The Rice Inversion Project

William W. Symes

Annual Review for Project Year 2005 - Jan 2006

The Rice Inversion Project

- University-industry research consortium in seismic inversion
- Directed since 1992 by W. W. Symes, Computational and Applied Mathematics
- Total research expenditure, industry funds: $\sim$ \$1.8 M
- 2005 Sponsors: Amerada Hess, ConocoPhillips, ExxonMobil, Landmark Graphics, Shell, Total
- `www.trip.caam.rice.edu`

Goals

- Contribute to the solution of mathematical and computational problems arising in seismic prospecting and its industrial applications.
- Train applied mathematicians at graduate and postdoctoral levels who can (i) communicate with scientists and engineers, (ii) identify mathematical and computational issues in scientific and engineering applications, and (iii) bring to bear appropriate mathematical and computational ideas and tools to solve them.

TRIP Alumnae, 1992-2004

- *Postdocs:* Kidane Araya, Philippe Ecoublet, Chaoming Zhang, Seongjai Kim, Michel Kern, Senren Liu, Lucio Santos, Alan Sei Huy Tran, Roelof Versteeg, Christiaan Stolk
- *PhD Students:* Clifford Nolan, Maissa abd El-Mageed, Joakim Blanch, Susan Minkoff, Jianliang Qian, Hua Song, Mark Gockenbach, Peng Shen, Eric Dussaud
- *MA Students:* Nate Winslow, Shannon Scott, Regina Hill, Hala Dajani

TRIP Contributions

- Differential semblance velocity analysis - concept and various implementations (WWS,...,Shen)
- Eikonal solvers: first fast upwind solvers, first anisotropic and high-order solvers (van Trier & WWS, Kim, Qian)
- Eulerian methods for multiarrival traveltime computation - slowness matching
- GRT modeling, migration, inversion with Eikonal-derived inputs
- FD methods for viscoelasticity with controlled numerical dispersion (Blanch & Robertsson)
- Iterative inversion for anelastic models (Blanch)
- Joint inversion for source and medium - concept and demonstration (Minkoff, Winslow)
- Stability of velocity analysis via interferometry (Dussaud)

TRIP Staff, 2005-6

- Director: William W. Symes
- Postdoc: TBN
- PhD Students: Eric Dussaud (stochastic effects in velocity analysis, PhD defended 12/12/05, currently Total E&P USA), Jintan Li (differential semblance velocity analysis via Kirchhoff migration, MS expected 5/06), Sichao Chen (globalizing nonlinear inversion, MS expected 5/06)
- Visiting Student: Alex Khoury (differential semblance velocity analysis using common azimuth migration, supported by Total E&P USA)

TRIP SEG Presentations, 2005

- ,Velocity analysis from interferometric data, Eric A. Dussaud and William W. Symes SPVA 1
- Differential semblance velocity analysis via shot profile migration, Peng Shen, William W. Symes, Scott Morton, and Henri Calendra, SPVA 1
- Fast interval velocity estimation via NMO-based differential semblance, Jintan Li and William W. Symes, SPVA 1
- Kinematics of prestack shot-geophone migration, Christiaan C. Stolk, Maarten de Hoop, and William W. Symes, SPMI 2

Today's Agenda

0900 - 0910: Welcome and Overview (William W. Symes)

0910 - 1000: Student Presentations (Jintan Li, Alex Khoury, Sichao Chen)

1000 - 1015: TRIP Software (Symes)

1015 - 1030: Break

1030 - 1115: Traveltime computation and tomography based on the Liouville equation (Jianliang Qian)

1115 - 1200: Velocity analysis and nonlinear inverse scattering (Symes)

1200 - 1300: Lunch, Cohen House

1300 - 1400: Coffee and Discussion of Project Accomplishments and Directions (1049 DH)

1400: meeting adjourns

TRIP Software

- Released today: DSVA-NMO package
- Early spring (?!): RTFD S-G migration, serial version
- Late spring (Jintan's MS thesis project): DSVA-Kirch
- In planning stages: RTFD S-G migraiton, parallel version + DSVA

All TRIP software is mixed-language (C/C++/F77) and depends on SU and on the *Rice Vector Library* = C++ framework for vector calculus, and on Tony Padula's *Algorithm* package = C++ framework for expression of iterative algorithms.

Each package comes with documentation (self-docs and/or web-browsable) and test/demo suites.

Access method for private TRIP releases: *blind web site*. See printed agenda for URL.

Velocity Analysis and Nonlinear Inverse Scattering

Versions of this talk given at

- American Geophysical Union spring meeting (May 05),
- Colorado School of Mines (June 05),
- Applied Inverse Problems 05 (June 05)
- University of British Columbia (August 05).
- TRIP sponsor labs.

Why Differential Semblance (1)

Velocity analysis driven by *flatness of gathers* - and this is measurable! Several approaches tried in the 80's:

- maximizing stack power / semblance - Toldi, Fowler, al-Yahya;
- output least squares - Tarantola

These optimize a *velocity-dependent quadratic form in inverted or migrated data* - but OLS is notoriously dysfunctional - *many local minima*. Stack power has similar properties.

