *inverse modulo smoothing operators* or *parametrix*.

Outline of proof

Express $F[v]^*F[v]$ as "Kirchhoff modeling" followed by "Kirchhoff migration"; (ii) introduce Fourier transform; (iii) approximate for large wavenumbers using stationary phase, leads to representation of $F[v]^*F[v]$ modulo smoothing error as *pseudodifferential operator* ("ΨDO"):

$$F[v]^*F[v]r(\mathbf{x}) \simeq p(\mathbf{x}, D)r(\mathbf{x}) \equiv \int d\xi \, p(\mathbf{x}, \boldsymbol{\xi})e^{i\mathbf{x}\cdot\boldsymbol{\xi}}\hat{r}(\boldsymbol{\xi})$$

in which $p \in C^\infty$, and for some m (the *order* of p), all multiindices α, β, and all compact $K \subset \mathbf{R}^n$, there exist constants $C_{\alpha,\beta,K} \geq 0$ for which

$$|D_{\mathbf{x}}^\alpha D_{\boldsymbol{\xi}}^\beta p(\mathbf{x}, \boldsymbol{\xi})| \leq C_{\alpha,\beta,K}(1 + |\boldsymbol{\xi}|)^{m-|\beta|}, \ \mathbf{x} \in K$$

Explicit computation of **symbol** p - for details, see Notes on Math Foundations.

Microlocal Propertyof ΨDOs

:

if $p(x,D)$ is a ΨDO, $u \in \mathcal{E}'(\mathbf{R}^n)$ then $WF(p(x,D)u) \subset WF(u)$.

Will prove this, from which imaging property of prestack Kirchhoff migration follows. First, a few other properties:

- differential operators are ΨDOs (easy - exercise)
- ΨDOs of order m form a module over $C^\infty(\mathbf{R}^n)$ (also easy)
- product of ΨDO order m, ΨDO order l = ΨDO order $\leq m+l$; adjoint of ΨDO order m is ΨDO order m (harder)

Complete accounts of theory, many apps: books of Duistermaat, Taylor, Nirenberg, Treves, Hörmander.

Proof of Microlocal Property

Suppose $(\mathbf{x}_0, \boldsymbol{\xi}_0) \notin WF(u)$, choose neighborhoods X, Ξ as in defn, with Ξ conic. Need to choose analogous nbhds for $P(x, D)u$. Pick $\delta > 0$ so that $B_{3\delta}(\mathbf{x}_0) \subset X$, set $X' = B_\delta(\mathbf{x}_0)$.

Similarly pick $0 < \epsilon < 1/3$ so that $B_{3\epsilon}(\boldsymbol{\xi}_0/|\boldsymbol{\xi}_0|) \subset \Xi$, and chose $\Xi' = \{\tau\boldsymbol{\xi} : \boldsymbol{\xi} \in B_\epsilon(\boldsymbol{\xi}_0/|\boldsymbol{\xi}_0|), \tau > 0\}$.

Need to choose $\phi \in \mathcal{E}'(X')$, estimate $\mathcal{F}(\phi P(\mathbf{x}, D)u)$. Choose $\psi \in \mathcal{E}(X)$ so that $\psi \equiv 1$ on $B_{2\delta}(\mathbf{x}_0)$.

NB: this implies that if $\mathbf{x} \in X'$, $\psi(\mathbf{y}) \neq 1$ then $|\mathbf{x} - \mathbf{y}| \geq \delta$.

Write $u = (1-\psi)u + \psi u$. Claim: $\phi P(\mathbf{x}, D)((1-\psi)u)$ is smooth.

$$\phi(\mathbf{x}) P(\mathbf{x}, D)((1-\psi)u))(\mathbf{x})$$

$$= \phi(\mathbf{x}) \int d\xi \, P(\mathbf{x}, \boldsymbol{\xi}) e^{i\mathbf{x}\cdot\boldsymbol{\xi}} \int dy \, (1-\psi(\mathbf{y}))u(\mathbf{y}) e^{-i\mathbf{y}\cdot\boldsymbol{\xi}}$$

$$= \int d\xi \int dy \, P(\mathbf{x}, \boldsymbol{\xi}) \phi(\mathbf{x})(1-\psi(\mathbf{y})) e^{i(\mathbf{x}-\mathbf{y})\cdot\boldsymbol{\xi}} u(\mathbf{y})$$

$$= \int d\xi \int dy \, (-\nabla_\xi^2)^M P(\mathbf{x}, \boldsymbol{\xi}) \phi(\mathbf{x})(1-\psi(\mathbf{y})) |\mathbf{x}-\mathbf{y}|^{-2M} e^{i(\mathbf{x}-\mathbf{y})\cdot\boldsymbol{\xi}} u(\mathbf{y})$$

using the identity

$$e^{i(\mathbf{x}-\mathbf{y})\cdot\boldsymbol{\xi}} = |\mathbf{x}-\mathbf{y}|^{-2}\left[-\nabla_\xi^2 e^{i(\mathbf{x}-\mathbf{y})\cdot\boldsymbol{\xi}}\right]$$

and integrating by parts $2M$ times in $\boldsymbol{\xi}$. This is permissible because $\phi(\mathbf{x})(1 - \psi(\mathbf{y})) \neq 0 \Rightarrow |\mathbf{x}-\mathbf{y}| > \delta$.

According to the definition of ΨDO,

$$|(-\nabla_\xi^2)^M P(\mathbf{x}, \boldsymbol{\xi})| \leq C|\boldsymbol{\xi}|^{m-2M}$$

For any K, the integral thus becomes absolutely convergent after K differentiations of the integrand, provided M is chosen large enough. Q.E.D. Claim.

This leaves us with $\phi P(\mathbf{x}, D)(\psi u)$. Pick $\eta \in \Xi'$ and w.l.o.g. scale $|\eta| = 1$.

