Optimal Scaling of Prestack Migration

William W. Symes[⋆] and Eric Dussaud[†]

[⋆]Rice University, [†]Total E&P USA

SEG 07

Agenda

1. Migration vs. Inversion

2. Acoustic Modeling and Migration

3. The Structure Theorem

4. From Migration to Approximate Inversion

5. Conclusions and Prospects

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Reverse Time Migration

Based on acoustic or p-wave model, velocity $v(x, z)$.

Basic concept: reflectors occur at coincidence of downgoing, upcoming fields (Claerbout, 1971).

Data: $\{d(x_s, z_s, x_r, , z_r, t)\}$, $(x_r, z_r) \in \mathcal{R}(x_s, z_s)$

Downgoing or reference or source or direct field: $S(x_s, z_s, x, z, t)$, solves initial value problem (forward in time)

$$\left[\frac{1}{v^2}\frac{\partial^2 S}{\partial t^2} - \nabla^2 S\right](x_s, z_s, x, z, t) = w(t)\delta(x - x_s)\delta(z - z_s)$$

$$S = 0,\ t << 0$$

Reverse Time Migration

Upcoming or adjoint or receiver field: $R(x_s, z_s, x, z, t)$, solves final value problem (backward in time)

$$\left[\frac{1}{v^2}\frac{\partial^2 R}{\partial t^2} - \nabla^2 R\right](x_s, z_s, x, z, t)$$

$$= \sum_{(x_r . z_r)\in\mathcal{R}(x_s,z_s)} d(x_s, z_s, x_r, z_r, t)\delta(x - x_r)\delta(z - z_r)$$

$$R = 0,\; t >> 0$$

Reverse Time Migration

Image output by *imaging condition*, of which there are several, for example:

Crosscorrelation of source, receiver fields

$$I_{CC}(x,z) = \sum_{x_s,z_s} \int dt\, S(x_s,z_s,x,z,t) R(x_s,z_s,x,z,t)$$

Adjoint State (explanation to come!)

$$I_{AD}(x,z) = 2v(x,z) \sum_{x_s,z_s} \int dt\, \nabla^2 S(x_s,z_s,x,z,t) R(x_s,z_s,x,z,t)$$

Reverse Time Migration

I_{CC}, I_{AD} are *images*: events placed correctly in depth, provided that

- d is primaries-only;
- v is kinematically correct (traveltimes to within 1/4 wavelength);
- v *nonreflecting*, i.e. slowly varying on wavelength scale (fixes available if not!).

What about image amplitudes, i.e. predicted reflector strengths?

Synthetic Example: Marmousmooth

Marmousi synthetic model: proposed in 1989 by IFP: "2D Earth" after actual play, offshore West Africa.

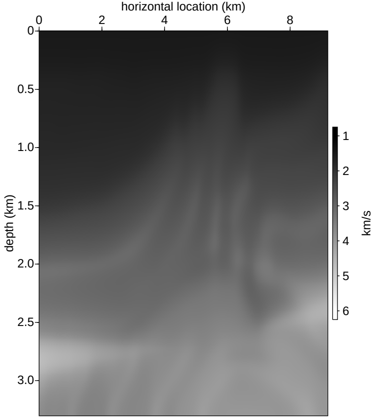
Modeled acquisition geometry similar to original. Same: 240 source points ($\mathbf{x}_s$) spaced 25 m apart, each recorded by 96 receivers also 25 m apart, min offset 150 m. Source depth = 6 m, receiver depth = 5 m.

Different: sources are point (rather than array), wavelet = zero-phase 5-13-40-55 Hz bandpass filter, absorbing boundary at $z = 0$.

"Primaries only" requirement realized via linearized ("Born") simulation: separate velocity into (i) smooth background v, smoothed by spatial Gaussian filter with half-power width of 80 m, and (ii) perturbation δv = difference of original and smoothing with 20 m Gaussian.

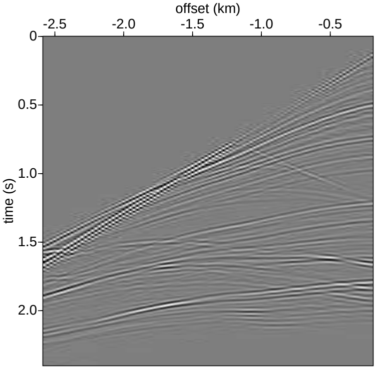
Computation: (2,4) centered difference scheme.

