Nonlinear Inverse Scattering and Velocity Analysis

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Global vs. local, linear vs. nonlinear

- Contemporary inversion of active source reflection data = migration + velocity analysis ("MVA") - give approximate solution of linearized ("Born", single scattering) inverse problem - cf. Stolk, this session.
- MVA limited by Born assumption (no multiple scattering!) but can create *large* velocity updates and move events many wavelengths to correct locations - *linear, global*.
- Nonlinear least squares inversion ("NLS") can include all features of wave physics (multiple scattering, elasticity, anelasticity,...), but needs initial velocity "accurate within a wavelength" to succeed - *nonlinear, local*

How to combine *large update* property of MVA with *nonlinear physics* of NLS to produce a *nonlinear, global* approach to seismic inverse scattering?

Shot-geophone prestack migration

Claerbout (1971): given velocity field $v(x, z)$ (ref. model), compute:

- *source wavefield* $S(x_s; z, \bar{x}_s, t)$ - continue the source at (z_s, x_s) to the "sunken source" at $(z, \bar{x}_s)$, $z > 0$ (i.e. solve wave eqn with *source* as source);
- *receiver wavefield* $R(x_s; z, \bar{x}_r, t)$ - continue recorded data for the source at (z_s, x_s) to the "sunken receiver" at $(z, \bar{x}_r)$ (solve wave eqn with *data* as source);
- *image volume* $\bar{I}(z, \bar{x}_r, \bar{x}_s)$ - time cross-correlate S and R at zero lag, same depth, sum over sources:

$$\bar{I}(z, \bar{x}_s, \bar{x}_r) = \int dx_s \int dt R(x_s, z, \bar{x}_r, t) S(x_s, z, \bar{x}_s, t)$$

- *Claerbout's imaging condition*: extract *image* $I(z, x)$ where sunken source and receiver coincide, $\bar{x}_r = \bar{x}_s = x$ ("zero offset"): $I(z, x) = \bar{I}(z, x, x)$ - related to ordinary Born inversion.

Objective velocity analysis

Based on *Claerbout's focusing principle:* Velocity correct $\Rightarrow$ image volume $\bar{I}[v]$ *focuses* at zero (subsurface) half offset $h = (\bar{x}_r - \bar{x}_s)/2$, i.e. exhibits essentially no energy at $|h| > 0$.

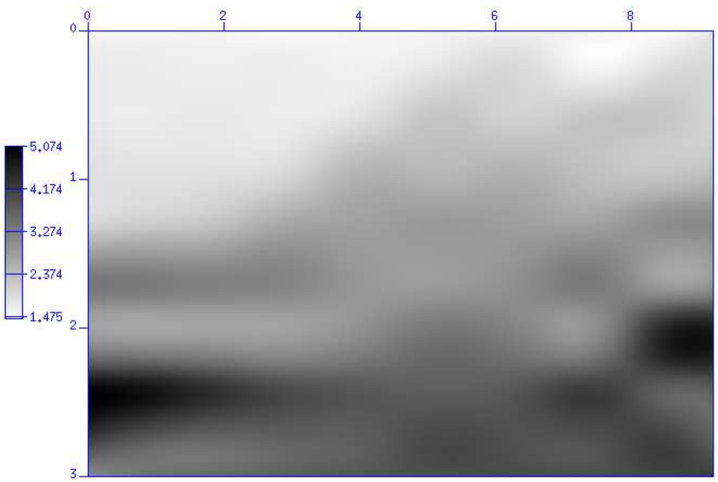
How to measure focusing at $h = 0$: multiply by h! If product is *big* RMS, image is unfocused, velocity is *wrong*. If product is *small* RMS, image is focused, velocity is *right*.

focusing as an optimization problem: minimize $\|h\bar{I}[v]\|^2$ over v ($\|\cdot\|$ = L^2 norm).

Stolk & S., IP 2003: this is essentially the only nontrivial quadratic form in image volume which (a) varies smoothly as function of v and d, and (b) vanishes for focused $\bar{I}$.

First results: Shen et al, SEG 2003. Following example also due to P. Shen.