Fourier transform:

$$\mathcal{F}(\phi P(\mathbf{x}, D)(\psi u))(\tau\eta) = \int dx \int d\xi \, P(\mathbf{x}, \boldsymbol{\xi})\phi(\mathbf{x})\hat{\psi u}(\xi) e^{i\mathbf{x}\cdot(\boldsymbol{\xi} - \tau\eta)}$$

Introduce $\tau\theta = \xi$, and rewrite this as

$$= \tau^n \int dx \int d\theta \, P(\mathbf{x}, \tau\theta)\phi(\mathbf{x})\hat{\psi u}(\tau\theta) e^{i\tau\mathbf{x}\cdot(\theta - \eta)}$$

Divide the domain of the inner integral into $\{\theta : |\theta - \eta| > \epsilon\}$ and its complement. Use

$$-\nabla_x^2 e^{i\tau\mathbf{x}\cdot(\theta-\eta)} = \tau^2 |\theta - \eta|^2 e^{i\tau\mathbf{x}\cdot(\theta-\eta)}$$

Integrate by parts $2M$ times to estimate the first integral:

$$\tau^{n-2M} \left| \int dx \int_{|\theta-\eta|>\epsilon} d\theta \, (-\nabla_x^2)^M [P(\mathbf{x}, \tau\theta)\phi(\mathbf{x})] \hat{\psi u}(\tau\theta) \right.$$

$$\left. \times \, |\theta - \eta|^{-2M} e^{i\tau\mathbf{x}\cdot(\theta-\eta)} \right|$$

$$\leq C\tau^{n+m-2M}$$

m being the order of P. Thus the first integral is rapidly decreasing in τ.

For the second integral, note that $|\theta - \eta| \leq \epsilon \Rightarrow \theta \in \Xi$, per the defn of Ξ'. Since $X \times \Xi$ is disjoint from the wavefront set of u, for a sequence of constants C_N, $|\hat{\psi u}(\tau\theta)| \leq C_N \tau^{-N}$ uniformly for θ in the (compact) domain of integration, whence the second integral is also rapidly decreasing in τ. **Q. E. D.**

And that's why Kirchhoff migration works, at least in the simple geometric optics regime.

Inversion aperture

$\Gamma[v] \subset \mathbf{R}^3 \times \mathbf{R}^3 - 0$:

if $WF(r) \subset \Gamma[v]$, then $WF(F[v]^*F[v]r) = WF(r)$ and $F[v]^*F[v]$ "acts invertible". [construction of $\Gamma[v]$ - later!]

Beylkin: with proper choice of amplitude $b(\mathbf{x}_r, t; \mathbf{x}_s)$, the modified Kirchhoff migration operator

$$F[v]^\dagger d(\mathbf{x}) = \int\int\int dx_r\, dx_s\, dt\, b(\mathbf{x}_r, t; \mathbf{x}_s)\delta(t - \tau(\mathbf{x}; \mathbf{x}_s) - \tau(\mathbf{x}; \mathbf{x}_r))d(\mathbf{x}_r, t; \mathbf{x}_s)$$

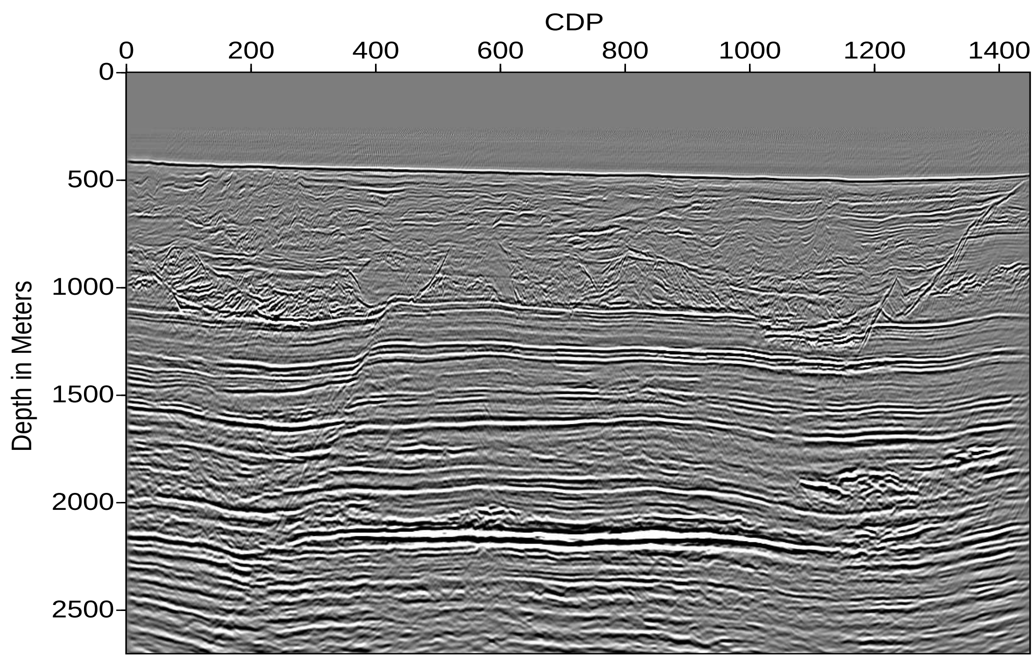
yields $F[v]^\dagger F[v] r \simeq r$ if $WF(r) \subset \Gamma[v]$

For details of Beylkin construction: Beylkin, 1985; Miller et al 1989; Bleistein, Cohen, and Stockwell 2000; WWS Math Foundations, MGSS notes 1998. All components are by-products of eikonal solution.

aka: Generalized Radon Transform ("GRT") inversion, Ray-Born inversion, migration/inversion, true amplitude migration,...

Many extensions, eg. to elasticity: Bleistein, Burridge, deHoop, Lambaré,...

Apparent limitation: construction relies on simple geometric optics (no multipathing) - is this really necessary?



Example of GRT Inversion (application of $F[v]^\dagger$): K. Araya (1995), "2.5D" inversion of marine streamer data from Gulf of Mexico: 500 source positions, 120 receiver channels, 750 Mb.

Why Beylkin isn't enough

The theory developed by Beylkin and others cannot be the end of the story:

- The "single ray" hypotheses generally fails in the presence of strong refraction.
- B. White, "The Stochastic Caustic" (1982): For "random but smooth" $v(\mathbf{x})$ with variance σ, points at distance $O(\sigma^{-2/3})$ from source have more than one ray connecting to source, with probability 1 - *multipathing* associated with formation of *caustics* = ray envelopes.
- Formation of caustics invalidates asymptotic analysis on which Beylkin result is based.

Why it matters

- Strong refraction leading to multipathing and caustic formation typical of salt (4-5 km/s) intrusion into sedimentary rock (2-3 km/s) (eg. Gulf of Mexico), also chalk tectonics in North Sea and elsewhere - some of the most promising petroleum provinces!

Escape from simplicity - the Canonical Relation

How do we get away from "simple geometric optics", SSR, DSR,... - all violated in sufficiently complex (and realistic) models? Rakesh *Comm. PDE* 1988, Nolan *Comm. PDE* 1997: global description of $F_\delta[v]$ as mapping reflectors $\mapsto$ reflections.

