The Rice Inversion Project

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Annual Review, October 2004

Agenda, Morning

0845 Welcome and logistical announcements

0900 J.-L. Qian, UCLA: Recent developments in level set methods for traveltime and related computations

0945 C. C. Stolk, U. Twente: Aspects of wave equation imaging

1030 break

1040 E. Dussaud, Explicit extrapolators and common azimuth migration

1110 W. W. Symes and F.-C. Gao, Rice U: HOCIGs and VOCIGs via two way reverse time migration

1130 E. Dussaud, Rice U: A sparse, bound-respecting parametrization of velocities

1140 W. W. Symes and J. Li, Rice U: NMO-based DSO: implementation and initial noise studies

Agenda, Afternoon

1200 Lunch, Cohen House

1300 E. Dussaud, Rice U: Velocity analysis in the presence of uncertainty

1320 P. Shen, Rice U and Total: Wave equation velocity analysis

1350 W. W. Symes, Rice U: Velocity analysis and nonlinear inverse scattering

1420 Discusson: immediate plans, future directions

1500 Adjournment

NMO-Based DSO

Objectives:

- automatic velocity analysis accounting for mild lateral heterogeneity
- accommodate both 2D and 3D data in standard input format (SEGY)
- produce velocity models in *depth* with controlled resolution, using PIGrid data structure

Working version: uses hyperbolic traveltimes, estimates isotropic P-wave velocity

NMO-Based DSO - Fundamentals

$d(t,h,\mathbf{m})$ = CMP gathers, $h,\mathbf{m}$ = (3D) half-offset, midpoint, $v=v(z,\mathbf{m})$ midpoint dependent interval velocity. NMO = layered medium approximation to migration:

$$d_{\mathrm{NMO}}[v](t_0,h,\mathbf{m}) = d(t[v](t_0,h),h,\mathbf{m})$$

Differential semblance measures flatness of nmo-corrected CMP:

$$s[v](t_0,h,\mathbf{m}) = \frac{\partial}{\partial h} d_{\mathrm{NMO}}[v](t_0,h,\mathbf{m})$$

Differential semblance optimization:

$$\min_v \left\{ J_{\mathrm{DSO}}[v,d] \equiv \sum_{t_0,h,\mathbf{m}} |s[v](t_0,h,\mathbf{m})|^2 \right\}$$

NMO-Based DSO - Implementation

- change of variables $t \mapsto t_0$ by *local cubic interpolation* - smooth enough (barely) for differentiation w.r.t. v.
- use Fortran for basic numerical kernels. Motivation: availability of *automatic differentiation* (TAMC) to produce derivatives and adjoints required for optimization.
- kernels wrapped in C++ to produce *Standard Vector Library* `Operator` subclasses
- SU and SEP data structures implemented as SVL `Space`, `DataContainer` subclasses
- linked to SVL implementation of limited-memory quasi-Newton optimization algorithm to produce final `NMOOpt.x` driver.
- SU-style self-doc provided.

NMO-Based DSO - Limitations

- Accounts only for isotropic P-wave (or single velocity) moveout
- Accounts only for *primary reflection data* from (near-)layered structure
- Sensitive to coherent noise: multiple reflections, mode conversions, etc. (see WWS and Gockenbach, SEG 99)

Jintan Li MA project: assess accuracy, ease of use, influence of various types of noise using synthetic and field data

NMO-Based DSO - Future

- will remain a tool for inversion of *primaries only* data - dependent on multiple suppression technology
- anisotropy accommodated through (a) approximate high-order corrections to hyperbolic TT, (b) ray trace TT (also interesting for isotropic case) via eikonal solvers
- multiple modes handled *without mode separation* through *concatenated annihilators* (see TRIP annual report 2000).
- for multiple reflections, we will pursue another path...

HOCIGs and VOCIGs

Biondi-Shan 2002, TRIP 2003, Biondi-Symes 2004: Reverse-time shot-geophone ("S-G") migration permits use of turning rays in prestack imaging.

This talk:

- Fuchun Gao: how to produce offset image gathers using *frequency domain* two-way migration, and their focussing property when DSR condition holds;
- in order to avoid imaging ambiguity when rays turn, image volume *must* include nonhorizontal offsets;
- midpoint dip filtering produces artifact-free horizontal and vertical offset CIGs - reduce cost by decimating midpoints, avoid midpoint dip filtering, and still eliminate artifacts;
- Details: paper *Reverse time shot-geophone migration* ("RTSGM")