Synthetic Example: Marmousmooth



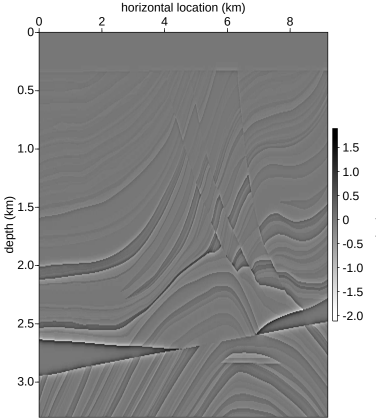
Left: Velocity (v) - 80 m smoothing of original. Right: Velocity perturbation (δv), difference of original Marmousi model and 20 m smoothing.

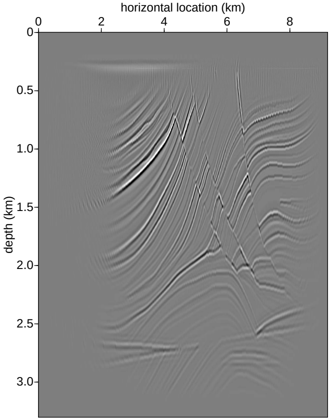
Synthetic Example: Marmousmooth



Primaries-only (linearized) shot gather at $x_s = 7500$ m (from west edge of model).

Synthetic Example: Marmousmooth





Left: Velocity perturbation (δv), difference of original Marmousi model and 20 m smoothing. Right: output of RTM, adjoint state version (I_{AD}). Note that amplitude trend, wavelet shape in migration output differs from that of "true" velocity perturbation, but structure is same.

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Acoustic Linearized Modeling

Divide velocity into smooth reference v and oscillatory / short-scale perturbation δv.

Linearized ("Born", single scattering) modeling operator $\mathbf{A}$ defined by

$$\mathbf{A}\delta v = \{\delta p(x_s, z_s, x_r, z_r, t) : (x_r, z_r) \in \mathcal{R}(x_s, z_s)\}$$

where p solves the wave equation with Born source:

$$\left[\frac{1}{v^2}\frac{\partial^2 \delta p}{\partial t^2} - \nabla^2 \delta p\right](x_s, z_s, x, z, t)$$

$$= 2\frac{\delta v(x,z)}{v(x,z)}\nabla^2 S(x_s, z_s, x, z, t);\; \delta p \equiv 0, t << 0$$

NB: A depends on v, source wavelet.

Acoustic Linearized Modeling

Lailly, Tarantola early 80's:

$$\mathbf{A}^T d = I_{AD}$$

If d approximates single-scattering data: $d \simeq \mathbf{A}\delta v$, then

$$I_{AD} \simeq \mathbf{A}^T \mathbf{A} \delta v$$

$\mathbf{A}^T \mathbf{A} =$ "normal operator".

Approximate inversion by scaling

Inversion: $\delta v \simeq (\mathbf{A}^T\mathbf{A})^{-1} I_{AD} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T d$.

Since relation $\delta v \mapsto I_{AD}$ appears to be *local*, try diagonal or near-diagonal approximation. Diagonal in space domain = *scaling* operator.

- Nemeth et al. 99, Shin et al. 01, Plessix & Mulder 04: use *diagonal* of $\mathbf{A}^T\mathbf{A}$ to construct approximate inverse;
- Chavent & Plessix 98, Valenciano et al 06: invert near-diagonal blocks $\mathbf{A}^T\mathbf{A}$;
- Claerbout & Nichols 94, Rickett 03: use "test model" to estimate best scale factor (least squares);
- Guitton 04: use "test model" to estimate best integral op with kernel supported near diagonal (spatially varying short filter).

Common drawbacks: diagonal (pure scaling) approximations not particularly accurate, near-diagonal approximations sometimes better - but when?

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Structure Theorem for Normal Operator

(Beylkin, 85; Rakesh, 86, 88; Nolan & S. 97; Smit, tenKroode, & Verdel 98; Stolk 00) For generic smooth v, monochromatic wave packet δv defined by envelope χ, phase ψ, frequency ω:

$$\mathbf{A}^T\mathbf{A}\chi(\mathbf{x})e^{i\omega\psi(\mathbf{x})} = \sigma(\mathbf{x},\omega\nabla\psi(\mathbf{x}))\chi(\mathbf{x})e^{i\omega\psi(\mathbf{x})} + O(|\omega|^{m-\beta}),$$

where $\sigma(\mathbf{x},\mathbf{k}) \geq 0$ is homogeneous of degree m in $\mathbf{k}$, and $\beta > 0$.