$Y = \{\mathbf{x}_s, t, \mathbf{x}_r\}$ (time $\times$ set of source-receiver pairs) submfd of $\mathbf{R}^7$ of dim. ≤ 5, $\Pi : T^*(\mathbf{R}^7) \rightarrow T^*Y$ the natural projection

$\mathrm{supp}\, r \subset X \subset \mathbf{R}^3$

Canonical relation $C_{F_\delta[v]} \subset T^*(X) - \{\mathbf{0}\} \times T^*(Y) - \{\mathbf{0}\}$ describes singularity mapping properties of F:

$$(\mathbf{x}, \xi, \mathbf{y}, \eta) \in C_{F_\delta[v]} \Leftrightarrow$$

$$\text{for some } u \in \mathcal{E}'(X),\ (\mathbf{x}, \xi) \in WF(u),\ \text{and } (\mathbf{y}, \eta) \in WF(Fu)$$

Rays Construction of the Relation

Rays of geometric optics: solutions of Hamiltonian system

$$\frac{d\mathbf{X}}{dt} = \nabla_{\mathbf{\Xi}} H(\mathbf{X}, \mathbf{\Xi}),\ \frac{d\mathbf{\Xi}}{dt} = -\nabla_{\mathbf{X}} H(\mathbf{X}, \mathbf{\Xi})$$

with $H(\mathbf{X}, \mathbf{\Xi}) = \frac{1}{2}[1 - v^2(\mathbf{X})|\mathbf{\Xi}|^2] = 0$ (*null bicharacteristics*).

Characterization of C_F:

$$((\mathbf{x}, \xi), (\mathbf{x}_s, t, \mathbf{x}_r, \xi_{\mathbf{s}}, \tau, \xi_{\mathbf{r}})) \in C_{F_\delta[v]} \subset T^*(X) - \{\mathbf{0}\} \times T^*(Y) - \{\mathbf{0}\}$$

$\Leftrightarrow$ there are *rays of geometric optics* $(\mathbf{X}_s, \mathbf{\Xi}_s)$, $(\mathbf{X}_r, \mathbf{\Xi}_r)$ and times t_s, t_r so that

$$\Pi(\mathbf{X}_s(0), t, \mathbf{X}_r(t), \mathbf{\Xi}_s(0), \tau, \mathbf{\Xi}_r(t)) = (\mathbf{x}_s, t, \mathbf{x}_r, \xi_s, \tau, \xi_r),$$

$$\mathbf{X}_s(t_s) = \mathbf{X}_r(t - t_r) = \mathbf{x},\ t_s + t_r = t,\ \mathbf{\Xi}_s(t_s) - \mathbf{\Xi}_r(t - t_r) || \xi$$

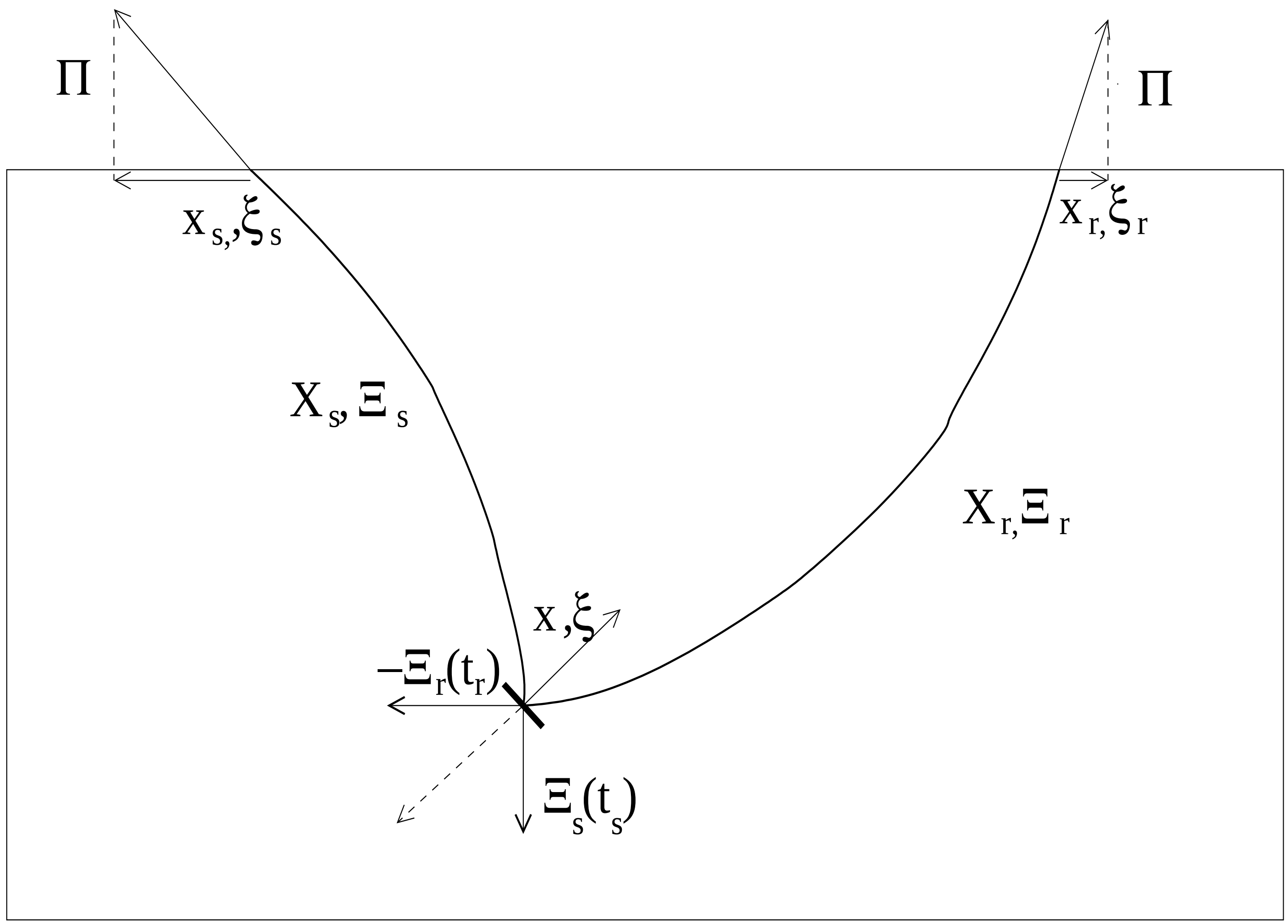
Since $\mathbf{\Xi}_s(t_s)$, $-\mathbf{\Xi}_r(t-t_r)$ have same length, sum = bisector $\Rightarrow$ *velocity vectors of incident ray from source and reflected ray from receiver (traced backwards in time) make equal angles with reflector at* $\mathbf{x}$ *with normal* ξ.