Objective velocity analysis in Marmousi: initial velocity, image



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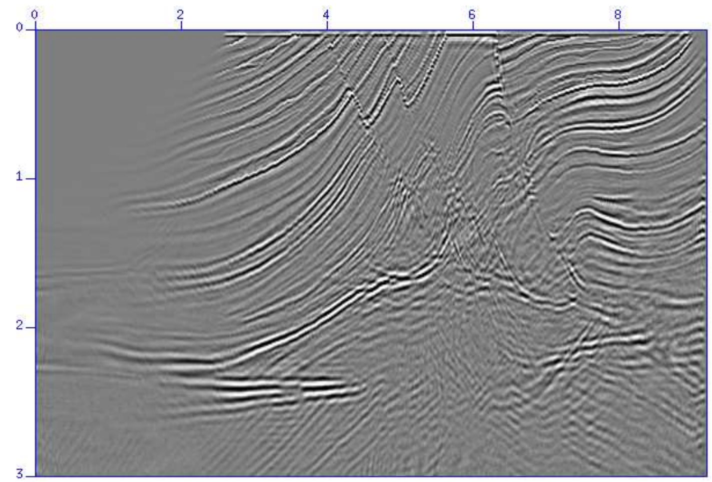
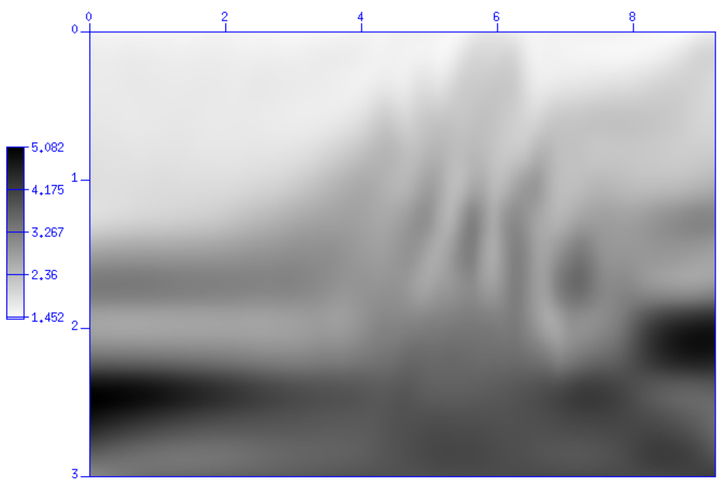


Left: Initial velocity **Right:** image ($h = 0$ section from volume)

Objective velocity analysis in Marmousi: final velocity, image



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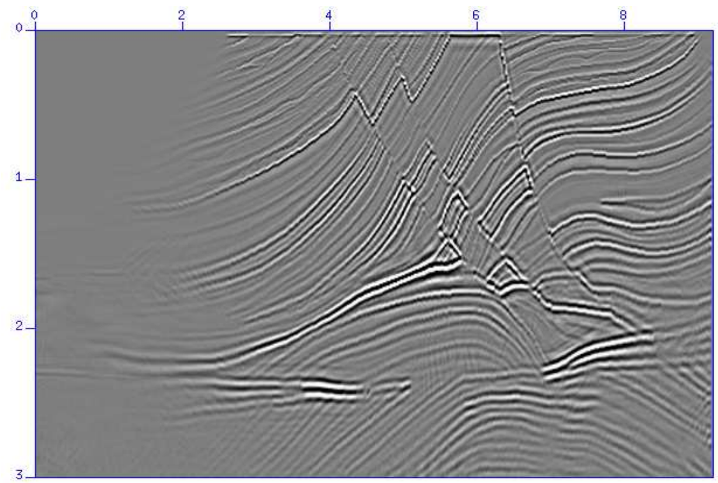
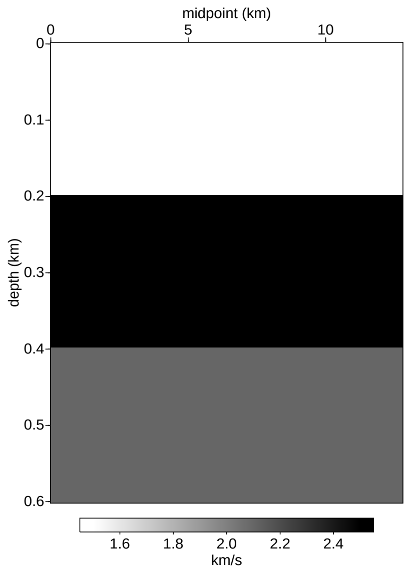


Left: Final estimated velocity **Right:** image ($h = 0$ section from image volume) after 47 iterations of LMBFGS. Pretty good image - but input is Born data!!!