Kinematics

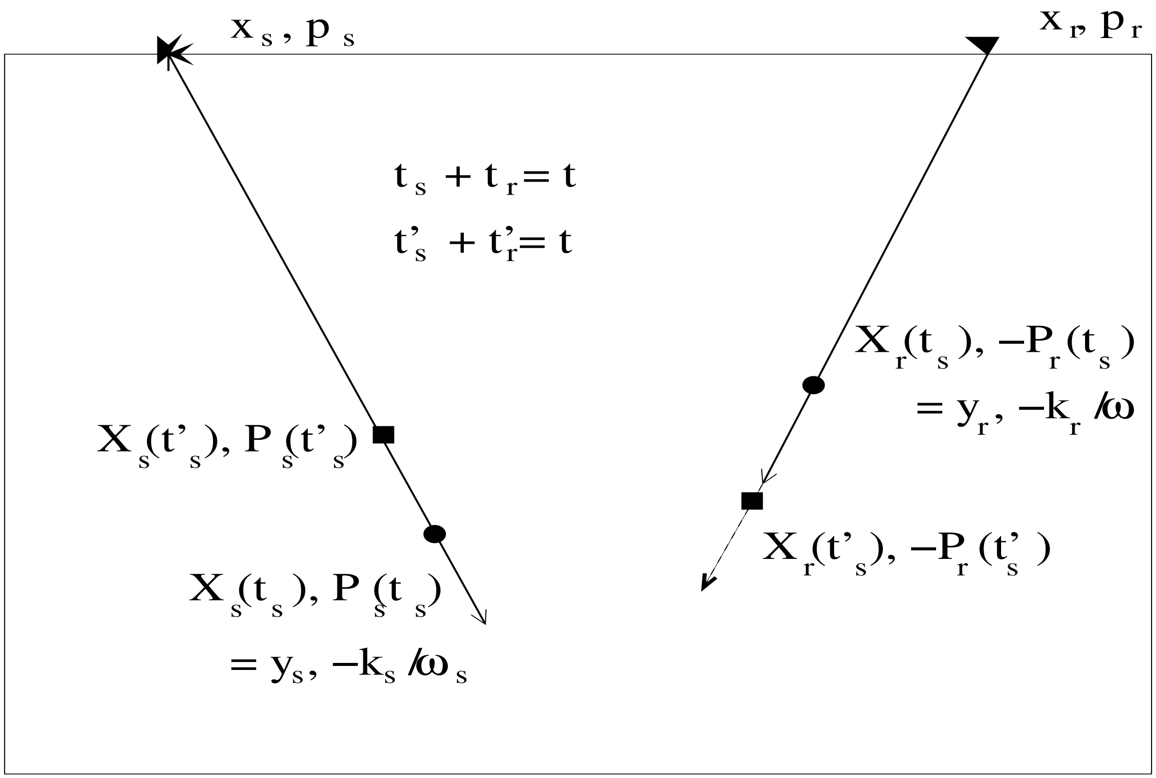
Phase space description: reflector has *location* $(\mathbf{y}_r, \mathbf{y}_s)$ and *dip* $(\mathbf{k}_r, \mathbf{k}_s)$.

Similarly, reflection event in data at location $(\mathbf{x}_r, t; \mathbf{x}_s)$ and dip $\omega(\mathbf{p}_r, 1; \mathbf{p}_s)$. Event slownesses $\mathbf{p}_r, \mathbf{p}_s$ determined by data for ”true 3D”, otherwise many data-compatible slownesses (eg. for idealized streamer geometry).

Kinematic Relation of S-G modeling/migration: reflection event $(\mathbf{x}_r, t; \mathbf{x}_s), \omega(\mathbf{p}_r, 1; \mathbf{p}_s)$ occurs $\Leftrightarrow$ reflector exists at $\mathbf{y}_r, \mathbf{y}_s, \mathbf{k}_r, \mathbf{k}_s$ **and**

- a ray begins at $\mathbf{x}_s$ with takeoff slowness $\mathbf{p}_s$ and reaches $\mathbf{y}_s$ with arrival slowness $\mathbf{k}_s/\omega$, in time t_s;
- a ray begins at $\mathbf{x}_r$ with takeoff slowness $\mathbf{p}_r$ and reaches $\mathbf{y}_r$ with arrival slowness $\mathbf{k}_r/\omega$, in time t_r;
- $t_s + t_r = t$

