Theory of True Amplitude Common-shot Migration

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Summary

We present a new formulation of common-shot migration. Theoretical analysis shows that the proposed method gives the same amplitude results as that of true amplitude Kirchhoff migration in the sense of the high frequency approximation. Therefore, a finite-difference implementation of the new method has advantages to maintain its high fidelity in imaging complex structures and carry correct dynamic behavior for a general velocity $v(x, y, z)$.

Introduction

The demands of imaging complex geological structures have led to growing popularity of prestack migrations based on one-way wavefield extrapolation. Common-shot migration is a candidate among such migration methods. It provides high imaging quality and relatively good computational efficiency with parallel computing. The standard formulation of common-shot migration (Claerbout, 1971) consists of two parts. The first part is the downward continuation of the wavefields from the source and receiver locations using a "wave equation" that splits the wavefields into downgoing and upgoing parts. The second part is the application of an imaging condition, namely the division of the downward continued receiver wavefield by the downward continued source wavefield at each image point. Unfortunately, the one-way "wave equations" used in the downward continuation are not equivalent to the acoustic wave equation whose behavior they are designed to mimic. This lack of equivalence makes a migrated wavefield questionable in both amplitude and phase behavior, even though it is kinematically correct. On the other hand, by exploration of Beylkin (1985), Bleistein (1987), Schleicher et al. (1993) and other researchers, Kirchhoff migration has been put on to a solid theoretical basis as an inversion method. In so called true amplitude Kirchhoff migration, the amplitudes in migrated images can be considered as estimated reflectivities which is desirable for geological interpretation.

Our previous work in Zhang et. al. (2001) and (2002) show that the conventional formulation of common-shot migration fails to preserve amplitude. The conclusion is obtained by expressing the downward continued wavefields asymptotically in a $v(z)$ medium and then comparing the migrated formulation with that of true amplitude Kirchhoff migration. This theoretical analysis leads to some remedies to the method for preserving amplitude in a constant or layered velocity. The remedies include changes to downward extrapolation equations, boundary conditions and the imaging condition. In this abstract, we push forward to generalize the new algorithm to a general velocity $v(x, y, z)$. When the new method is applied, the migration produces images whose amplitudes and phases agree with true amplitude Kirchhoff migration. These corrections are inexpensive to implement, and they do not compromise the migration's structural imaging fidelity.

Theory

We begin with 3D common-shot migration. Given an acoustic wave-field p with source excitation at $\vec{x_s} = (x_s, y_s, 0)$ and $t = 0$,

$$\left(\frac{\omega^2}{v^2} + \frac{\partial^2}{\partial z^2} + \triangle\right) p(x, y, z; t) = -\delta(\vec{x} - \vec{x_s}), \tag{1}$$

(where $\triangle = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$), we record the surface data Q:

$$p(x_r, y_r, z = 0; \omega) = Q(x_r, y_r; \omega). \tag{2}$$

According to Bleistein et al.'s (2001) work on inversion, the true amplitude common-shot Kirchhoff inversion formula is (Hanitzsch, 1997)

$$\begin{aligned} R(x, y, z) \sim \quad & \iiint i\omega \frac{\cos\alpha_{r0}}{v(\mathbf{x}_r)} \frac{A(\mathbf{x}_r, \mathbf{x})}{A(\mathbf{x}, \mathbf{x}_s)} \\ & e^{i\omega(\tau_s + \tau_r)} Q(x_r, y_r; \omega) dx_r dy_r d\omega, \end{aligned} \tag{3}$$

where $A(\mathbf{x}, \mathbf{y})$ is the amplitude of the Green's function with source at $\mathbf{y}$ and observation point at $\mathbf{x}$, τ_s (τ_r) is the traveltime between source (receiver) and image point , and α_{r0} is the ray angle at the receiver relative to the vertical at the surface.

For conventional common-shot migration, both shot and receiver wavefields are downward continued:

$$\begin{cases} \left(\dfrac{\partial}{\partial z} + \Lambda\right) D = 0, \\ D(x, y, z = 0; \omega) = \delta(\vec{x} - \vec{x_s}), \end{cases} \tag{4}$$

and

$$\begin{cases} \left(\dfrac{\partial}{\partial z} - \Lambda\right) U = 0, \\ U(x, y, z = 0; \omega) = Q(x, y; \omega) \end{cases} \tag{5}$$

where D and U are the downgoing and upgoing waves (Claerbout, 1985), respectively, and

$$\Lambda = i\frac{\omega}{v}\sqrt{1 + \frac{v^2}{\omega^2}\triangle}$$

is the square-root operator. To produce the image, the following imaging condition is used

$$R(x,y,z) = \int \frac{U(x,y,z;\omega)}{D(x,y,z;\omega)} d\omega. \tag{6}$$

It has been shown in Zhang et al. (2001) by asymptotic analysis, that the algorithm (4-6) cannot provide the same true amplitude image as that given by (3); even the phase term $i\omega$ in (3) is missing. A remedy to correct the amplitude for constant velocity was given in the above-cited paper, and then was generalized to a $v(z)$ medium in Zhang et al. (2002).