MVA does not account for multiple reflections



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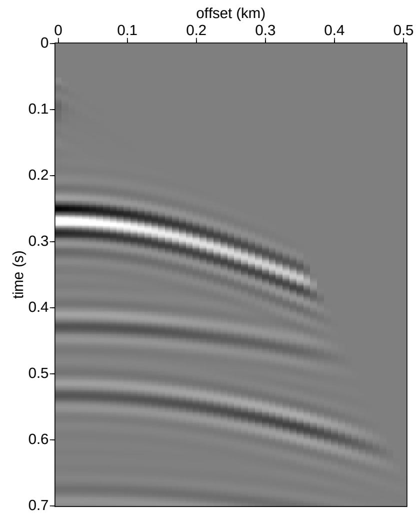

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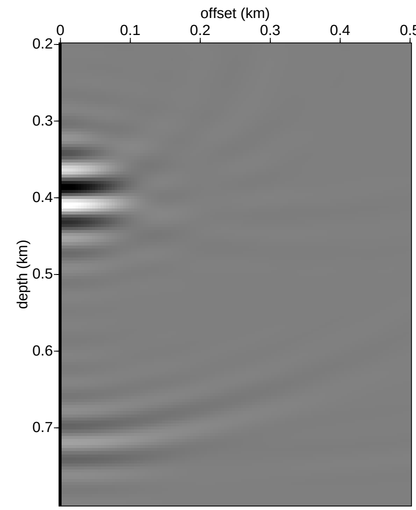


Left: Three layer model **Center:** Data - source wavelet = 4-10-30-40 Hz bandpass. Free surface multiple is about same size as second primary. **Right:** S-G Migration at good v - note focused primary, defocused image of surface multiple.

Nonlinear Least Squares Inversion

Modeling operator give by $v \to D$, where $D(x_s; x_r, t) = P(x_s; z, x, t)_{z=z_r, x=x_r}$

$$u(z,x)\frac{\partial^2 P}{\partial t^2}(x_s; z, x, t) - \nabla^2_{z,x} P(x_s; z, x, t) = w(t)\delta(z - z_s)\delta(x - x_s)$$

and $u = v^{-2}$ (*square slowness*).

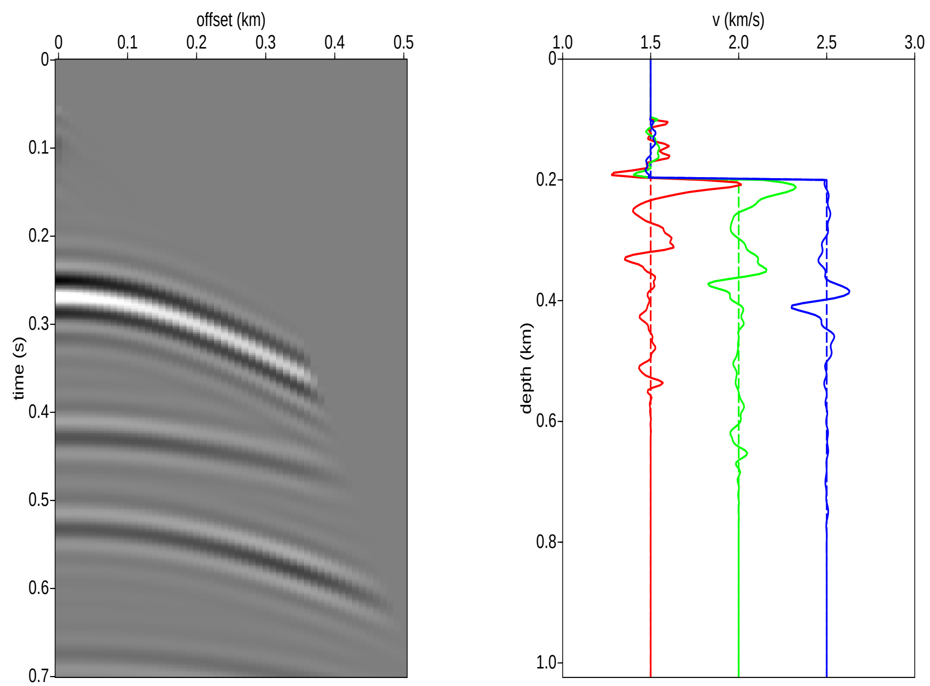
Least squares inversion: given data D^{obs}, adjust v (or u) so that predicted data $D[u]$ fits observed data as well as possible:

