Exact and approximate weights for Kirchhoff migration

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Summary

Drawing on work by Bleistein et al. (1998) and others, we present formulas for 2-D, 2.5-D, and 3-D Kirchhoff migration weights in a depth-varying ($v(z)$) medium. These formulas agree with migration weights presented by Winbow and Schneider (1999), although our versions allow us to attach physical significance to some of the terms, and further allow us to investigate the effect of dropping some of the terms in the interest of computational efficiency. The constant-velocity versions of these weights can be simplified to provide approximate, efficiently-calculated weights for depth migration. In numerical examples, we show that the simplified weights give well controlled amplitude for different dips, depth and offset events.

Introduction

Kirchhoff prestack time and depth migration are widely used in 3-D seismic data processing. To build the migrated image at a diffractor location, each input trace contributes samples whose time values correspond to the source-to-diffractor to receiver traveltimes. Arbitrary weights can be applied to the various input trace samples during their contribution to the image but, depending on the goal of the migration, some choices of weight are better than others. For example, an unweighted migration is usually unsuitable even for stratigraphic imaging because the lack of proper weights causes migration artifacts to appear on the image, hindering stratigraphic interpretation. In general, some control of the migration amplitudes is desirable for analyzing migrated amplitudes, either after stack or as a function of opening angle.

Several "true-amplitude" Kirchhoff migration weight functions have been developed for various recording geometries in two , two-and-a-half, and three dimensions (2-D, 2.5-D, and 3-D) (Bleistein et al., 1998; Schleicher et al., 1993; Hanitzsch, 1997). Since all these weights are somewhat complicated, containing square roots, numerical derivatives, and/or ray quantities, their direct implementation greatly increases the computational expense of Kirchhoff migration, and may cause numerical inaccuracy in processing. By contrast, Dellinger et al. (2000) have suggested a simplified 2.5-D weight function to facilitate the computation of depth-migrated images. When the velocity is constant, this weight is correct for all dips at zero offset, and for flat dips at all offsets. In this paper, we seek a simple weight function that will remain valid in more general velocity/structural settings. Our approach is to derive a set of analytical $v(z)$ weight functions from the formulation of Bleistein et al. (1998), and then to see how far these weight functions can be simplified to meet the needs of practical seismic processing. Based on the approach of Schleicher et al. (1993), Winbow and Schneider (1999) obtained similar results. While our expressions appear to be different from those of Winbow and Schneider, they are actually identical; however, their different forms might offer some computational advantages. For prestack depth migration, we present a corrected weight that handles amplitudes on steeply dipping events somewhat better than those of Dellinger et al.

Theory and method

1. Basic formulas

Bleistein et al. (1998) present a general formula for 3-D prestack Kirchhoff migration:

$$\beta(\mathbf{x}) \sim \frac{1}{8\pi^3} \int w(\mathbf{x}, \boldsymbol{\xi}) e^{i\omega(\tau_s+\tau_r)} u_s(\mathbf{x_r}, \mathbf{x_s}, \omega) i\omega d\omega d^2\boldsymbol{\xi},$$

where

$$w(\mathbf{x}, \boldsymbol{\xi}) = \frac{|h(\mathbf{x}, \boldsymbol{\xi})|}{A(\mathbf{x}, \mathbf{x}_s) A(\mathbf{x}_r, \mathbf{x}) |\nabla(\tau_s + \tau_r)|^2} \qquad (1)$$

is the migration weight. In (1), $A(\mathbf{x}, \mathbf{y})$ is amplitude of Green's function with source at $\mathbf{y}$ and observation point at $\mathbf{x}$, τ_s (τ_r) is traveltime between source (receiver) and image point , $\mathbf{x}_s(\boldsymbol{\xi})$ and $\mathbf{x}_r(\boldsymbol{\xi})$ are source and receiver points, respectively, $\boldsymbol{\xi} = (\xi_1, \xi_2)$ is the parameter labeling source and receiver points, and

$$h(\mathbf{x}, \boldsymbol{\xi}) = \det \begin{vmatrix} \nabla(\tau_s + \tau_r) \\ \frac{\partial}{\partial \xi_1} \nabla(\tau_s + \tau_r) \\ \frac{\partial}{\partial \xi_2} \nabla(\tau_s + \tau_r) \end{vmatrix} \qquad (2)$$

is the Beylkin determinant, the Jacobian of the transformation from subsurface coordinates (where the imaging takes place) to surface coordinates (where the integration is performed). For a given image point the Beylkin determinant serves to normalize by directional fold, where "directional fold" means the total number of contributions from all the input traces arriving at the image point from a particular direction, namely the average of the ray directions from the source and receiver locations.