Necessary for good optimization behaviour with Newton-type methods: objective should be

- smooth in both velocity and data
- unimodal: no spurious stationary points

Why Differential Semblance (2)

- Stolk & WWS, *Inverse Problems* 03, TRIP 02: DS is *only* v-dependent quadratic form in data which is (i) smooth under *arbitrary finite energy data perturbations* and spatially smooth perturbations in v. Stack power and OLS are not smooth in this sense.
- Drop quadratic form requirement: *any* smooth function of (finite energy) data and smooth v, minimized at correct v for noise-free data, *must* be tangent to DS to 2nd order.
- WWS, TRIP 99 & 01: Layered medium DS (based on linearized wave equation or its approximations, eg. HF asymptotics): under reasonable circumstances, DS is *unimodal* - this is true also for finite bandwidth data, for which SP and OLS are (formally) smooth but not unimodal.

What is the best imaging approach for DSVA in laterally heterogeneous media?

Shot-geophone prestack migration

Claerbout (1971): given velocity field $v(x,z)$ (ref. model), compute:

- *source wavefield* $S(x_s; z, \bar{x}_s, t)$ - continue the source at (z_s, x_s) to the "sunken source" at $(z, \bar{x}_s)$, $z > 0$ (i.e. solve wave eqn with *source* as source);
- *receiver wavefield* $R(x_s; z, \bar{x}_r, t)$ - continue recorded data for the source at (z_s, x_s) to the "sunken receiver" at $(z, \bar{x}_r)$ (solve wave eqn with *data* as source);
- *image volume* $\bar{I}(z, \bar{x}_r, \bar{x}_s)$ - time cross-correlate S and R at zero lag, same depth, sum over sources:

$$\bar{I}(z, \bar{x}_s, \bar{x}_r) = \int dx_s \int dt R(x_s, z, \bar{x}_r, t) S(x_s, z, \bar{x}_s, t)$$

- *Claerbout's imaging condition*: extract *image* $I(z,x)$ where sunken source and receiver coincide, $\bar{x}_r = \bar{x}_s = x$ ("zero offset"): $I(z,x) = \bar{I}(z,x,x)$ - related to ordinary Born inversion.

Why S-G? (1)

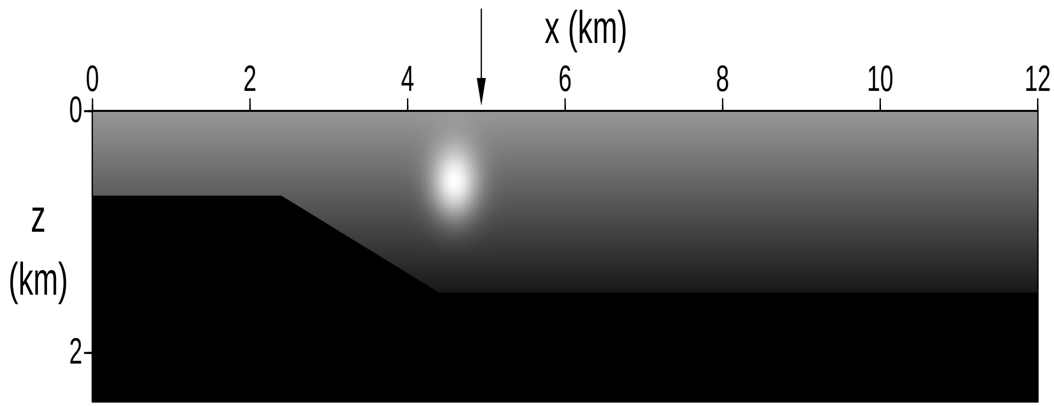
The competition = common bin migration:

- common shot
- common offset
- common scattering angle

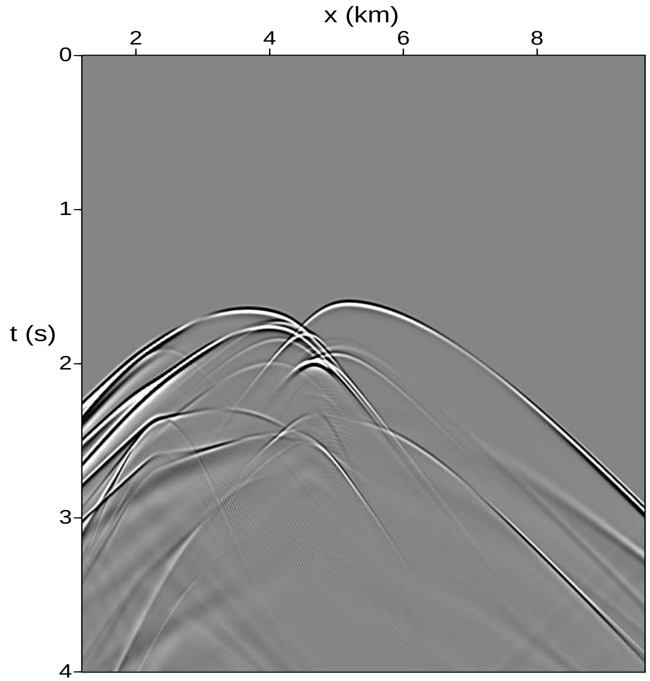
Each data bin imaged independently - for CO and CSA, only Kirchhoff approach is feasible.

Nolan & WWS (TRIP 95, SEG 96, Comm. PDE 97), Stolk & WWS (Geophys. 04): *All* of these methods produce *kinematic artifacts* when v is complex enough to produce multipathing. That is: gathers are *not flat at correct* v.