$$\mathrm{minimize}_u \, \|D[u] - D^{\mathrm{obs}}\|^2$$

(some regularization usually a good idea).

Upshot of much work from 80's on: LS inversion (i) handles multiples, i.e. does not produce artifact images due to multiples, but (ii) requires a very good initial estimate - *domain of attraction* of global minimizer has small "measure".

NLS does not make large velocity updates



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Left: Data from three layer model **Right:** Inversions (solid lines) from three initial v's (dashed lines). 30-40 its of LMBFGS, redn in gradient length by 10^{-2}.

Nonlinearizing MVA, step 1

Get from linear to nonlinear in two steps:

(1) recognize that shot-geophone prestack migration op $d \rightarrow \bar{I}$ is *adjoint* of *extended Born modeling op* $\bar{I} \rightarrow d$; modeling op given by $d(x_s; x_r, t) = p(x_s; z, x, t)_{z=z_r, x=x_r}$ where z_r = recvr depth and

$$\left(\frac{1}{v(z,x)^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2_{z,x} p\right)(x_s; z, x, t) = \int d\bar{x} S(x_s; z, \bar{x}, t) \bar{I}(z, x, \bar{x})$$

(this is best seen using Green's functions to represent solutions);

If $\bar{I}$ is *physical* = *focused* at zero offset, i.e. $\bar{I}(z, x, \bar{x}) = I(z, x)\delta(x - \bar{x})$ with $I = 2\delta v / v^3$, then extended Born modeling specializes to ordinary Born modeling.

Nonlinearizing MVA, step 2

(2) recognize that preceding eqn is *perturbation equation* of *extended model*

$$\int d\bar{x}U(z,x,\bar{x})\frac{\partial^2 P}{\partial t^2}(x_s;z,\bar{x},t) - \nabla^2_{z,x}P(x_s;z,x,t) = w(t)\delta(z-z_s)\delta(x-x_s)$$

That is, *replace velocity (or square slowness) with SPD bounded operator* - existence theory for such problems due to Lions, late 60's.

If $U(z,x,\bar{x}) \simeq v(z,x)^{-2}\delta(x-\bar{x}) + \bar{I}(z,x,\bar{x})$, then $P(x_s;z,x,t) \simeq S(...) + p(...)$.

In particular, if U is *physical* = *focused* at zero offset, i.e. $U(z,x,\bar{x}) = v^{-2}(z,x)\delta(x-\bar{x})$, then the extended model becomes the ordinary acoustic model.

Extended NLS

Given *observed data* $D^{\rm obs}(x_r, x_s, t)$, find extended sqr slowness $U(z, x, \bar{x})$ so that *predicted data* $D[U](x_s; x_r, t) = P(x_s; z, x, t)_{z=z_r, x=x_r}$ fits observed data: $D[U] \simeq D^{\rm obs}$. [Formulate as least squares, use nonlinear optimization, blah, blah, blah....]