When the velocity varies only with depth ($v(z)$), we can obtain the following expression for all the factors that make up w in (1):

$$A_s A_r = \sqrt{\frac{v_0 v}{\cos\alpha_{s0} \psi_s \sigma_s}} \cdot \sqrt{\frac{v_0 v}{\cos\alpha_{r0} \psi_r \sigma_r}}, \qquad (3)$$

2-D	shot	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_s}{\psi_r}}$
	receiver	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_r}{\psi_s}}$
	offset	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\left(\sqrt{\frac{\psi_s}{\psi_r}}+\sqrt{\frac{\psi_r}{\psi_s}}\right)$
2.5-D	shot	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_s}{\psi_r}}\sqrt{\sigma_s+\sigma_r}$
	receiver	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_r}{\psi_s}}\sqrt{\sigma_s+\sigma_r}$
	offset	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\left(\sqrt{\frac{\psi_s}{\psi_r}}+\sqrt{\frac{\psi_r}{\psi_s}}\right)\sqrt{\sigma_s+\sigma_r}$
3-D	shot	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_s}{\psi_r}}\sqrt{\frac{\sigma_s}{\sigma_r}}$
	receiver	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\sqrt{\frac{\psi_r}{\psi_s}}\sqrt{\frac{\sigma_r}{\sigma_s}}$
	offset	$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\left[\left(\sqrt{\frac{\psi_s}{\psi_r}}+\sqrt{\frac{\psi_r}{\psi_s}}\right)\left(\sqrt{\frac{\sigma_s}{\sigma_r}}+\sqrt{\frac{\sigma_r}{\sigma_s}}\right)+\frac{\sin^2\gamma}{2\cos^2\theta}L\right]$ $L=(\cos\alpha_s+\cos\alpha_r)\left(\sqrt{\frac{\psi_s\psi_r}{\sigma_s\sigma_r}}+\sqrt{\frac{\sigma_s\sigma_r}{\psi_s\psi_r}}\right)-(1+\cos\alpha_s\cos\alpha_r)\left(\sqrt{\frac{\psi_s\sigma_r}{\psi_r\sigma_s}}+\sqrt{\frac{\psi_r\sigma_s}{\psi_s\sigma_r}}\right)$

Table 1: The $v(z)$ Kirchhoff weights for common-shot, common-receiver and common-offset recording geometries.

$$|\nabla(\tau_s+\tau_r)| = \frac{2}{v}\cos\theta,$$

where α_{s0} and α_{r0} are the ray angles for source and receiver relative to the vertical at the surface, and θ is the reflection angle. In (3), ψ_s and σ_s are in-plane and out-of-plane spreading terms from the source, defined by

$$\begin{aligned}\psi_s &= \cos\alpha_s\int_0^z \frac{v(\varsigma)}{\cos^3\alpha_s(\varsigma)}d\varsigma = \cos\alpha_s\frac{\partial\rho_s}{\partial p_s},\\ \sigma_s &= \int_0^z \frac{v(\varsigma)}{\cos\alpha_s(\varsigma)}d\varsigma = \frac{\rho_s}{p_s},\end{aligned}$$

where α_s is the angle along the ray path from source relative to the vertical, $\rho_s = \sqrt{(x-x_s)^2+(y-y_s)^2}$, $p_s = (\sin\alpha_s)/v(z)$. ψ_r and σ_r are in-plane and out-of-plane spreading terms from receiver, defined similarly. We can also derive expressions for the Beylkin determinant h for common-shot, common-receiver and common-offset geometries in 3-D. In Table 1, we present $v(z)$ migration weights in 2-D, 2.5-D and 3-D.