Upshot: canonical relation of $F_\delta[v]$ simply enforces the equal-angles law of reflection.

Further, *rays carry high-frequency energy*, in exactly the fashion that seismologists imagine.

Finally, *Rakesh's characterization of* C_F *is global:* no assumptions about ray geometry, other than no forward scattering and no grazing incidence on the acquisition surface Y, are needed.

The Picture



Proof: Plan of attack

Recall that

$$F[v]r(\mathbf{x}_r, t; \mathbf{x}_s) = \frac{\partial \delta u}{\partial t}(\mathbf{x}_r, t; \mathbf{x}_s)$$

where

$$\frac{1}{v^2}\frac{\partial^2 \delta u}{\partial t^2} - \nabla^2 \delta u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} r$$

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = \delta(t)\delta(\mathbf{x} - \mathbf{x}_s)$$

and $u, \delta u \equiv 0,\ t < 0$.

Need to understand (1) $WF(u)$, (2) relation $WF(r) \leftrightarrow WF(ru)$, (3) WF of soln of WE in terms of WF of RHS (this also gives (1)!).

Singularities of the Acoustic Potential Field

Main tool: **Propagation of Singularities** theorem of Hörmander (1970).

Given symbol $p(\mathbf{x},\boldsymbol{\xi})$, order m, with asymptotic expansion, define *null bicharateristics* (= rays) as solutions $(\mathbf{x}(t),\boldsymbol{\xi}(t))$ of Hamiltonian system

$$\frac{d\mathbf{x}}{dt}=\frac{\partial p}{\partial \boldsymbol{\xi}}(\mathbf{x},\boldsymbol{\xi}),\ \ \frac{d\boldsymbol{\xi}}{dt}=-\frac{\partial p}{\partial \mathbf{x}}(\mathbf{x},\boldsymbol{\xi})$$

with $p(\mathbf{x}(t),\boldsymbol{\xi}(t))\equiv 0$.

Theorem: Suppose $p(\mathbf{x},D)u=f$, and suppose that for $t_0\le t\le t_1$, $(\mathbf{x}(t),\boldsymbol{\xi}(t))\notin WF(f)$. Then either $\{(\mathbf{x}(t),\boldsymbol{\xi}(t)) : t_0\le t\le t_1\}\subset WF(u)$ or $\{(\mathbf{x}(t),\boldsymbol{\xi}(t)) : t_0\le t\le t_1\}\subset T^*(\mathbf{R}^n)-WF(u)$.

Source to Field

RHS of wave equation for u = δ function in $\mathbf{x}, t$. WF set = $\{(\mathbf{x}, t, \boldsymbol{\xi}, \tau) : \mathbf{x} = \mathbf{x}_s, t = 0\}$ - i.e. no restriction on covector part.

$\Rightarrow$ $(\mathbf{x}, t, \boldsymbol{\xi}, \tau) \in WF(u)$ iff a ray starting at $(\mathbf{x}_s, 0)$ passes over $(\mathbf{x}, t)$ - i.e. $(\mathbf{x}, t)$ lies on the "light cone" with vertex at $(\mathbf{x}_x, 0)$. Symbol for wave op is $p(\mathbf{x}, t, \boldsymbol{\xi}, \tau) = \frac{1}{2}(\tau^2 - v^2(\mathbf{x})|\boldsymbol{\xi}|^2)$, so Hamilton's equations for null bicharacteristics are

$$\frac{d\mathbf{X}}{dt} = -v^2(\mathbf{X})\boldsymbol{\Xi}, \;\; \frac{d\boldsymbol{\Xi}}{dt} = \nabla \log v(\mathbf{X})$$

Thus $\boldsymbol{\xi}$ is proportional to velocity vector of ray.

[$(\boldsymbol{\xi}, \tau)$ *normal* to light cone.]

Singularities of Products

To compute $WF(ru)$ from $WF(r)$ and $WF(u)$, use *Gabor calculus* (Duistermaat, Ch. 1)

Here r is really $(r \circ \pi)u$, where $\pi(\mathbf{x}, t) = \mathbf{x}$. Choose bump function ϕ localized near $(\mathbf{x}, t)$

$$\phi \widehat{(r \circ \pi)} u(\boldsymbol{\xi}, \tau) = \int d\boldsymbol{\xi}' \, d\tau' \widehat{\phi r}(\boldsymbol{\xi}') \delta(\tau') \widehat{u}(\boldsymbol{\xi} - \boldsymbol{\xi}', \tau - \tau')$$

$$= \int d\boldsymbol{\xi}' \widehat{\phi r}(\boldsymbol{\xi}') \widehat{u}(\boldsymbol{\xi} - \boldsymbol{\xi}', \tau)$$

This will decay rapidly as $|(\boldsymbol{\xi},\tau)| \to \infty$ unless (i) you can find $(\mathbf{x}',\boldsymbol{\xi}') \in WF(r)$ so that $\mathbf{x},\mathbf{x}' \in \pi(\mathrm{supp}\phi)$, $\boldsymbol{\xi}-\boldsymbol{\xi}' \in WF(u)$, i.e. $(\boldsymbol{\xi},\tau) \in WF(r\circ\pi)+WF(u)$, or (ii) $\boldsymbol{\xi} \in WF(r)$ or $(\boldsymbol{\xi},\tau) \in WF(u)$.

Possibility (ii) will not contribute, so effectively

$$WF((r\circ\pi)u) = \{(\mathbf{x},t_s,\boldsymbol{\xi}+\boldsymbol{\Xi}_s(t_s),\cdot) : (\mathbf{x},\boldsymbol{\xi}) \in WF(r),\ \mathbf{x}=\mathbf{X}_s(t_s)$$

for a ray $(\mathbf{X}_s,\boldsymbol{\Xi}_s)$ with $\mathbf{X}_s(0)=x_s$, some τ.

Wavefront set of Scattered Field

Once again use propagation of singularities: $(\mathbf{x}_r, t, \boldsymbol{\xi}_r, \tau_r) \in WF(\delta u) \Leftrightarrow$ on ray $(\mathbf{X}_r, \boldsymbol{\Xi}_r)$ passing through $WF(ru)$. Can argue that time of intersection is $t - t_r < t$.