Kinematics



Kinematic relation of S-G modeling/migration

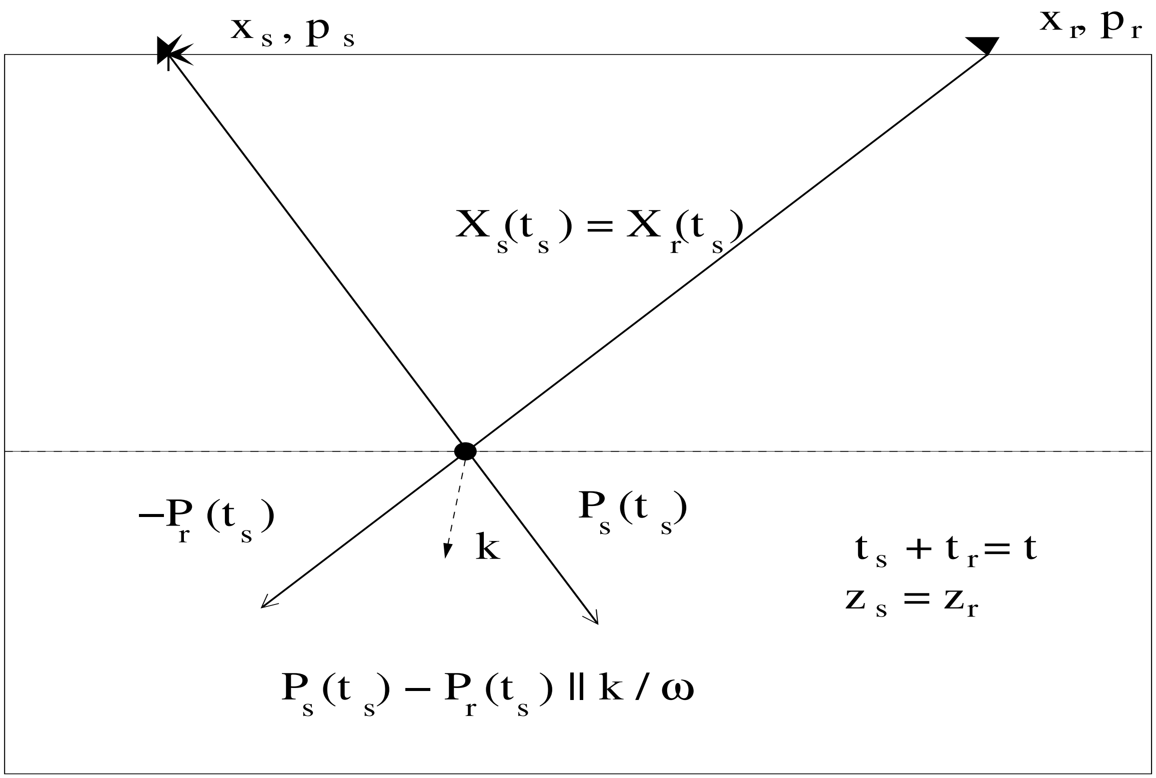
Too many image points!

Note: for any given reflection event in data, *many corresponding (double) reflectors*: all points on rays from source, receiver with correct total time.

$\Rightarrow$ gross imaging ambiguity

The ”traditional” fix: (1) DSR assumption, i.e. no turning rays; (2) ”sunken offset” vector *horizontal*

DSR, good $v \Rightarrow$ focus at $h = 0$



Kinematic relation of S-G modeling/migration + DSR + horizontal offset: NO IMAGING AMBIGUITY (Stolk-deHoop 2001)

Q. Why drop DSR?

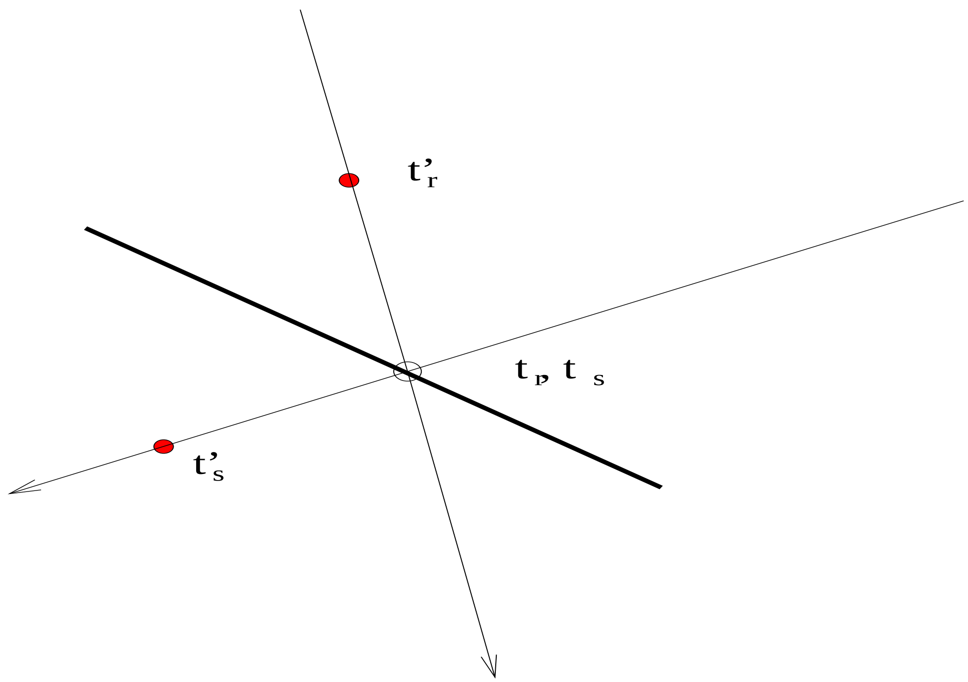
A. Because in complex structure, rays turn.

Q. Why drop horizontal offsets? A. Because reflectors structures may be vertical or near-vertical, and then horizontal offset images will be *smeared* (i.e. ambiguous reflector locations!)

Nonvertical reflector $\Rightarrow$ total traveltime determines reflection point uniquely when velocity is correct and *horizontal* offset assumed.