Actually, the split system (4) and (5) only preserves the kinematics of the acoustic equation (1). Zhang (1993) proposed that to maintain both kinematics and dynamics of (1), we have to use the following one-way wave equations

$$\left\{ \begin{array}{l} \left(\frac{\partial}{\partial z} + \Lambda\right) D + \Gamma D = 0, \\ \left(\frac{\partial}{\partial z} - \Lambda\right) U + \Gamma U = 0 \end{array} \right. \tag{7}$$

where the operator

$$\Gamma = \frac{v_z}{2v}\left[1 - \left(\frac{\omega^2}{v^2} + \triangle\right)^{-1} \triangle\right],$$

D and U are connected with full wave field p by the relationship

$$D = \frac{1}{2}\left(\Lambda - \frac{\partial}{\partial z}\right) p,$$

$$U = \frac{1}{2}\left(\Lambda + \frac{\partial}{\partial z}\right) p,$$

and

$$D + U = \Lambda p.$$

It can be proved that the split wave system (7) is equivalent to full wave equation (1) in the sense that they share the same traveltime (τ) and first order amplitude (A).

Based on Zhang's wave splitting formulation (7), we propose the following true amplitude common-shot migration algorithm: Instead of solving for D and U, we solve for pressure fields $p_D = \Lambda^{-1} D$ and $p_U = \Lambda^{-1} U$, which satisfy the following equations and boundary conditions

$$\left\{ \begin{array}{l} \left(\frac{\partial}{\partial z} + \Lambda - \Gamma\right) p_D(x,y,z;\omega) = 0, \\ p_D(x,y,z=0;\omega) = \frac{1}{2}\Lambda^{-1}\delta(\vec{x} - \vec{x}_s), \end{array} \right. \tag{8}$$

and

$$\left\{ \begin{array}{l} \left(\frac{\partial}{\partial z} - \Lambda - \Gamma\right) p_U(x,y,z;\omega) = 0, \\ p_U(x,y,z=0;\omega) = Q(x,y;\omega). \end{array} \right. \tag{9}$$

Also, we modify the imaging condition (6) to be the quotient of the wave fields p_D and p_U:

$$R(x,y,z) = \int \frac{p_U(x,y,z;\omega)}{p_D(x,y,z;\omega)} d\omega. \tag{10}$$

We have mathematically proven that equations (8) and (9), together with imaging condition (10), give the same true amplitude result as (3) in the sense of high frequency approximation.

Implementation

For a $v(z)$ medium, the implementation of algorithm (8-10) is straightforward as a modified

common-shot phase-shift migration (Zhang et al., 2002) if we notice the operators Λ and Γ can be simply expressed in frequency and wave-number domain as

$$\lambda = i\frac{\omega}{v}\sqrt{1 - \frac{v^2(k_x^2 + k_y^2)}{\omega^2}},$$

and

$$\gamma = \frac{v_z}{2v}\left[1 + \frac{v^2(k_x^2 + k_y^2)}{\omega^2 - v^2(k_x^2 + k_y^2)}\right].$$

For a general velocity $v(x,y,z)$, Λ and Γ should be interpreted as pseudo-differential operators. Zhang (1993) gave Λ an explicit expression as a differential-integral operator

$$\Lambda = i\frac{\omega}{v}\left[I + \frac{1}{\pi}\int_{-1}^{1}\sqrt{1 - s^2}\left(\frac{\omega^2}{v^2} + s^2\triangle\right)^{-1} \triangle ds\right]. \tag{11}$$

For convenient numerical computation, the integral in (11) can be approximated by the summation and solved by finite difference

$$\Lambda_n = i\frac{\omega}{v}\left[I + \sum_{l=1}^{n}\left(\frac{\omega^2}{v^2} + \beta_{n,l}\triangle\right)^{-1} \alpha_{n,l}\triangle\right],. \tag{12}$$

Here, the coefficients $\alpha_{n,l}$ and $\beta_{n,l}$ can be chosen as

$$\alpha_{n,l} = \frac{1}{n+1}\sin^2\left(\frac{l\pi}{n+1}\right),$$

and

$$\beta_{n,l} = \cos\left(\frac{l\pi}{n+1}\right).$$

Some optimized $\alpha_{n,l}$ and $\beta_{n,l}$ were given in Lee et al. (1985). If we replace Λ in (4) and (5) with Λ_n , the special cases for $n = 1$ and $n = 2$ give classical 15° and 45° migration equations. Generally, larger n produces more accurate high dipping reflectors in the imaging. Also, many hybrid techniques have been developed to solve operator Λ, such as PSPI (phase shift plus interpolation) (Gazdag and Sguazzero, 1984), SSF (split-step

Fourier method) (Stoffa et al., 1990) , FFD (Fourier finite-difference) (Ristow and Ruhl, 1994) and GS (generalized screen) (Le Rousseau and De Hoop, 2001), etc..

Numerical results

To show how true amplitude common-shot migration works, we apply it to a 2-D horizontal reflector model in a medium with velocity $v = 2000 + 0.3z$. The input data (Figure 1) is a single shot record over four horizontal reflectors from density contrast generated by applying geometrical spreading to equal-amplitude Ricker wavelets with analytical traveltime.