That is,

$$\mathbf{X}_r(t) = \mathbf{x}_r, \mathbf{X}_r(t - t_r) = \mathbf{X}_s(t_s) = x,$$

$t = t_r + t_s$, and

$$\boldsymbol{\Xi}_r(t_s) = \boldsymbol{\xi} + \boldsymbol{\Xi}_s(t_s)$$

for some $\boldsymbol{\xi} \in WF(r)$. **Q. E. D.**

Rakesh's Thesis

Rakesh also showed that $F[v]$ is a *Fourier Integral Operator* = class of oscillatory integral operators, introduced by Hörmander and others in the '70s to describe the solutions of nonelliptic PDEs.

Phases and amplitudes of FIOs satisfy certain restrictive conditions. Canonical relations have geometric description similar to that of $F[v]$. Adjoint of FIO is FIO with inverse canonical relation.

ΨDOs are special FIOs, as are GRTs.

Composition of FIOs does *not* yield an FIO in general. Beylkin had shown that $F[v]^* F[v]$ is FIO (ΨDO, actually) under simple ray geometry hypothesis - but this is only sufficient. Rakesh noted that this follows from general results of Hörmander: *simple ray geometry* $\Leftrightarrow$ *canonical relation is graph of ext. deriv. of phase function*.

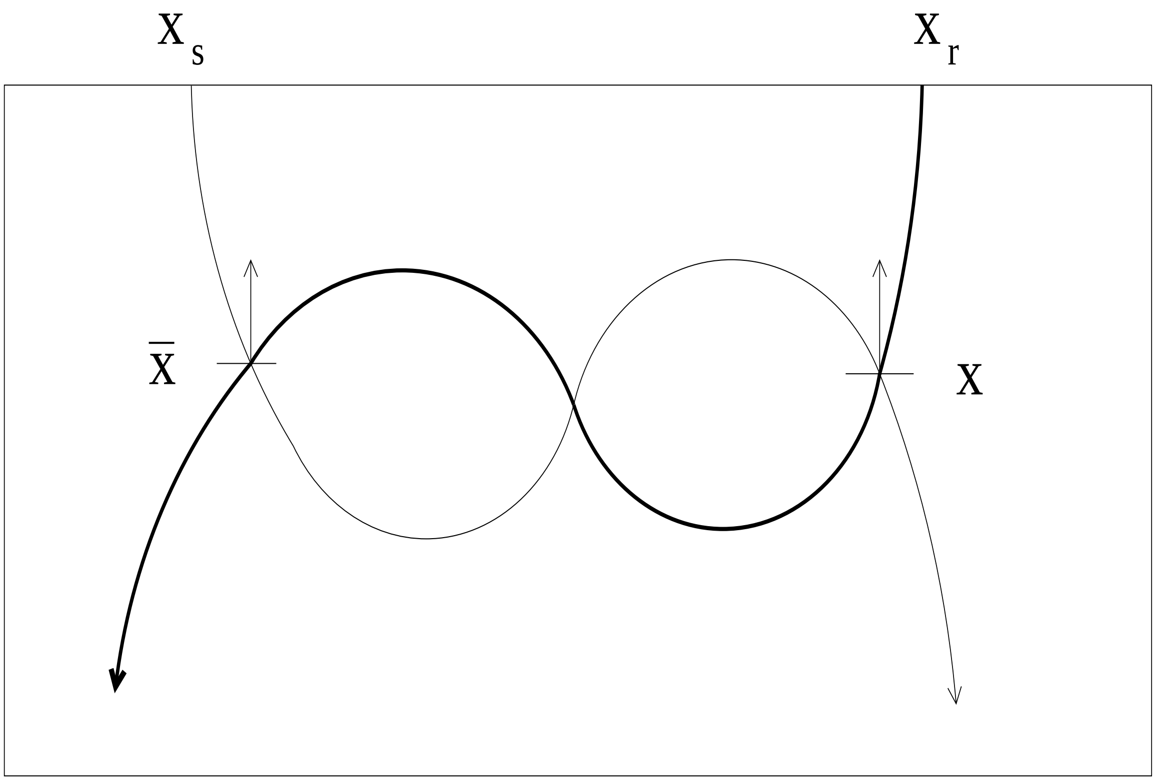
The Shell Guys and TIC

Smit, tenKroode and Verdel (1998): provided that

- source, receiver positions $(\mathbf{x}_s, \mathbf{x}_r)$ form an *open* 4D manifold ("complete coverage" - all source, receiver positions at least locally), and
- the *Traveltime Injectivity Condition* ("TIC") holds: $C_{F[v]}^{-1} \subset T^*Y - \{0\} \times T^*X - \{0\}$ is a *function* - that is, initial data for source and receiver rays and total travel time together determine reflector uniquely.

then $F[v]^*F[v]$ is ΨDO $\Rightarrow$ application of $F[v]^*$ produces image, and $F[v]^*F[v]$ has microlocal parametrix ("asymptotic inversion").

TIC is a nontrivial constraint!



Symmetric waveguide: time ($\mathbf{x}_s \rightarrow \bar{\mathbf{x}} \rightarrow \mathbf{x}_r$) same as time ($\mathbf{x}_s \rightarrow \mathbf{x} \rightarrow \mathbf{x}_r$), so TIC fails.

Stolk's Thesis

Stolk (2000): under "complete coverage" hypothesis, v for which $F[v]^*F[v]$ is = [ΨDO + rel. smoothing op] form open, dense set (without assuming TIC!).

NB: application of $F[v]^*$ involves accounting for *all* rays connecting source and receiver with reflectors. Standard practice still attempts imaging with single choice of ray pair (shortest time, max energy,...). Operto et al (2000) give nice illustration that all rays must be included.

Nolan's Thesis

Limitation of Smit-tenKroode-Verdel: most idealized data acquisition geometries violate "complete coverage": for example, idealized marine streamer geometry (src-recvr submfd is 3D)

Nolan (1997): result remains true without "complete coverage" condition: requires only TIC plus addl condition so that projection $C_{F[v]} \to T^*Y$ is embedding - but examples violating TIC are much easier to construct when source-receiver submfd has positive codim.

Sinister Implication: When data is just a single gather - common shot, common offset - image may contain *artifacts*, i.e. spurious reflectors not present in model.