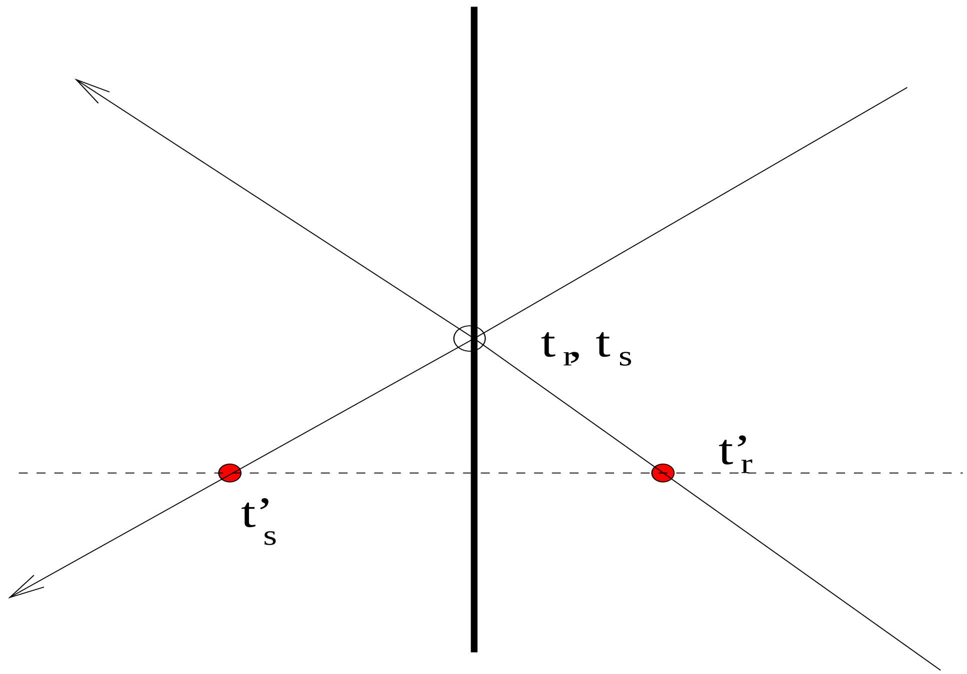
Vertical reflector $\Rightarrow$ many different (double) reflectors correspond to single physical reflector, all having same traveltimes and horizontal offset.

Nonvertical Reflector



Nonvertical reflector: $t_r + t_s = t'_r + t'_s$, but depths can *only* be the same at one point (which must be the physical reflection point, if velocity is correct, by S-deH).

Vertical Reflector



(Near) vertical reflector: $t_r + t_s = t'_r + t'_s$, and depths can be the same at a continuum of points, besides the physical reflection point $\Rightarrow$ reflector is smeared, location ambiguous.

Horizontal and vertical offsets via filtering

Suggested approach (differs from Biondi-Symes 2004): create HO and VO image volumes, then *filter in midpoint dip* (i.e. in x, z, not in h): remove near-vertical reflector components from HO volume, near-horizontal reflector components from VO volume.

See paper RTSGM for details.

DIfficulty: computation of (HO) image volume

$$I(x,z,h) = \int dt \int dx_s u(x_s, x-h, z, t) v(x_s, x+h, z, t)$$

requires $N_t N_s N_x N_h N_z$ flops - *and this can overwhelm the cost of solving the wave equation* if all axes are sampled densely!

Reasonable cost requires *decimation in midpoint*, i.e. compute only a relatively small number of HOCIGs, VOCIGs.

Horizontal and vertical offsets via filtering

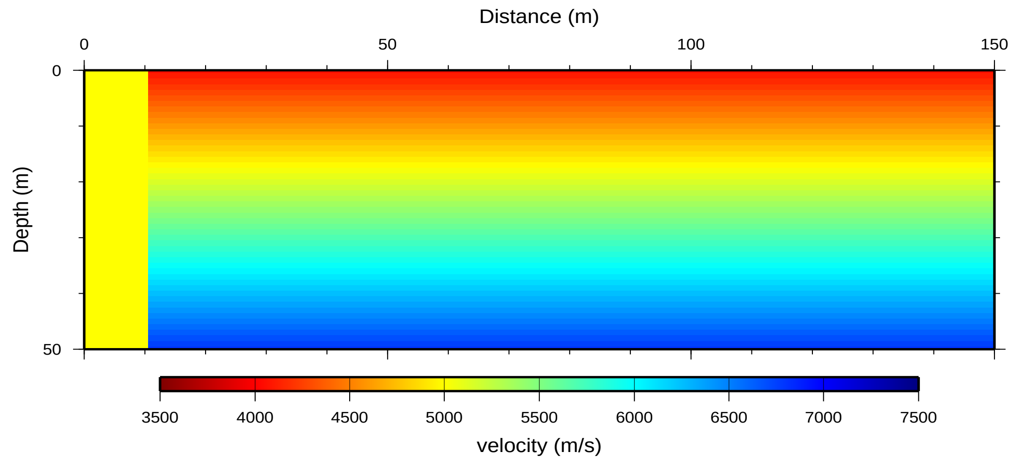
Decimated midpoints $\Rightarrow$ can't filter in midpoint dip.