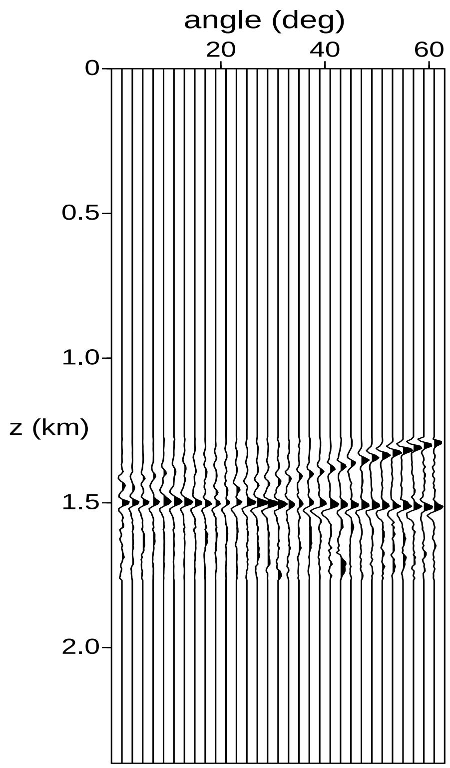
Valhall cartoon

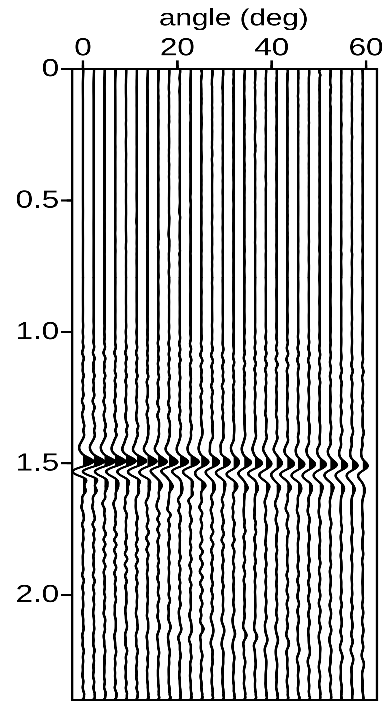


Typical Shot Record



Kirchhoff vs. S-G Angle Gathers





Why S-G? (2)

Kinematic artifact = coherent noise due to energy migrating along wrong ray pair.

Known $v \Rightarrow$ can process these out - but with inexact v can't tell noise from signal - disaster for automatic VA!

Stolk & de Hoop 01 MSRI prepring, Stolk, de Hoop, & WWS 05 (subm. to Geophys.): provided that rays carrying significant energy

- do not turn (necessary for depth extrap. implementation!)
- are determined by data phases (always in 2D and "true" 3D, sometimes for narrow azimuth)

S-G image volumes are *free of kinematic artifacts*: offset gathers are focussed, angle gathers are flat when velocity is correct.

Objective velocity analysis via S-G

Based on *Claerbout's focusing principle:* Velocity correct $\Rightarrow$ image volume $\bar{I}[v]$ *focuses* at zero (subsurface) half offset $h = (\bar{x}_r - \bar{x}_s)/2$, i.e. exhibits essentially no energy at $|h| > 0$.

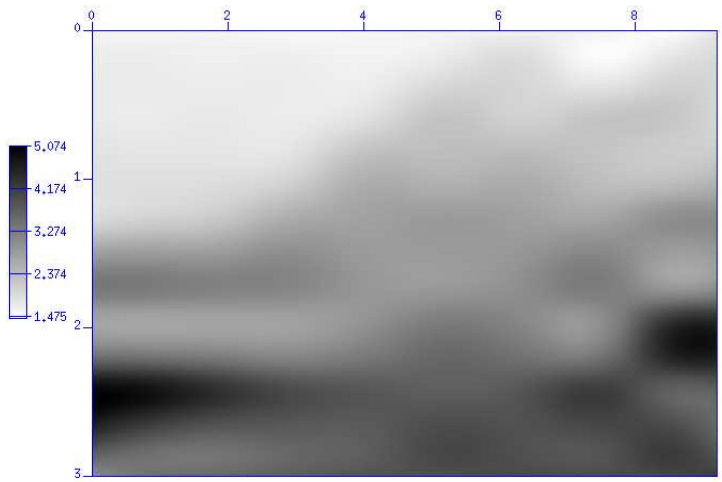
How to measure focusing at $h = 0$: multiply by h! If product is *big* RMS, image is unfocused, velocity is *wrong*. If product is *small* RMS, image is focused, velocity is *right*.

Focusing as an optimization problem: minimize $\|h\bar{I}[v]\|^2$ over v ($\|\cdot\|$ = L^2 norm).

Minor extension of Stolk & S., IP 2003: this is essentially the only nontrivial quadratic form in image volume which (a) varies smoothly as function of v and d, and (b) vanishes for focused $\bar{I}$.

First results: Shen et al, SEG 2003. Following example also due to P. Shen.