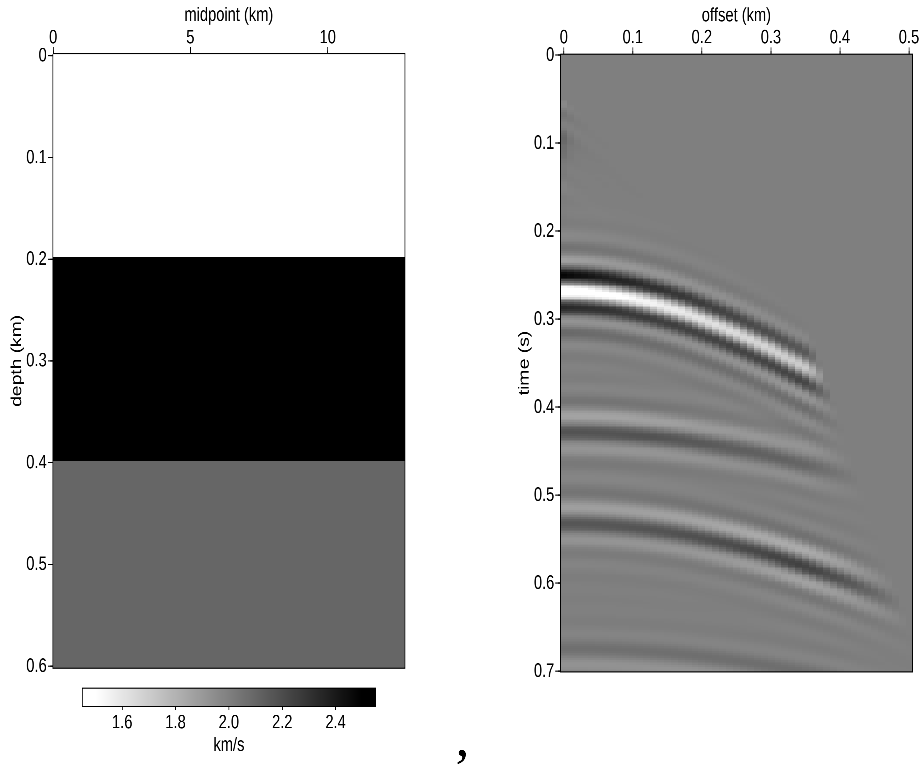
This problem is *underdetermined*: can fit data equally well with many extended sqr slownesses.

BUT: *physical* sqr slownesses *focuses at zero offset* [Claerbout redux!]:
$U(z, x, \bar{x}) \simeq v(z, x)^{-2}\delta(x - \bar{x})$

Hypothesis: focusing of extended square slowness at zero offset $\Leftrightarrow$ correct kinematics for primary reflections (like MVA), multiple energy assigned to primary reflectors ("multiples suppressed in image", like NLS).

Initial numerical exploration: layered models $\Rightarrow$ U is convolution op in x variables $\Rightarrow$ diagonalized by cosine transform $\Rightarrow$ cheap!

Model and data



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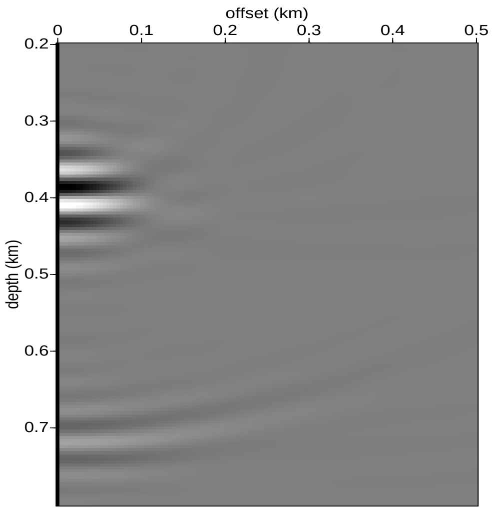
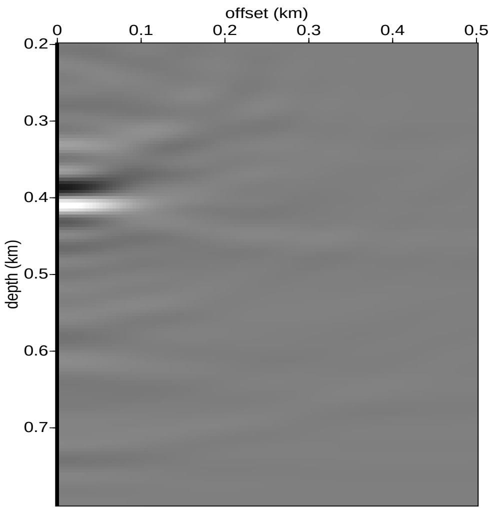
Left: Three layer model **Right:** Data - source wavelet = 4-10-30-40 Hz bandpass. Free surface multiple is about same size as second primary.