Table 1 shows some interesting connections among the various weight functions. For example, each of the weights is a combination of in-plane and out-of-plane spreading terms. 2.5-D migration weights differ from 2-D weights only in the presence of the out-of-plane terms. In both 2-D and 2.5-D, the common-offset weight equals the sum of common shot weight plus the common-receiver weight. In 3-D, however, the common-offset weight differs from the sum of common-shot weight plus common-receiver weight because the 3-D common-offset (common-azimuth) recording geometry is not a perfect generalization of the 2-D common-offset geometry. In fact, the 3-D common-offset weight function is by far the most complicated expression in Table 1, and we shall concentrate on this weight, and discuss possible simplifications in the interest of computational efficiency.

2. Prestack time migration

In the 3-D common-offset weight, by far the most complicated term is the final term, involving $\frac{\sin^2\gamma}{2\cos^2\theta}L$, where γ is the angle between the projections of source and receiver rays to the surface. If this term is very small compared with the others, then we can ignore it with very little error. Indeed, this term is zero if the velocity is constant ($L=0$), or if the offset is zero ($\gamma=0$), or if the structure is invariant in the crossline direction, in which case all specular reflection occurs in the plane beneath the source and receiver ($\gamma=\pi$).

After dropping the last term, we get the simplfied prestack time Kirchhoff migration weight

$$\frac{\sqrt{\cos\alpha_{s0}\cos\alpha_{r0}}}{v_0}\left(\sqrt{\frac{\psi_s}{\psi_r}}+\sqrt{\frac{\psi_r}{\psi_s}}\right)\left(\sqrt{\frac{\sigma_s}{\sigma_r}}+\sqrt{\frac{\sigma_r}{\sigma_s}}\right). \quad (4)$$

The factors $\sqrt{\cos\alpha_0}$, $\sqrt{\psi}$ and $\sqrt{\sigma}$ can be precalculated when computing travel time, saving a lot of computations in the innermost loop. In Figure 1, we show the error introduced by dropping the last term of the common-offset weight. In this example, the velocity varies from $2000m/s$ on the surface down to $4100m/s$ ($7000m$ deep). The subsurface reflector has 30 degree dip in a direction perpendicular to the direction of the source-receiver azimuth (the worst case). We see the error is small (about 5%) even for a large opening angle. Figure 2 shows the result of using the simplified prestack time weight (4) to migrate five dipping events ($0°$, $10°$, $20°$, $30°$ and $45°$, see Fig. 2a) with equal reflectivities. In this example, we chose a constant gradient velocity $v = 2000 + 0.3z$ and migrated down to $5s$ ($7446m$). The image is displayed in

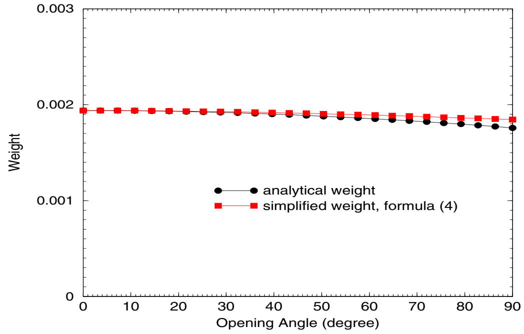

Fig. 1: Comparison of the 3-D analytical common-offset weight with the simplified weight (4).

		Exact Weight	Simplified
2-D	shot	$\frac{z}{v^2}\frac{1}{\sqrt{t_s t_r}}\frac{t_s}{t_r}$	
	receiver	$\frac{z}{v^2}\frac{1}{\sqrt{t_s t_r}}\frac{t_r}{t_s}$	
	offset	$\frac{z}{v^2}\frac{1}{\sqrt{t_s t_r}}\left(\frac{t_r}{t_s}+\frac{t_s}{t_r}\right)$	$\frac{4z}{v^2 t}$
2.5-D	shot	$\frac{z}{v}\sqrt{\frac{t_s+t_r}{t_s t_r}}\frac{t_s}{t_r}$	
	receiver	$\frac{z}{v}\sqrt{\frac{t_s+t_r}{t_s t_r}}\frac{t_r}{t_s}$	
	offset	$\frac{z}{v}\sqrt{\frac{t_s+t_r}{t_s t_r}}\left(\frac{t_r}{t_s}+\frac{t_s}{t_r}\right)$	$\frac{4z}{v\sqrt{t}}$
3-D	shot	$\frac{z}{v^2}\frac{t_s}{t_r^2}$	
	receiver	$\frac{z}{v^2}\frac{t_r}{t_s^2}$	
	offset	$\frac{z}{v^2}\left(\frac{t_s}{t_r}+\frac{t_r}{t_s}\right)\left(\frac{1}{t_r}+\frac{1}{t_s}\right)$	$\frac{8z}{v^2 t}$