Alternate process: *high-cut filter*

- HOCIGs in z
- VOCIGs in x

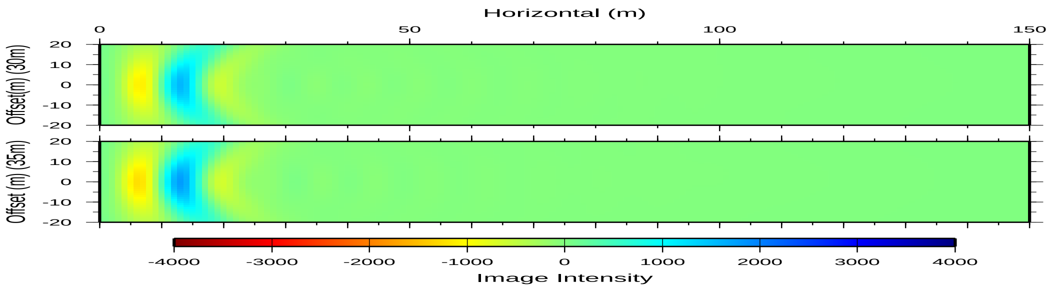
Also removes horizontal dips from HOCIGs, vertical dips from VOCIGs, but carried out *per midpoint*, i.e. fixed x for HOCIGs, fixed z for VOCIGs - compatible with decimated midpoints.

Example



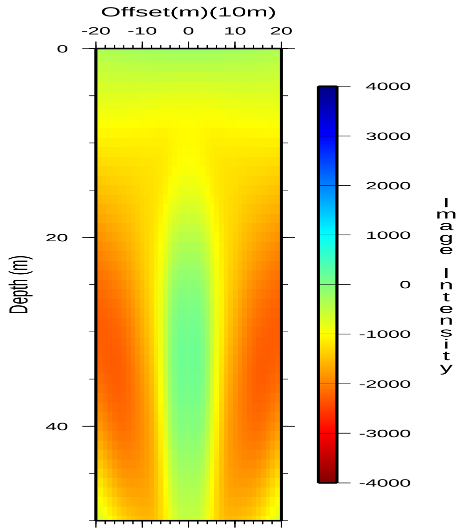
Velocity model with velocity increasing with depth, generating turning rays, and vertical reflector.

Example



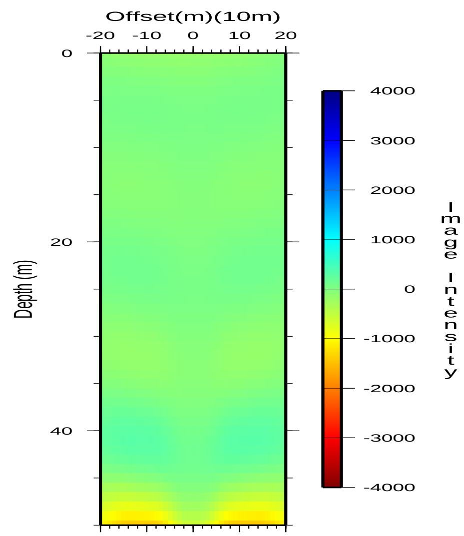
VOCIGs ($z = 30$ m, 35 m) are artifact-free - no imaging ambiguity

Example



HOCIG at reflector midpoint has substantial low freq component - *smearing*

Example



Filtered HOCIG at reflector midpoint has horizontal dip / LF components removed.

Focussing property of HO/VO image volume

Regard prestack image as

- filtered HOCIGs + VOCIGs

Then: at correct velocity, energy is focussed at zero offset in both HOCIGs and VOCIGs within an offset "corridor" of width $h_{\min}$ - depends on amount of ray bending, qualitative version of TIC assumption.

Proof: see RTSGM.

Note that apparently image artifacts may exist at large enough offsets, in contrast to DSR case. Future project: illustrate the existence, extent of such artifacts, explore implications for VA.

Velocity Analysis and Nonlinear Inverse Scattering

Overview of past, present, planned TRIP efforts on velocities

- A common framework for VA
- Differential semblance
- Nonlinear inverse scattering via an analogue of standard MVA
- A nonlinear version of S-G MVA

A common framework for VA

Constant Density Acoustic Model

acoustic potential $u(\mathbf{x}, t)$, *sound velocity* $c(\mathbf{x})$ related to pressure p and particle velocity $\mathbf{v}$ by

$$p = \frac{\partial u}{\partial t}, \;\; \mathbf{v} = \frac{1}{\rho}\nabla u$$

Second order wave equation for potential

$$\left(\frac{1}{c(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u(\mathbf{x}, t) = w(t)\delta(\mathbf{x} - \mathbf{x}_s)$$

plus initial, boundary conditions.

Forward map: $\mathcal{F}[c] \equiv p|_Y$, where $Y = \{(t, \mathbf{x}_r, \mathbf{x}_s) : 0 \leq t \leq T, ...\}$ is *acquisition manifold*.