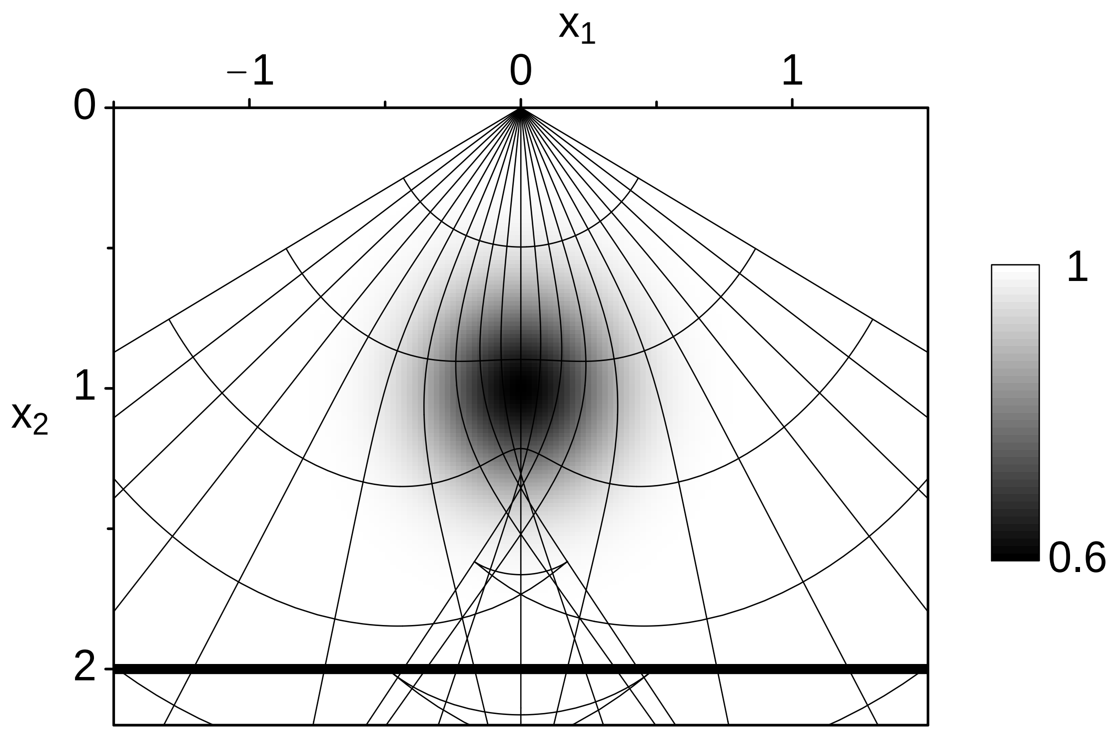
Horrible Example I

Synthetic 2D Example (Stolk and WWS, 2001 - *Geophysics* 2004)

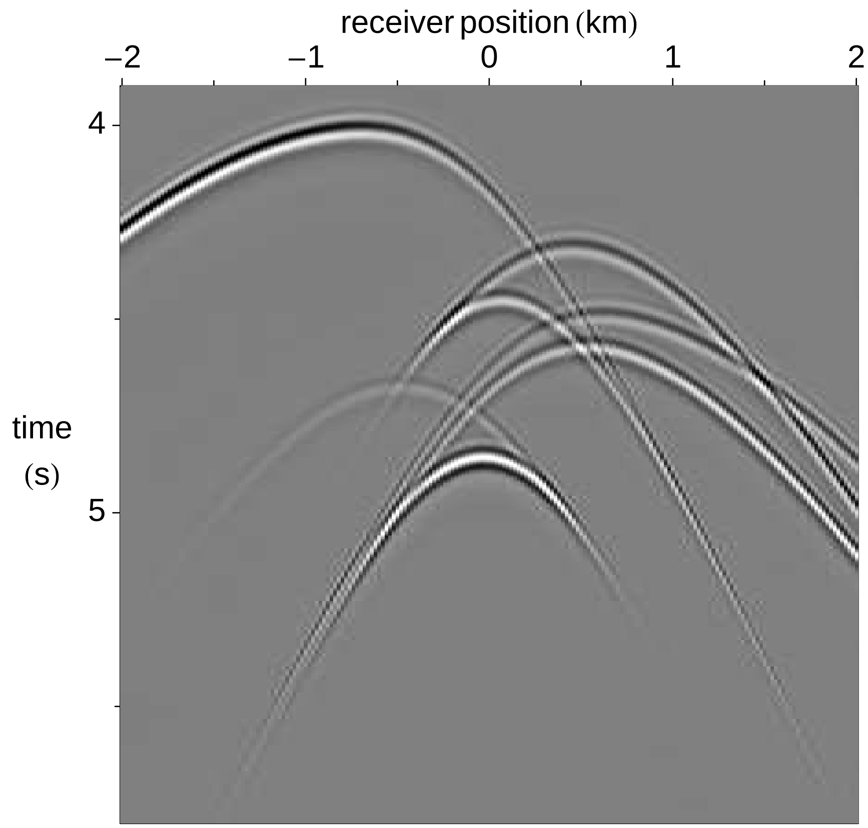
Strongly refracting acoustic lens (v) over horizontal reflector (r), $d = F[v]r$.

(i) for open source-receiver set, $F[v]^*d$ = good image of reflector - within limits of finite frequency implied by numerical method, $F[v]^*F[v]$ acts like ΨDO;