Objective velocity analysis in Marmousi: initial velocity, image



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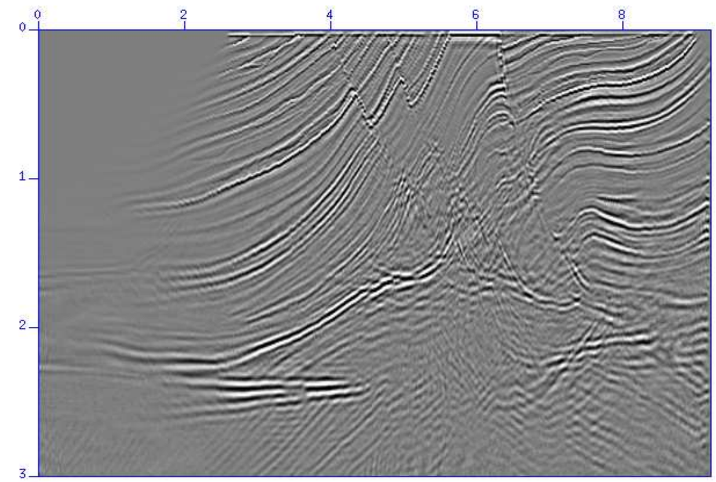
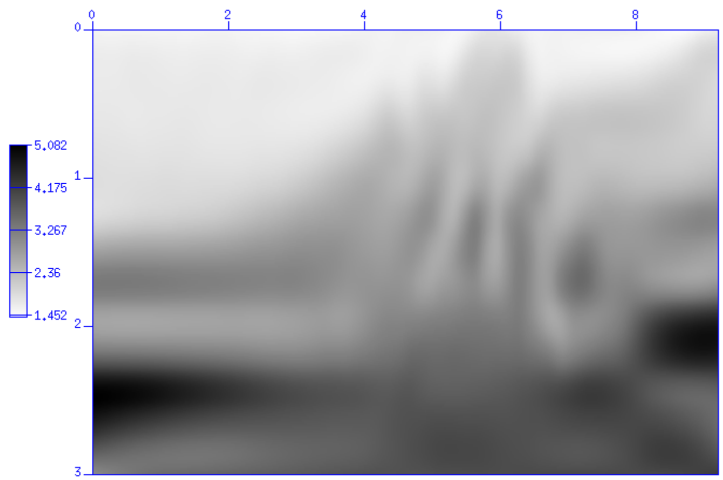


Left: Initial velocity **Right:** image ($h = 0$ section from volume)

Objective velocity analysis in Marmousi: final velocity, image



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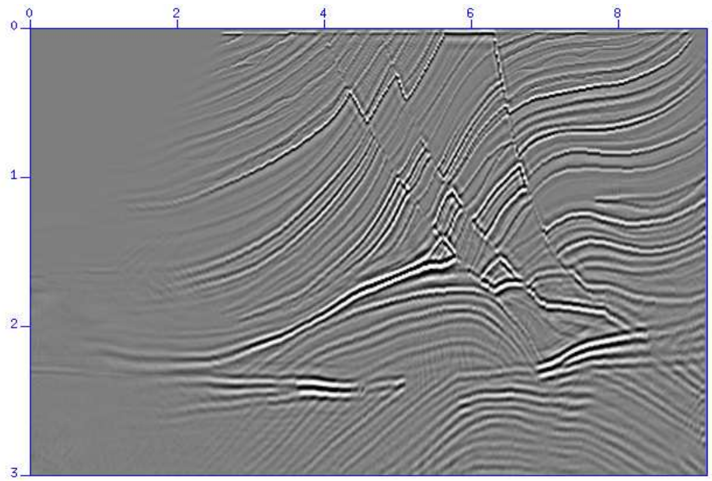
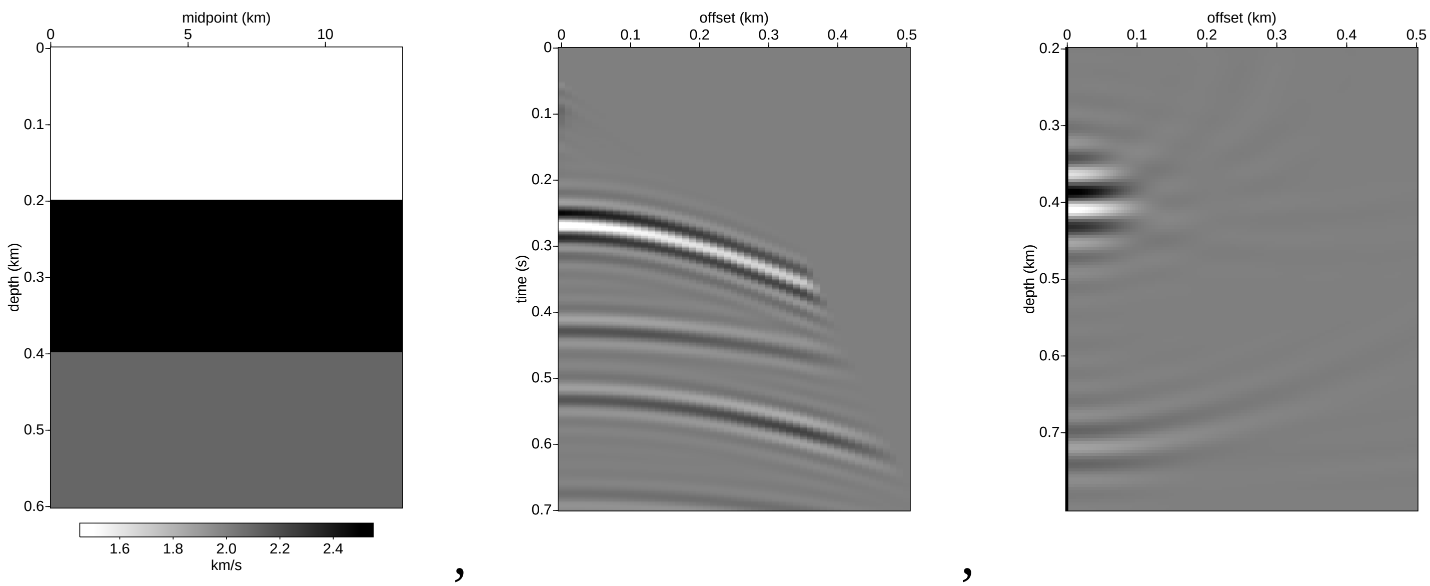


Left: Final estimated velocity **Right:** image ($h = 0$ section from image volume) after 47 iterations of LMBFGS. Pretty good image - but input is Born data!!!