Table 2: Exact weights for constant velocity and their simplification in 2-D, 2.5-D and 3D. t_s (t_r) is the traveltime from the source (receiver) location to the image point and $t = t_s + t_r$.

time, not in depth. The amplitude is almost uniform for different dips, offsets and depths, except for some amplitude loss on the 45° dipping reflector, which is mainly due to the anti-aliasing ($dx = dy = 25m$). The noise around $4 - 4.25s$ is due to the limited aperture.

3. Prestack depth migration

In principle, the weights for depth migration are much more complicated than the weights for time migration. For completely correct amplitude treatment of amplitudes in depth migration, we must calculate ray amplitude information in addition to ray traveltimes, and we must combine all this information with the Beylkin determinant (2). We must also incorporate phase rotations as the wavefield energy encounters caustics between the source locations and the image points, or between the image points and the receiver locations. All this work can easily be more expensive to perform than the already expensive diffraction stack at the heart of the migration. In addition, it is not clear that this extra work will be beneficial if the velocity field is very complicated, since it is easy to envision that any errors in migration velocity estimation might have a significant effect on our calculated wavefield amplitude and phase information. These distortions will be translated directly into amplitude distortions, possibly serious, on our migrated images.

As a result of the theoretical and practical difficulties of including detailed weights in Kirchhoff depth migration, Dellinger et al. (2000) have suggested that we might actually be better off using simplified (constant-velocity) weights, appropriately chosen for a given acquisition geometry. Accordingly, we present in Table 2 migration weights obtained by applying the same approximations as made by Dellinger et al. In practice, we have found these weights to be efficient to apply and to yield reasonable amplitude behavior on our depth-migrated images. For 3-D, the simplified common-offset weight is

$$w_1 = \frac{z}{8v_0^2 t}. \tag{5}$$

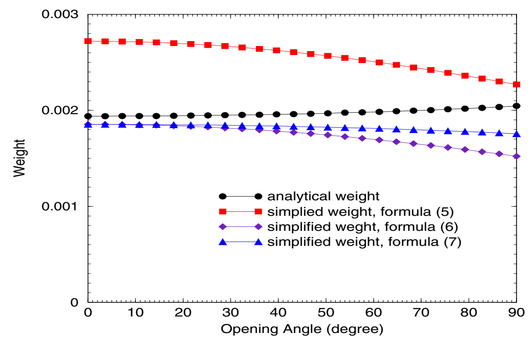

Fig. 3: Comparison of the 3-D common-offset analytical weight with its simplified version (5), (6) and (7).

The nice thing for (5) is that here, both z and two-way traveltime t can be evaluated as we read the input and write the output traces. By doing this, we save significantly on the cost of Kirchhoff migration. As mentioned in the Introduction, when the velocity is constant, weight (5) is correct for all dips at zero offset, and for flat dips at all offsets. For variable velocity and steeply dipping events, (5) may give a relatively large error even when the opening angle is small; see figure 3. If we want a more correct weight, we can use

$$w_2 = \frac{z}{8v_0(r_s + r_r)}, \tag{6}$$

or

$$w_3 = \frac{z}{v_0}\left(\frac{1}{t_s r_s} + \frac{1}{t_r r_r}\right) t. \tag{7}$$

For (7), we can play the same trick as in (5), moving the scalers z and t out of the innermost loop. With moderate additional computation, we hope weight (7) provides improved amplitude for steeply dipping events. In figure 3