(Partly) linearized inverse scattering

Formally, $\mathcal{F}[v(1+r)] \simeq \mathcal{F}[v] + F[v]r$ where $F[\cdot]$ is *linearized forward map* defined by

$$\left(\frac{1}{v(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\delta G(\mathbf{x}_s,\mathbf{x},t) = 2\frac{r(\mathbf{x})}{v^2(\mathbf{x})}\frac{\partial^2 G}{\partial t^2}(\mathbf{x}_s,\mathbf{x},t)$$

$$F[v]r = \left.\frac{\partial \delta G}{\partial t}\right|_Y$$

- basis of most practical data processing procedures.
- v is no more known than r, inverse problem for $[v,r]$ still nonlinear!
- linearization error contains many effects observable in field data, notably **multiple reflections**, which can be quite strong, or even dominant - *so major open issue in this subject is how to go beyond linearization!!!*

Extended models

Extension of $F[v]$ (aka *extended model*): manifold $\bar{X}$ and maps $\chi : \mathcal{E}'(X) \to \mathcal{E}'(\bar{X})$, $\bar{F}[v] : \mathcal{E}'(\bar{X}) \to \mathcal{D}'(Y)$ so that

$$\begin{array}{ccccc} & & \bar{F}[v] & & \\ & \mathcal{E}'(\bar{X}) & \rightarrow & \mathcal{D}'(Y) & \\ \chi & \uparrow & & \uparrow & \mathrm{id} \\ & \mathcal{E}'(X) & \rightarrow & \mathcal{D}'(Y) & \\ & & F[v] & & \end{array}$$

commutes, i.e.

$$\bar{F}[v]\chi r = F[v]r$$

Extension is "invertible" iff $\bar{F}[v]$ has a *right parametrix* $\bar{G}[v]$, i.e. $I - \bar{F}[v]\bar{G}[v]$is smoothing, or more generally if $\bar{F}[v]\bar{G}[v]$ is pseudodifferential ("inverse except for wrong amplitudes"). Also require existence of a left inverse η for χ: $\eta\chi = \mathrm{id}$.

NB: The trivial extension - $\bar{X} = X, \bar{F} = F$ - is virtually never invertible.

Grand Example

The Standard Extended Model: $\bar{X} = X \times H$, $H =$ offset range.

$\chi r(\mathbf{x}, \mathbf{h}) = r(\mathbf{x})$, $\eta \bar{r}(\mathbf{x}) = \frac{1}{|H|} \int_H dh\, \bar{r}(\mathbf{x}, \mathbf{h})$ ("stack").

$\bar{r} \in$ range of $\chi \Leftrightarrow$ plots of $\bar{r}(\cdot, \cdot, z, \mathbf{h})$ ("(prestack) image gathers") appear *flat*.

$$\bar{F}[v]\bar{r}(\mathbf{x}_r, \mathbf{x}_s, t) = \frac{\partial^2}{\partial t^2} \int dx \int d\tau\, G(\mathbf{x}, \mathbf{x}_r, t - \tau) G(\mathbf{x}, \mathbf{x}_s, \tau) \frac{2\bar{r}(\mathbf{x}, \mathbf{h})}{v^2(\mathbf{x})}$$

(recall $\mathbf{h} = (\mathbf{x}_r - \mathbf{x}_s)/2$)

NB: $\bar{F}$ is "block diagonal" - family of operators (FIOs) parametrized by $\mathbf{h}$.

Reformulation of inverse problem

Given d, find v so that $\bar{G}[v]d \in$ the range of χ.

Claim: if v is so chosen, then $[v, r]$ solves partially linearized inverse problem with $r = \eta\bar{G}[v]d$.

Proof: Hypothesis means

$$\bar{G}[v]d = \chi r$$

for some r (whence necessarily $r = \eta\bar{G}[v]d$), so

$$d \simeq \bar{F}[v]\bar{G}[v]d = \bar{F}[v]\chi r = F[v]r$$

Q. E. D.

Application: Migration Velocity Analysis

Membership in range of χ is *visually evident*

$\Rightarrow$ industrial practice: adjust parameters of v *by hand* (!) until visual characteristics of $\mathcal{R}(\chi)$ satisfied - "flatten the image gathers".

For the Standard Extended Model, this means: until $\bar{G}[v]d$ is independent of $\mathbf{h}$.

Practically: insist only that $\bar{F}[v]\bar{G}[v]$ be pseudodifferential, so adjust v until $\bar{G}[v]d$ is "smooth" in $\mathbf{h}$.

Differential semblance

Automating the reformulation

Suppose $W : \mathcal{E}'(\bar{X}) \to \mathcal{D}'(Z)$ annihilates range of χ:

$$\mathcal{E}'(X) \overset{\chi}{\to} \mathcal{E}'(\bar{X}) \overset{W}{\to} \mathcal{D}'(Z) \to 0$$

and moreover W is bounded on $L^2(\bar{X})$. Then

$$J[v;d] = \frac{1}{2}\|W\bar{G}[v]d\|^2$$

minimized when $[v, \eta\bar{G}[v]d]$ solves partially linearized inverse problem.