MVA does not account for multiple reflections



Left: Three layer model **Center:** Data - source wavelet = 4-10-30-40 Hz bandpass. Free surface multiple is about same size as second primary. **Right:** S-G Migration at good v - note focused primary, defocused image of surface multiple.

What to do about multiply reflected energy?

- suppress it: the traditional option - predictive decon,...., SRME. Not always easy!
- suppress on the basis of moveout *during VA*:
  - Gockenbach & WWS, TRIP 98, SEG 99: *extremal regularizaton* (constrained opt problem to perturb data as well as velocity);
  - Mulder & Plessix 01: multiples are *slower* - can be discriminated from primaries in *image volume* via Fourier/dip filter [Demo using DSVA-NMO...]
- but multiple reflection is physical - why not include it in the model that drives VA?

Nonlinear Least Squares Inversion

Modeling operator give by $v \to D$, where $D(x_s; x_r, t) = P(x_s; z, x, t)_{z=z_r, x=x_r}$

$$u(z,x)\frac{\partial^2 P}{\partial t^2}(x_s; z, x, t) - \nabla^2_{z,x} P(x_s; z, x, t) = w(t)\delta(z - z_s)\delta(x - x_s)$$

and $u = v^{-2}$ (*square slowness*).

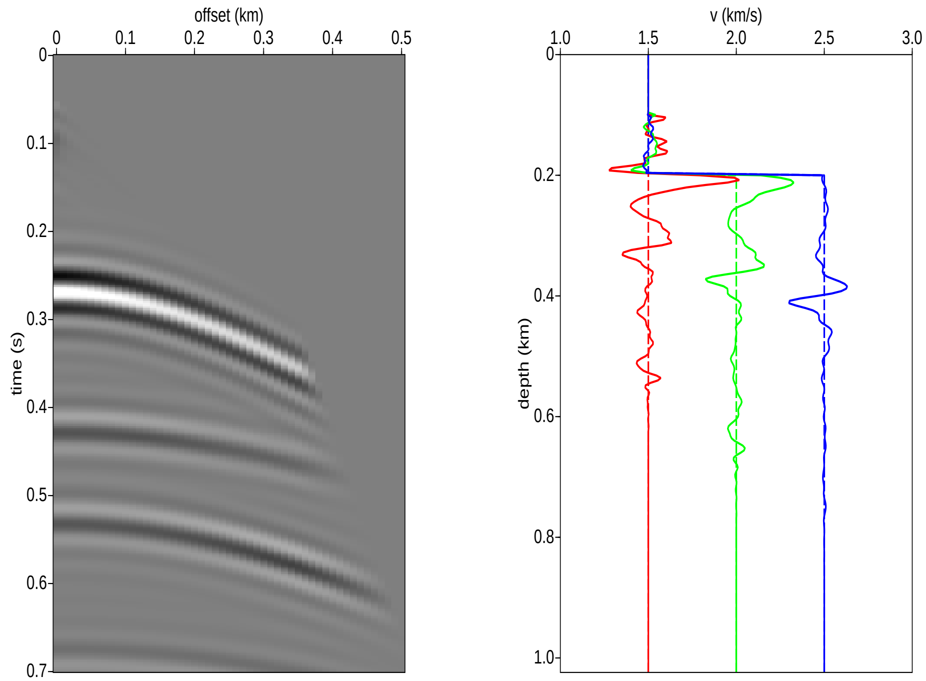
Least squares inversion: given data D^{obs}, adjust v (or u) so that predicted data $D[u]$ fits observed data as well as possible:

$$\text{minimize}_u \, \|D[u] - D^{\text{obs}}\|^2$$

(some regularization usually a good idea).

Upshot of much work from 80's on: LS inversion (i) handles multiples, i.e. does not produce artifact images due to multiples, but (ii) requires a very good initial estimate - *domain of attraction* of global minimizer has small "measure".

NLS does not make large velocity updates



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Left: Data from three layer model **Right:** Inversions (solid lines) from three initial v's (dashed lines). 30-40 its of LMBFGS, redn in gradient length by 10^{-2}.

Nonlinearizing MVA, step 1

Get from linear to nonlinear in two steps:

(1) recognize that shot-geophone prestack migration op $d \rightarrow \bar{I}$ is *adjoint* of *extended Born modeling op* $\bar{I} \rightarrow d$; modeling op given by $d(x_s; x_r, t) = p(x_s; z, x, t)_{z=z_r, x=x_r}$ where z_r = recvr depth and

$$\left(\frac{1}{v(z,x)^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2_{z,x} p\right)(x_s; z, x, t) = \int d\bar{x} S(x_s; z, \bar{x}, t)\bar{I}(z, x, \bar{x})$$

(this is best seen using Green's functions to represent solutions);

If $\bar{I}$ is *physical* = *focused* at zero offset, i.e. $\bar{I}(z, x, \bar{x}) = I(z, x)\delta(x - \bar{x})$ with $I = 2\delta v / v^3$, then extended Born modeling specializes to ordinary Born modeling.

Nonlinearizing MVA, step 2

(2) recognize that preceding eqn is *perturbation equation* of *extended model*

$$\int d\bar{x} U(z,x,\bar{x}) \frac{\partial^2 P}{\partial t^2}(x_s;z,\bar{x},t) - \nabla^2_{z,x} P(x_s;z,x,t) = w(t)\delta(z-z_s)\delta(x-x_s)$$

That is, *replace velocity (or square slowness) with SPD bounded operator* - existence theory for such problems due to Lions, late 60's.