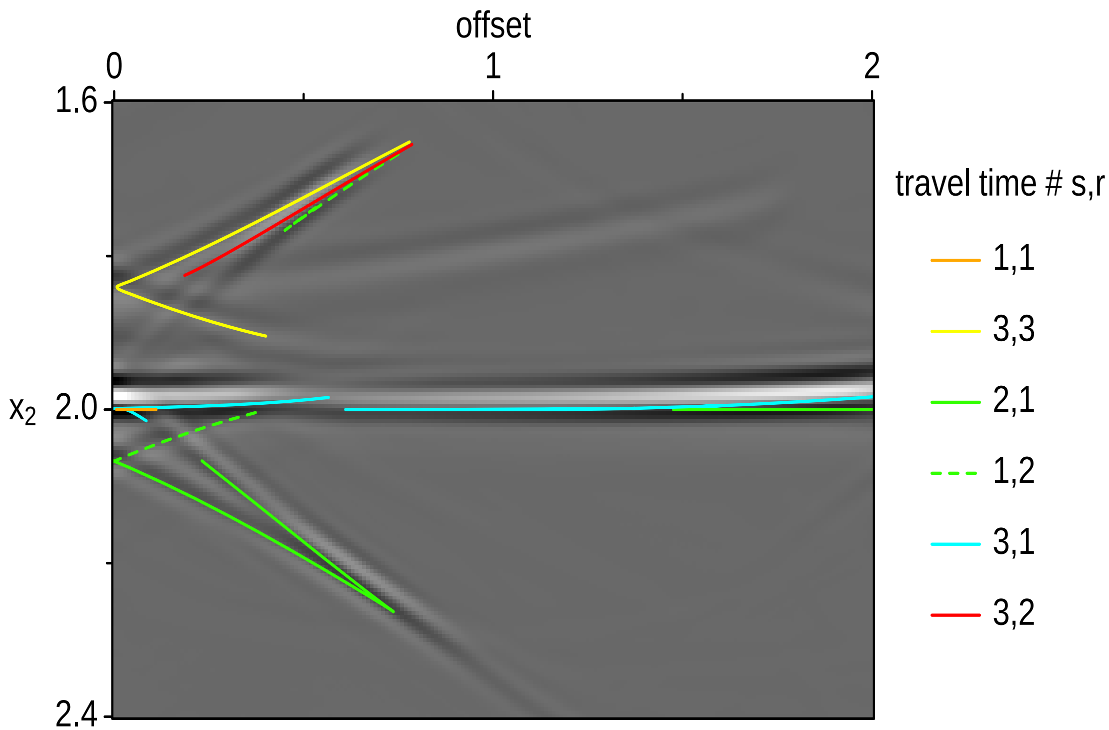
(ii) for *common offset* submfd (codim 1), TIC is violated and $WF(F[v]^*d)$ is larger than $WF(r)$.



Gaussian lens velocity model, flat reflector at depth 2 km, overlain with rays and wavefronts (Stolk & S. 2002 SEG).



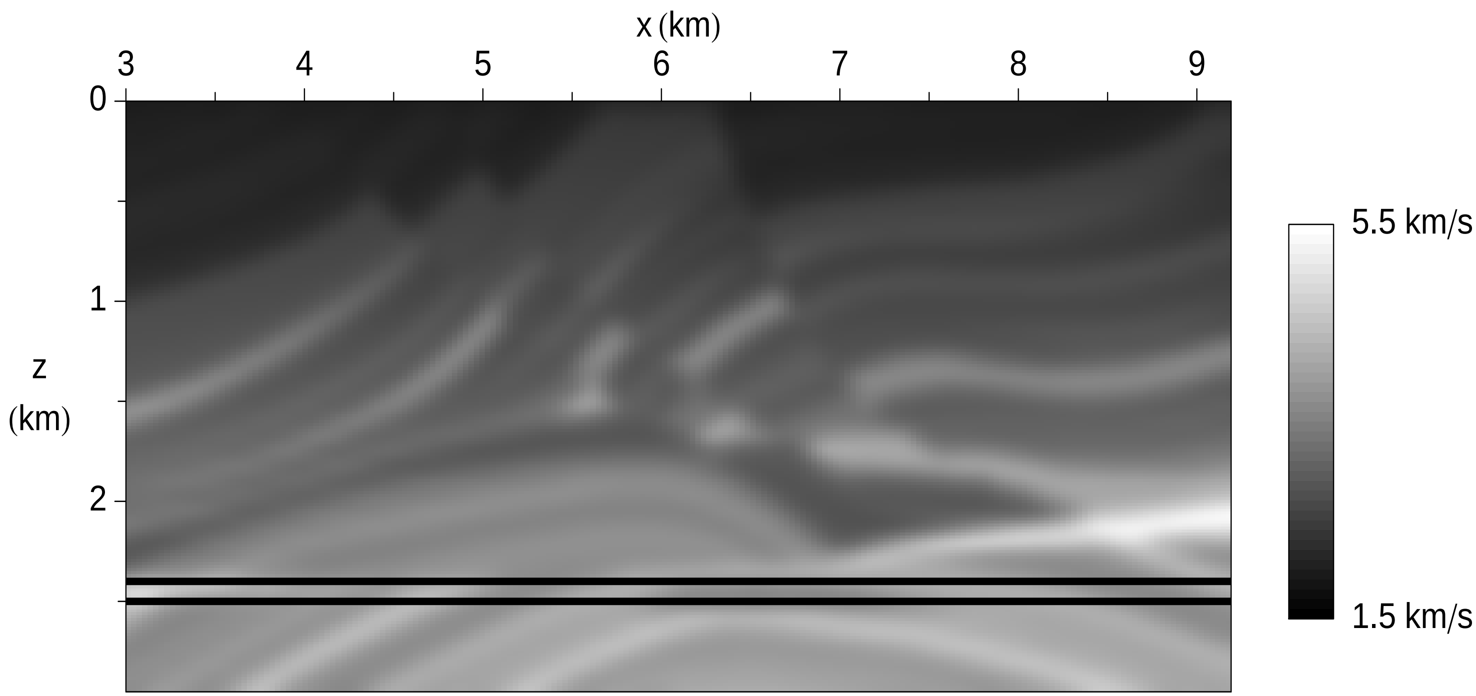
Typical shot gather - lots of arrivals

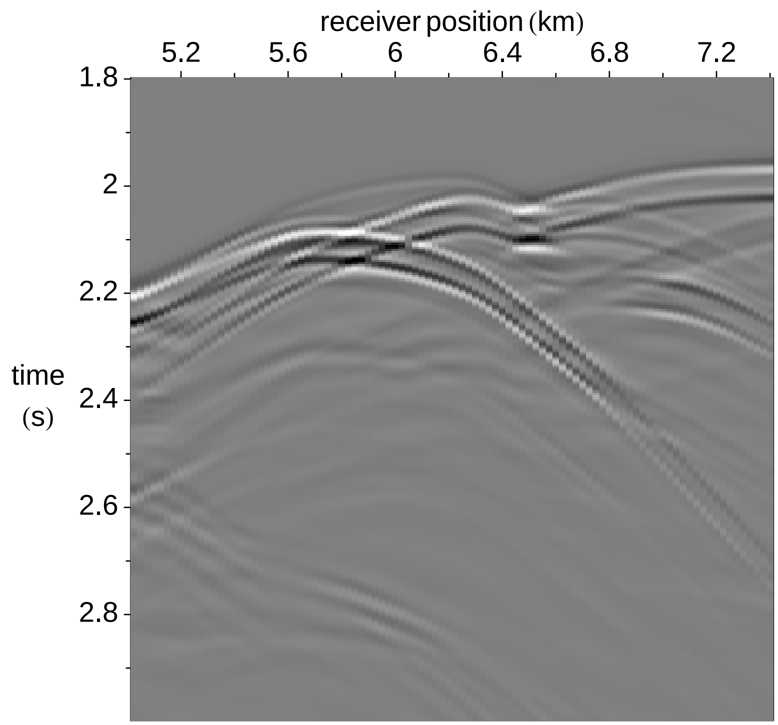


Offset common image gather (slice of $\tilde{F}[v]^*d$), with kinematically predicted reflector images overlain.