Migration vs. Inversion





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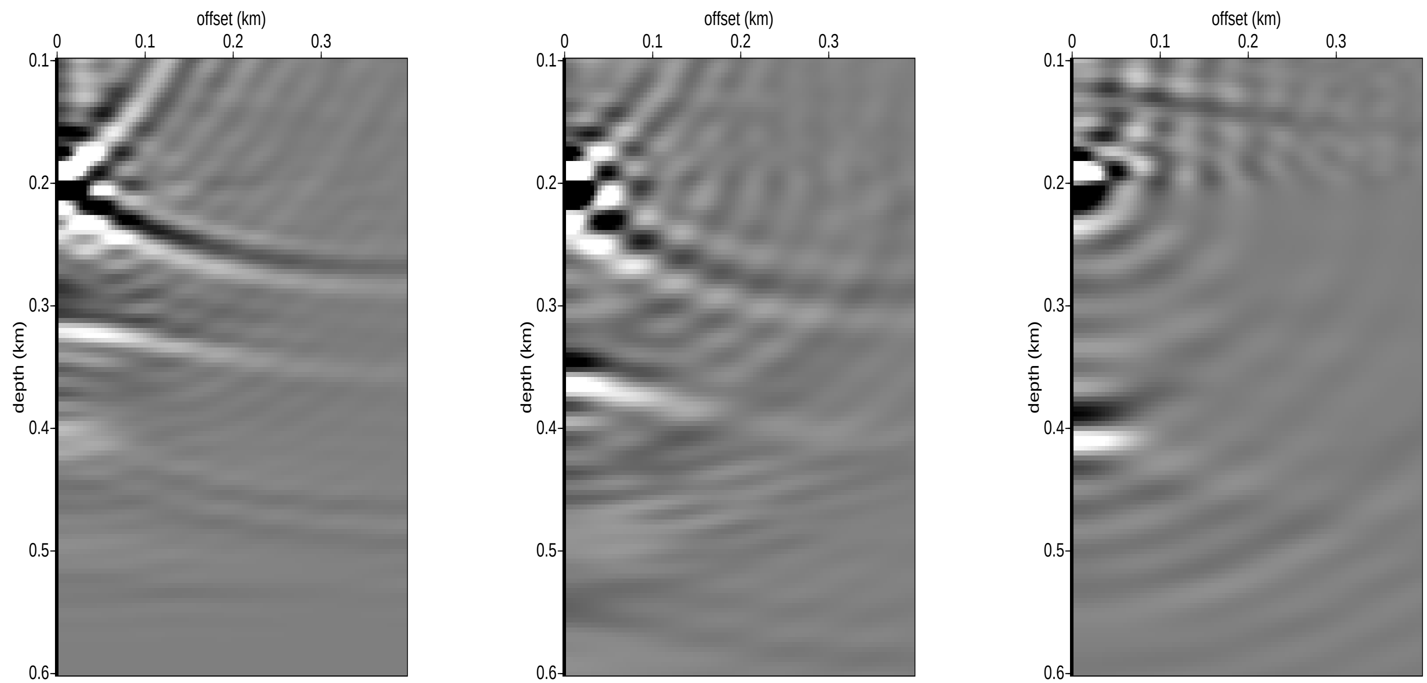
Left: Migration of three-layer data at $v = 1.5$ km/s for $z < 0.2km$, else $= 2.5$ km/s. **Right:** inversion, $\sim$ 40 LMBFGS iterations beginning at migration v. Note disappearance of migrated multiple.