If $U(z,x,\bar{x}) \simeq v(z,x)^{-2}\delta(x-\bar{x}) + \bar{I}(z,x,\bar{x})$, then $P(x_s;z,x,t) \simeq S(...) + p(...)$.

In particular, if U is *physical* = *focused* at zero offset, i.e. $U(z,x,\bar{x}) = v^{-2}(z,x)\delta(x-\bar{x})$, then the extended model becomes the ordinary acoustic model.

Extended NLS

Given *observed data* $D^{\rm obs}(x_r, x_s, t)$, find extended sqr slowness $U(z, x, \bar{x})$ so that *predicted data* $D[U](x_s; x_r, t) = P(x_s; z, x, t)_{z=z_r, x=x_r}$ fits observed data: $D[U] \simeq D^{\rm obs}$. [Formulate as least squares, use nonlinear optimization, blah, blah, blah....]

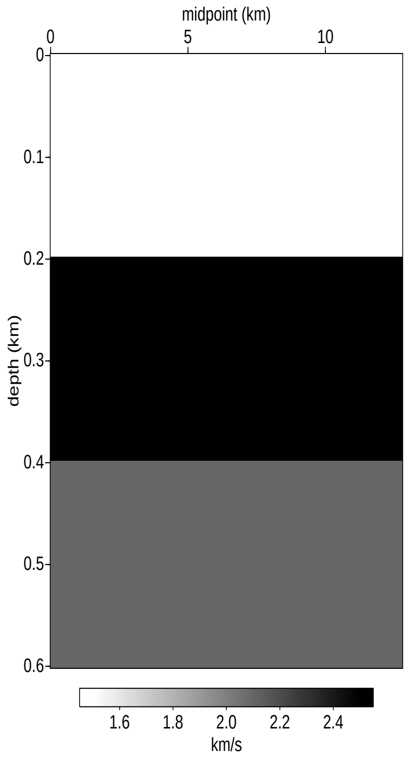
This problem is *underdetermined*: can fit data equally well with many extended sqr slownesses.

BUT: *physical* sqr slownesses *focuses at zero offset* [Claerbout redux!]:
$U(z, x, \bar{x}) \simeq v(z, x)^{-2}\delta(x - \bar{x})$

Hypothesis: focusing of extended square slowness at zero offset $\Leftrightarrow$ correct kinematics for primary reflections (like MVA), multiple energy assigned to primary reflectors ("multiples suppressed in image", like NLS).

Initial numerical exploration: layered models $\Rightarrow$ U is convolution op in x variables $\Rightarrow$ diagonalized by cosine transform $\Rightarrow$ cheap!

Model and data



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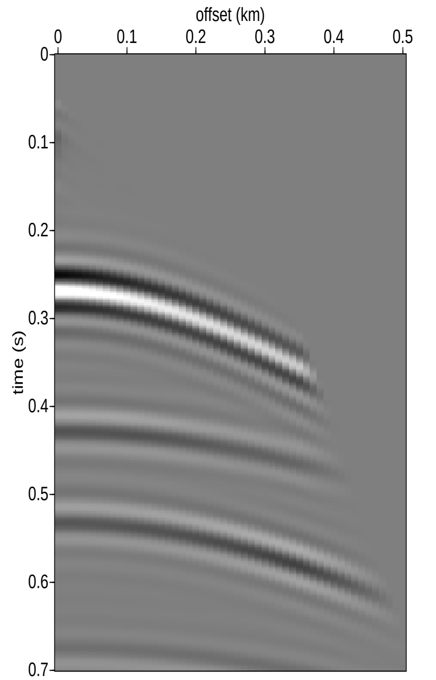
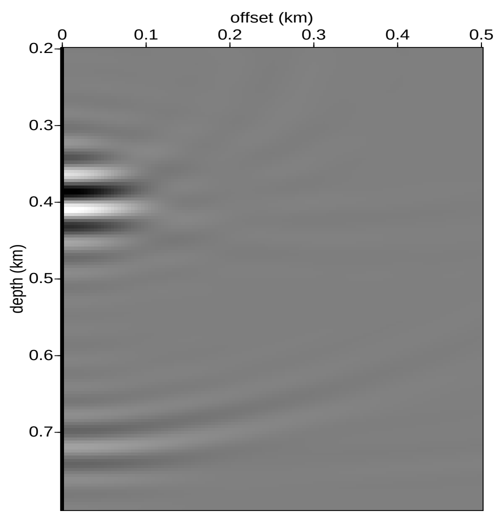
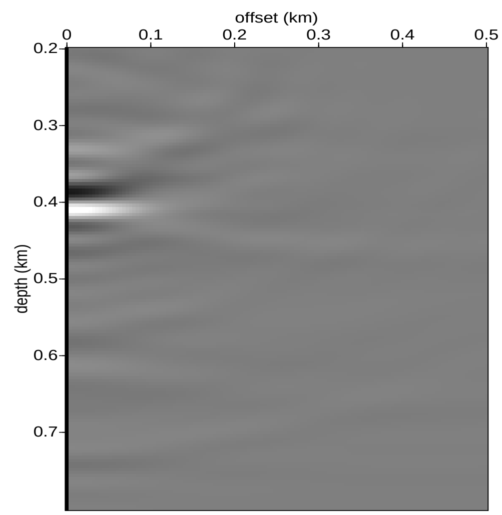


Left: Three layer model **Right:** Data - source wavelet = 4-10-30-40 Hz bandpass. Free surface multiple is about same size as second primary.