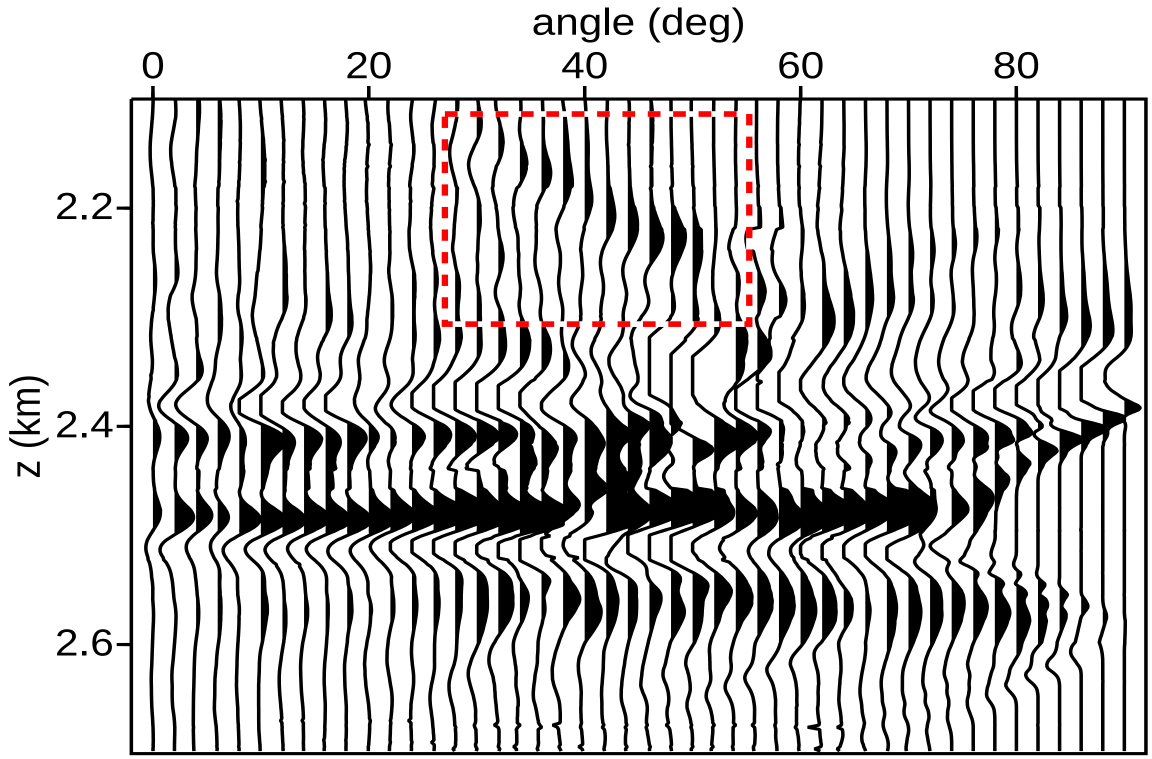
Horrible Example II

Stolk and Symes, *Geophysics* 2004: "Marmouflat" model = smoothed Marmousi (Versteeg & Grau 1991) with two flat reflectors.

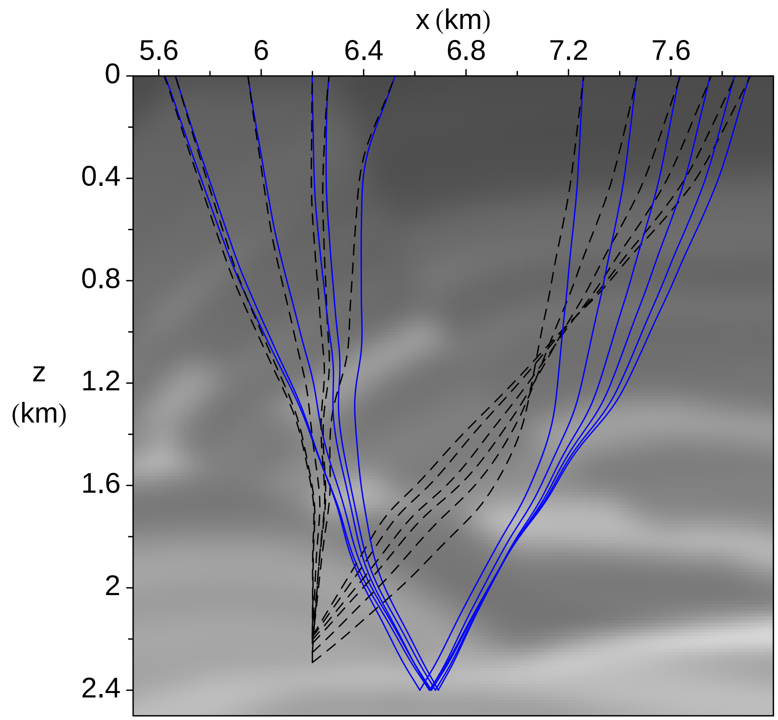




Typical shot gather: much evidence of multipathing, caustic formation.



Typical common scattering angle image gather: note nonflat event in box - results from data event migrating along *wrong ray pair*.



Blue rays = energy path producing data event. Black rays: energy path for migration, resulting in displaced, angle-dependent image artifact.

What it all means

Note that a gather scheme makes the scattering operator block-diagonal: for example with data sorted into common offset gathers $h = (x_r - x_s)/2$,

$$F[v] = [F_{h_1}[v], ..., F_{h_N}[v]]^T, \;\; d = [d_{h_1}, ..., d_{h_N}]^T$$

Thus $F[v]^* d = \sum_i F_{h_i}[v]^* d_{h_i}$. Otherwise put: to form image, **migrate** ith gather (apply migration operator $F_{h_i}[v]^*$, then **stack** individual migrated images (hence *prestack migration*).

Horrible Examples show that individual migrated images may contain nonphysical apparent reflectors (artifacts).

Smit-tenKroode-Verdel, Nolan, Stolk: if TIC holds, then these artifacts are not stationary with respect to the gather parameter, hence *stack out* (interfere destructively) in final image.