Three extended NLS inversions



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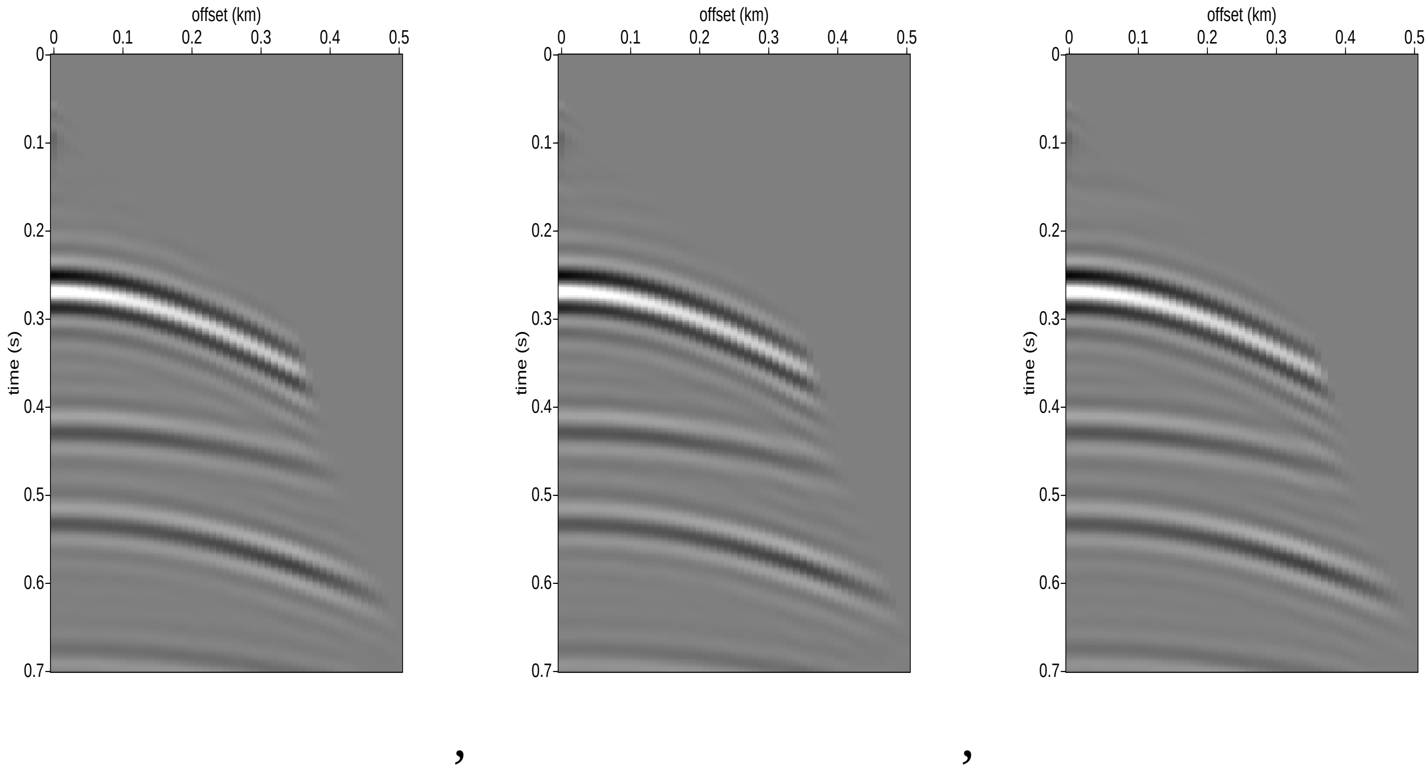
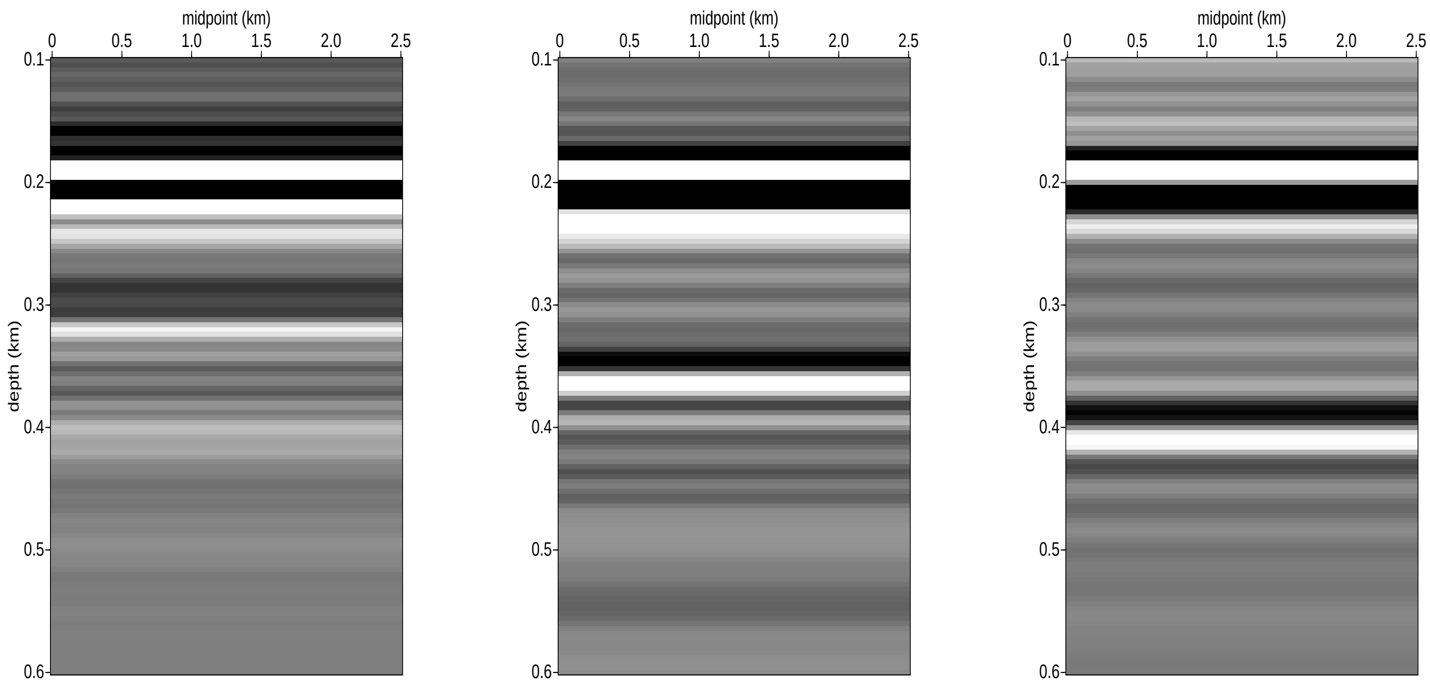
Different initial estimates of extended square slowness U (same as for NLS example), then LMBFGS until fit error reduced to $< 10^{-2} \times \|D^{\rm obs}\|$.