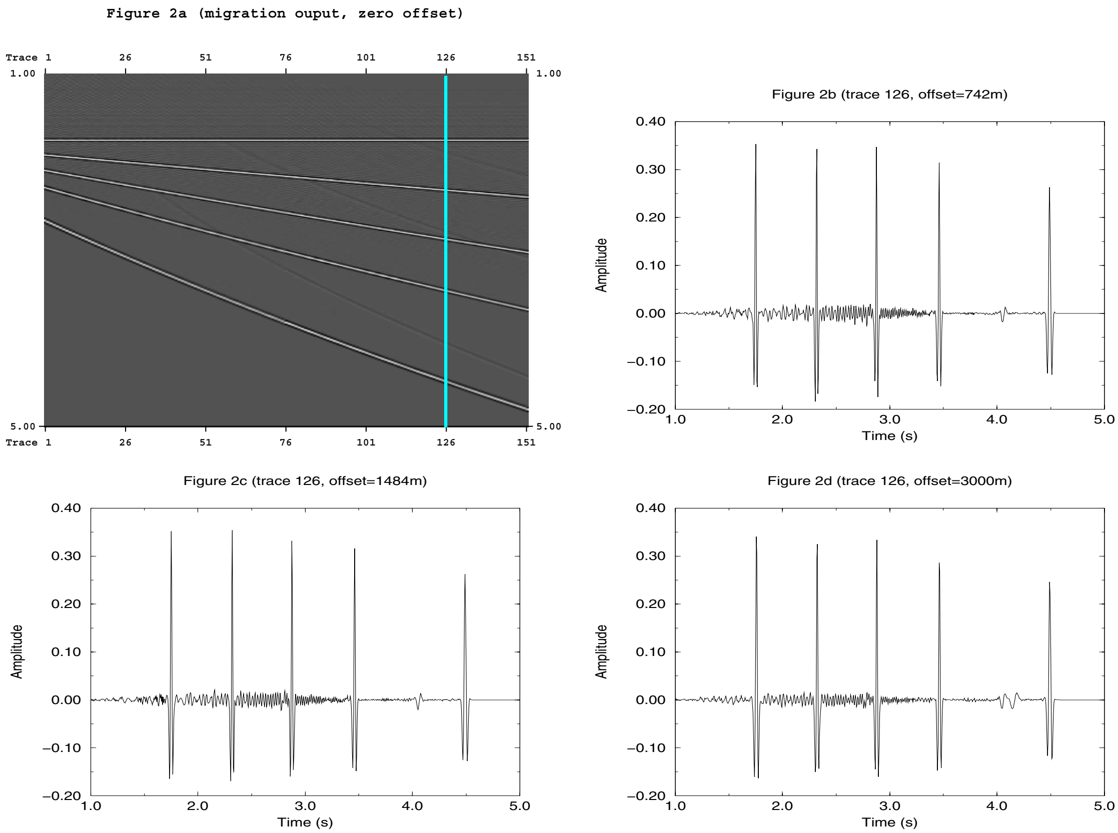

Fig. 2: Time migration using weight (4). Figure 2a: five dipping events, 0°, 10°, 20°, 30° and 45°. Figure 2b-d: amplitude on migrated trace ♯126 at offset $742m$, $1484m$ and $3000m$, respectively.

we compare the simplified weights (5), (6) and (7) with the theoretical weight. In this example, the velocity is the same as that in Figure 1, but the subsurface reflector has a 30° dip in the inline direction. For such a velocity variation, weights (6) and (7) follow the correct trend for small opening angles better than (5) does. When the opening angle is large, the error drops about 50% by using formula (7), compared with (6).

Conclusions

Migration amplitudes are easier to control in areas of moderate geologic complexity than in areas of extreme geologic complexity. In particular, for dipping reflectors in $v(z)$ media, such as Gulf of Mexico sediments, accurate migration amplitudes should result in reliable estimates of amplitude-vs-offset measurements. We have derived accurate migration weight functions for $v(z)$ media, and we have presented an approximation to the 3-D weight that provides accurate migrated amplitudes with a significant reduction of migration expense.

References

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