Figure 2 left shows the migrated shot record using the conventional common-shot migration algorithm (4-6). The peak amplitudes along the four migrated reflectors are shown in the right. The phase error of $i\omega$ has been corrected during the migration. However, the migrated amplitudes are poor, especially on the reflector at depth $z = 1000$m along which the reflection angles vary over a wide range. Figure 3 shows results of true amplitude common-shot migration (8-10). From the right plot, we clearly see that true amplitude algorithm recovers the reflectivity accurately, aside from the edge effects and small jitters caused by interference with wraparound artifacts.

Conclusions

Common-shot migrations offer good potential of imaging complex structures, but the conventional formulations of such migrations produce incorrect migrated amplitudes. The migration method we proposed in this abstract calibrate common-shot migrations by correcting both their amplitude and phase behavior. The new method actually builds a bridge between true amplitude Kirchhoff migration and the migrations based on one-way wavefield extrapolation.

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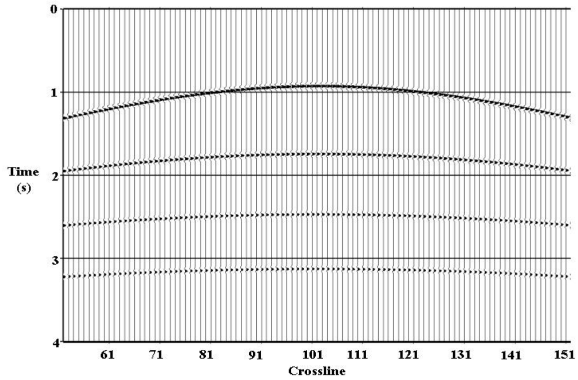

Fig. 1: 2-D shot record from four horizontal reflectors in a medium with velocity $v = 2000 + 0.3z$.

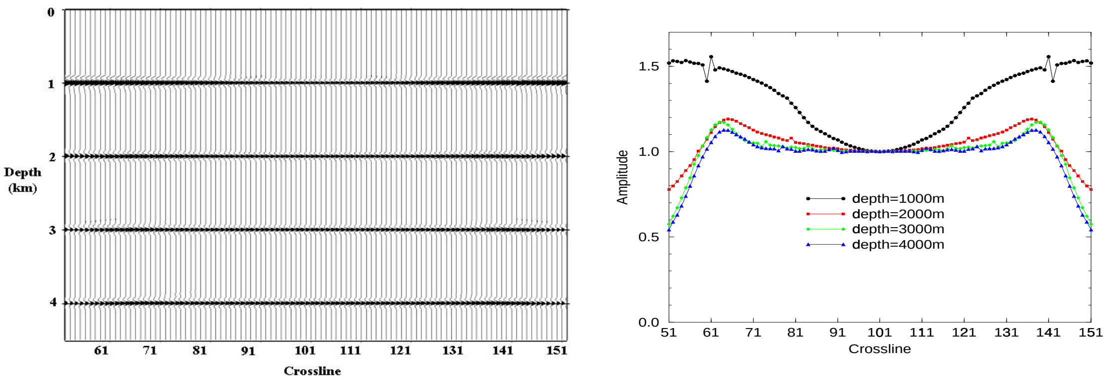

Fig. 2: Left: migrated shot record using algorithm (4- 6). Right: Normalized peak amplitudes along the migrated reflectors in the left plot.

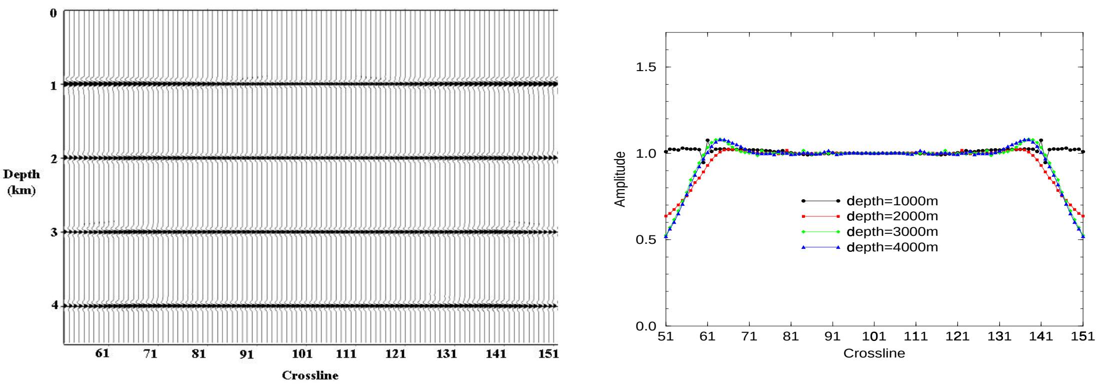

Fig. 3: Left: migrated shot record using true amplitude common-shot migration algorithm (8-10). Right: Normalized peak amplitudes along the migrated reflectors in the left plot.