Resimulations (predicted data)



All three fit the data equally well...

Images (filtered zero offset sections)



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But only focused extended velocity produces image with correct reflector depths and multiple energy assigned to primaries.

Summary

- MVA can be formulated as optimization problem amenable to Newton: can recover from large initial errors in v, but based on Born approximation $\Rightarrow$ degraded by nonlinear effects in data (multiple scattering).
- NLS accomodates any modeled physics, linear or nonlinear, but cannot recover from large initial errors in v.
- Migration operator = adjoint to *extended* linearized modeling operator
- Focusing criterion for *nonlinear extended model* generalizes both MVA and NLS.

Outlook

Automation: apply focusing condition via constrained least squares (as in MVA):

$$\mathrm{minimize}_U \|hU\|^2 \text{ subject to } \|D[U] - D^{\mathrm{obs}}\| \leq \epsilon$$

Two major obstacles to making this work (“research opportunities”):

(1) simulation: can’t afford full matrix multiply in every time step - must find basis in which U is *sparse*, analog of cosine transform for layered models - windowed Fourier / curvelets?

(2) optimization: in MVA, (smooth) velocity parametrizes solutions, allows efficient *reduced basis* approach, long steps within very curvy *feasible set* of models fitting data. What is replacement in nonlinear setting?

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Peng Shen and Total E&P USA for Marmousi expl.

The organizers, for inviting me

All of you, for listening

`http://www.trip.caam.rice.edu`