Migration vs. Inversion





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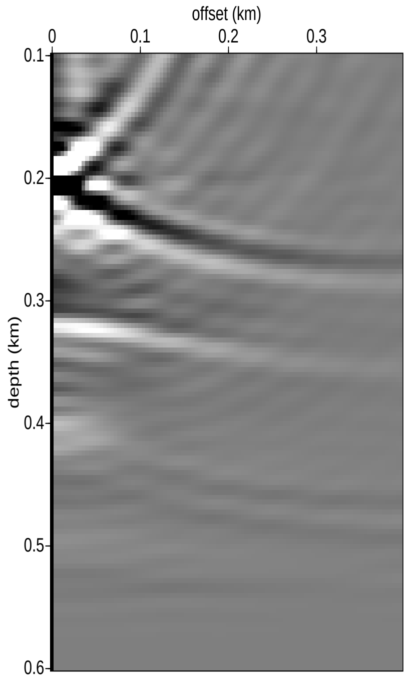
Left: Migration of three-layer data at $v = 1.5$ km/s for $z < 0.2km$, else $= 2.5$ km/s. **Right:** inversion, $\sim$ 40 LMBFGS iterations beginning at migration v. Note disappearance of migrated multiple.

Three extended NLS inversions



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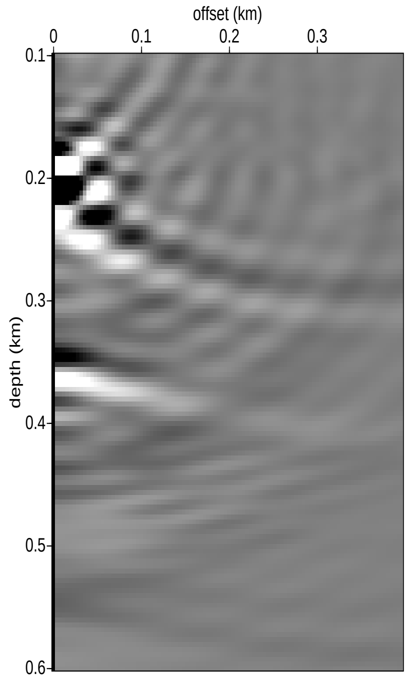

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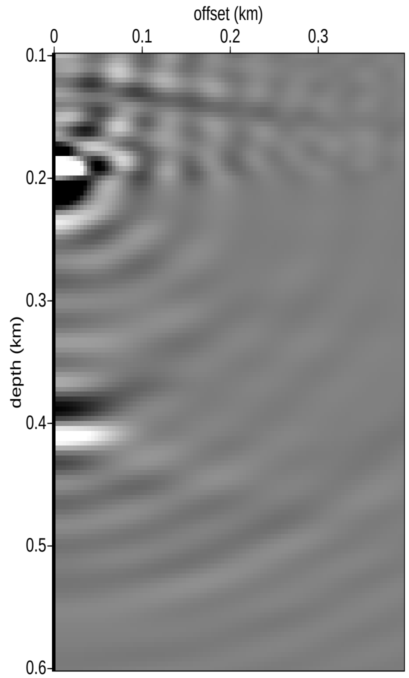
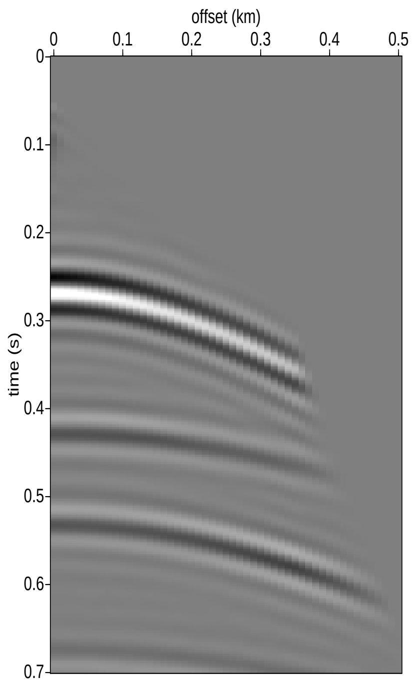


Different initial estimates of extended square slowness U (same as for NLS example), then LMBFGS until fit error reduced to $< 10^{-2} \times \|D^{\rm obs}\|$.

Resimulations (predicted data)



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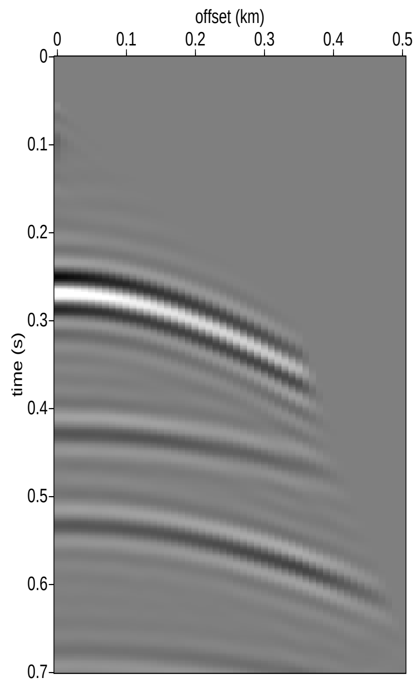