Construction of *annihilator* of $\mathcal{R}(F[v])$ (Guillemin, 1985):

$$d \in \mathcal{R}(F[v]) \Leftrightarrow \bar{G}[v]d \in \mathcal{R}(\chi) \Leftrightarrow W\bar{G}[v]d = 0$$

Annihilators, annihilators everywhere...

For Standard Extended Model, several popular choices:

- $$W = (I - \Delta)^{-\frac{1}{2}}\nabla_{\mathbf{h}}$$

  ("differential semblance" - WWS, 1986)

- $$W = I - \frac{1}{|H|}\int dh$$

  ("stack power" - Toldi, 1985)

- $$W = I - \chi F[v]^{\dagger}\bar{F}[v]$$

  $\Rightarrow$ minimizing $J[v,d]$ equivalent to reduced least squares.

But not many are good for much...

Since *problem is huge and data is noisy*, only W giving rise to differentiable $v, d \mapsto J[v, d]$ are useful - must be able to use Newton!!! Once again, idealize $w(t) = \delta(t)$.

Theorem (Stolk & WWS, 2003): $v, d \mapsto J[v, d]$ smooth $\Leftrightarrow$ W pseudodifferential.

i.e. only *differential semblance* gives rise to smooth optimization problem even with noisy data.

Some theory, many successful numerical tests of differential semblance using synthetic and field data: WWS et al., Chauris & Noble 2001, Mulder & tenKroode 2002. deHoop et al. 2004.

Nonlinear inverse scattering via an analogue of standard MVA

A nonlinear common-shot extension

Simply replace $\bar{F}$ by an extension of $\mathcal{F}$:

$$\left(\frac{1}{\bar{c}(\mathbf{x},\mathbf{x}_s)^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u(\mathbf{x},t) = w(t)\delta(\mathbf{x}-\mathbf{x}_s)$$

plus initial, boundary conditions.

Extended Forward map: $\bar{\mathcal{F}}[\bar{c}] \equiv p|_Y$, where $Y = \{(t,\mathbf{x}_r,\mathbf{x}_s) : 0 \le t \le T, ...\}$ is *acquisition manifold*.

Extension map: same as for partially linearized common shot extension, i.e. $\chi[c](\mathbf{x},\mathbf{x}_s) = c(\mathbf{x})$.

Q: What replaces the right inverse of the linear extended operator?

Nonlinear common-shot DS

A: Inverse scattering, what else.

A em feasible model $\bar{c}$ at noise level ϵ satisfies

$$\|\bar{\mathcal{F}}[\bar{c}] - d\| \le \epsilon \|d\|$$

Feasible points are easy to find, for extended models!!!

The natural common-shot differential semblance operator is $W = \partial/\partial x_s$.

Nonlinear differential semblance, common shot version:

$$\min_{\bar{c}} \|W\bar{c}\| \,\mathrm{subj}\, \|\bar{\mathcal{F}}[\bar{c}] - d\| \le \epsilon \|d\|$$

Nonlinear common-shot DS - implementation

$$\min_{\bar{c}} \|W\bar{c}\| \text{ subj } \|\bar{\mathcal{F}}[\bar{c}] - d\| \leq \epsilon \|d\|$$

Inequality constrained optimization problem, (relatively) easy access to feasible points $\Rightarrow$ interior point method.

Classic IPM = *log-barrier* method (Fiacco & McCormack 1967): (1) initialize penalty parameter μ; (2) while (not satisfied) (i) minimize log-barrier function

$$\|W\bar{c}\|^2 - \mu \log(\epsilon \|d\|^2 - \|\bar{\mathcal{F}}[\bar{c}] - d\|^2)$$

(ii) when gradient of log-barrier function small enough, reduce μ and do it again.

Status: log-barrier method implemented, being tested. Next: couple to already-implemented operator, gradient computations.

A nonlinear version of S-G MVA

Invertible Extensions

Beylkin (1985), Rakesh (1988): if $\|\nabla^2 v\|_{C^0}$ "not too big" (no caustics appear), then the Standard Extension is invertible.

Nolan & WWS 1997, Stolk & WWS 2004: if $\|\nabla^2 v\|_{C^0}$ is too big (caustics, multi-pathing), Standard Extension is **not** invertible! Not in any version - common offset, common source, common scattering angle,...

Brings the whole program to a screeching halt, unless there are *other, inequivalent extensions*.

Claerbout's extension

$\chi r(\mathbf{x},\mathbf{h}) = r(\mathbf{x})\delta(\mathbf{h})$, $\eta\bar{r}(\mathbf{x})$ "=" $\bar{r}(\mathbf{x},\mathbf{0})$ (Claerbout's zero-offset imaging condition)

$\bar{r} \in$ range of $\chi \Leftrightarrow$ plots of $\bar{r}(\cdot,\cdot,z,h)$ (i.e. *image gathers*) appear *focussed* at $\mathbf{h}=0$

$$\bar{F}[v]\bar{r}(\mathbf{x}_r,\mathbf{x}_s,t) = \frac{\partial^2}{\partial t^2}\int dx \int dh \int d\tau\, G(\mathbf{x}+\mathbf{h},\mathbf{x}_r,t-\tau)G(\mathbf{x}-\mathbf{h},\mathbf{x}_s,\tau)\frac{2\bar{r}(\mathbf{x},\mathbf{h})}{v^2(\mathbf{x})}$$

This extension is invertible, assuming (i) $\bar{r}(\mathbf{x},\mathbf{h}) = \hat{r}(\mathbf{x},h_1,h_2)\delta(h_3)$ (horizontal offset only) and (ii) "DSR hypothesis": waves propagate up and down, not sideways ("rays do not turn") [Stolk-DeHoop 2001] and sometimes under more general conditions [RTSGM].

Differential Semblance for Claerbout's Extension

$$W\bar{r}(\mathbf{x},\mathbf{h}) = \mathbf{h}\bar{r}(\mathbf{x},\mathbf{h}),\;\; J[v,d] = \frac{1}{2}\|W\bar{G}[v]d\|^2$$

Same smoothness properties as DS for Standard Extension.

P. Shen (2004): implementation, optimization via quasi-Newton algorithm, synthetic and field data.

Conclusion: successfully estimates v in settings (strong refraction) in which Standard Extension based DS fails.

Claerbout's Extension as a linearization

Write differential equation for $\bar{F}[v]$, by applying wave operator to both sides of integral representation: $\bar{F}[v]r = \delta\bar{u}|_Y$ where

$$\left(v^{-2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\delta\bar{u}(\mathbf{x},\mathbf{x}_s,t) = \int_H dh\, 2\bar{r}(\mathbf{x}-\mathbf{h},\mathbf{h})v^{-2}(\mathbf{x}-\mathbf{h})\frac{\partial^2 G}{\partial t^2}(\mathbf{x}-2\mathbf{h},\mathbf{x}_s,t)$$

Observe that this equation describes the linearization of the system

$$V^{-2}\left[\frac{\partial^2 u}{\partial t^2}\right] - \nabla^2 u(\mathbf{x},\mathbf{x}_s,t) = w(t)\delta(\mathbf{x}-\mathbf{x}_s),$$

in which the "velocity" V is an *operator*: formally,

$$Vw(\mathbf{x}) = \int_H dh\, K_V(\mathbf{x}-\mathbf{h},\mathbf{h})w(\mathbf{x}-2\mathbf{h})$$

and the linearization takes place at V with $K_V(\mathbf{x},\mathbf{h}) = v(\mathbf{x})\delta(\mathbf{h}) = \chi v(\mathbf{x},\mathbf{h})$.

The Nonlinear Claerbout Extension

That is, you can view Claerbout's extension of the linearized scattering problem as the linearization of an extension of the original scattering problem:

$$v^{-2}\left[\frac{\partial^2 u}{\partial t^2}\right] - \nabla^2 u(\mathbf{x}, \mathbf{x}_s, t) = w(t)\delta(\mathbf{x} - \mathbf{x}_s),$$

where v is the operator of multiplication by the positive function v, *versus*

$$V^{-2}\left[\frac{\partial^2 u}{\partial t^2}\right] - \nabla^2 u(\mathbf{x}, \mathbf{x}_s, t) = w(t)\delta(\mathbf{x} - \mathbf{x}_s),$$

with *self-adjoint positive* V.

This generalized nonlinear scattering problem makes sense: J.-L. Lions showed in the late '60s how to demonstrate the well-posedness of the initial value problem for operators like the above, with self-adjoint positive operator coefficients [also Stolk 2000].

Extended Inverse Scattering

The extended inverse scattering problem takes the place of the right inverse map $\bar{G}$ of the linear Claerbout extension: define the *extended forward map* $\bar{\mathcal{F}}$ by $\bar{\mathcal{F}}[V] = u|_Y$, where u solves

$$V^{-2}\left[\frac{\partial^2 u}{\partial t^2}\right] - \nabla^2 u(\mathbf{x}, \mathbf{x}_s, t) = w(t)\delta(\mathbf{x} - \mathbf{x}_s),$$

plus appropriate initial and boundary conditions. Given a nominal noise level ϵ, an ϵ-solution of the extended inverse scattering problem is a positive self-adjoint V so that

$$\|\bar{\mathcal{F}}[V] - d\| \leq \epsilon\|d\| \qquad (1)$$

In itself, this problem is grossly underdetermined - so use it as a constraint!

Nonlinear Differential Semblance

Natural differential semblance op for Claerbout extension: W = multiply by $\mathbf{h}$. The *nonlinear differential semblance* problem is: given d, ϵ, find V to minimize

$$\min_V \|WK_V\|^2 \,\mathrm{subj}\, \|\bar{\mathcal{F}}[V] - d\| \leq \epsilon \|d\|$$

where K_V is the distribution kernel of V.

Many open questions to be studied in near future, for instance:

- What is a good class of operators? Must have well-behaved kernels!
- How to sensibly define the norm on WK_V.
- Economical implementation?