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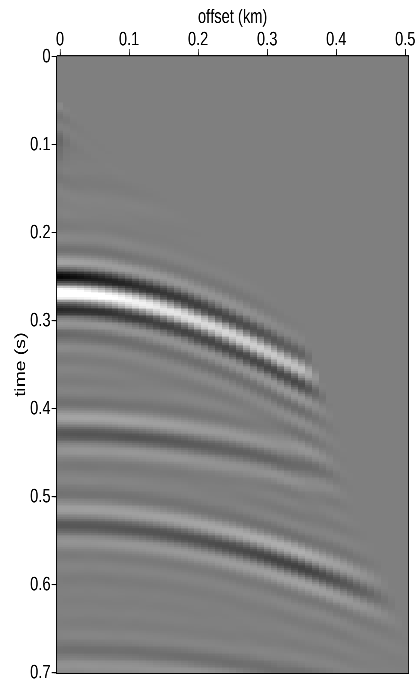
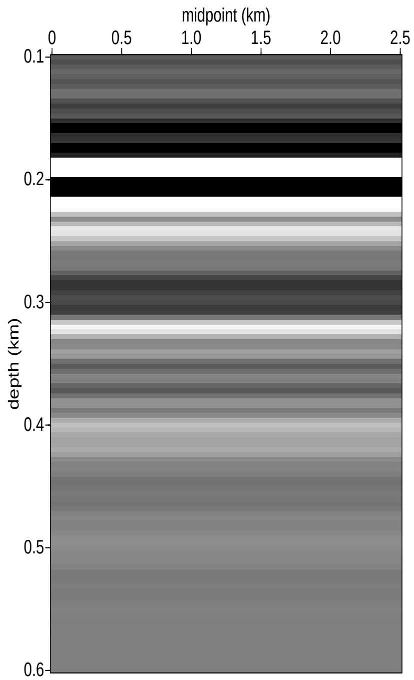


All three fit the data equally well...

Images (filtered zero offset sections)



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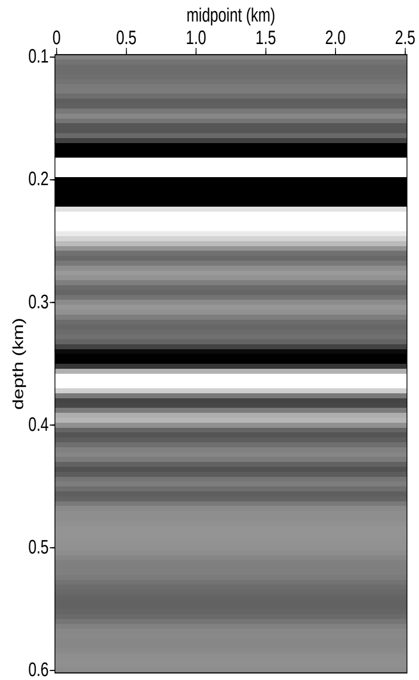

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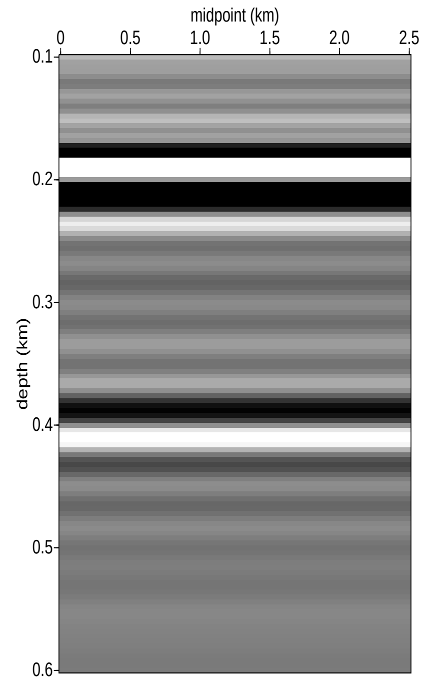

But only focused extended velocity produces image with correct reflector depths and multiple energy assigned to primaries.

Summary

- MVA can be formulated as optimization problem amenable to Newton: can recover from large initial errors in v, but based on Born approximation $\Rightarrow$ degraded by nonlinear effects in data (multiple scattering).
- NLS accomodates any modeled physics, linear or nonlinear, but cannot recover from large initial errors in v.
- Migration operator = adjoint to *extended* linearized modeling operator
- Focusing criterion for *nonlinear extended model* generalizes both MVA and NLS - and does this by generalizing S-G migration [there are simpler variants generalizing binwise migration], so some hope for complex structure!

Outlook

Automation: apply focusing condition via constrained least squares (as in MVA):

$$\mathrm{minimize}_U \, \|hU\|^2 \text{ subject to } \|D[U] - D^{\mathrm{obs}}\| \leq \epsilon$$

Two major obstacles to making this work ("research opportunities"):

(1) simulation: can't afford full matrix multiply in every time step - must find basis in which U is *sparse*, analog of cosine transform for layered models - windowed Fourier / curvelets?

(2) optimization: in MVA, (smooth) velocity parametrizes solutions, allows efficient *reduced basis* approach, long steps within very curvy *feasible set* of models fitting data. What is replacement in nonlinear setting?

Plans

- Jintan project: DSVA via CO Kirchhoff and collaboration with J-L Qian - good for mild structure
- Sichao project: nonlinear p-bin DS for layered acoustics - sandbox for theory
- Alex project: DSVA via DSR/Common Azimuth migration - practical?
- Multidimensional (linear) DSVA theory, either binwise or S-G - can we generalize layered result?
- Implementation of RT S-G DSVA - parallelization necessary
- Nonlinear DSVA: more numerical experimentation with layered case.
- Nonlinear theory: What is needed to get a handle on properties of nonlinear extension? Need *much* better understanding of wavefield response to *discontinuous* velocity perturbations. Even layered medium question is interesting (mathematically).