That is: normal op acts as a multiplier on monochromatic wave packets, to leading order in frequency.

Typical of *pseudodifferential* ("ΨDO") operator: normal op is ΨDO plus relatively smoothing error. *Order* is $m = d - 1$ in space dimension d, σ is *principal symbol* - computable by ray tracing (asymptotic inversion).

Structure Theorem and Scaling

Scaling strategy: use Structure Theorem to construct approximate inverse *without any ray-tracing whatsoever.*

Calculus of ΨDO's: $\Rightarrow (-\nabla^2)^{-\frac{m}{2}}\mathbf{A}^T\mathbf{A}$ is an operator of order 0.

Operators of order zero act as *frequency independent* multipliers (i.e. scaling operators) on monochromatic pulses:

$$(-\nabla^2)^{-\frac{m}{2}}\mathbf{A}^T\mathbf{A}\chi(\mathbf{x})e^{i\omega\psi(\mathbf{x})} = \bar{\sigma}(\mathbf{x})\chi(\mathbf{x})e^{i\omega\psi(\mathbf{x})} + O(|\omega|^{-\beta})$$

where $\bar{\sigma}(\mathbf{x}) = \|\nabla\psi(\mathbf{x})\|^{-m}\sigma(\mathbf{x}, \nabla\psi(\mathbf{x}))$.

Seismic images (migration outputs) tend to be local Fourier sums of monochromatic pulses with same phase (well-defined dip). So: in most places, $(-\nabla^2)^{-\frac{m}{2}}\mathbf{A}^T\mathbf{A}$ acts as scaling op.

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A Practical Approximate Inverse Method

- migrate data, then filter migrated data (I_{AD}) by $(-\nabla^2)^{-\frac{m}{2}}$ - gives (approx.) δv multiplied by $\bar{\sigma}$;
- apply $\mathbf{A}$ to the migrated data, apply $\mathbf{A}^T$ to remodeled data, then filter with $(-\nabla^2)^{-\frac{m}{2}}$ again - migrated data multiplied by same $\bar{\sigma}$.
- find $\bar{\sigma}^{-1}$ by dividing migrated data by filtered, remigrated data.
- then recover δv by multiplying filtered, migrated data by $\bar{\sigma}^{-1}$.

[Algorithm proposed by Claerbout-Nichols (1994), Rickett (2003) is *same*, except leave out filter (Laplacian power). Similar to Guitton (2004) but structure of filter is completely specified.]

A Practical Approximate Inverse Method

Details of implementation:

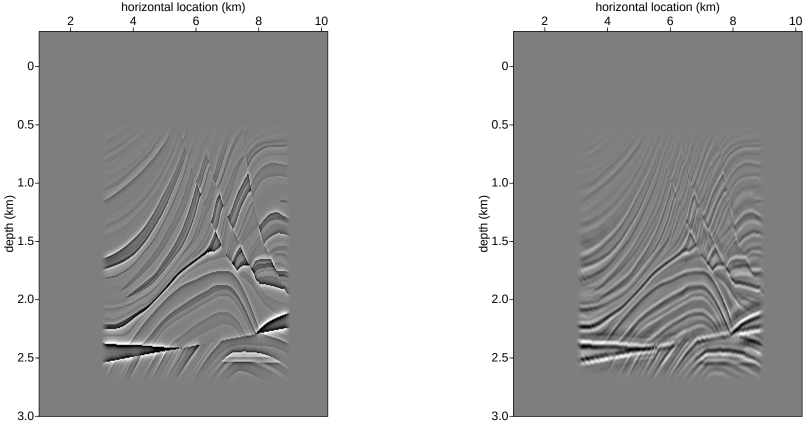
1. (2,4) FD scheme for linearized modeling and adjoint state (RTM) computations.

2. Laplace filter $(-\nabla)^{-\frac{1}{2}}$ implemented via 2D FFT.

3. Determine regularized division by $\bar{\sigma}$ via nonlinear least squares problem for $\tau = -\log(\bar{\sigma})$ - simple device to ensure that computed $\bar{\sigma}^{-1}$ is positive definite. Use standard quasi-Newton method (LBFGS).

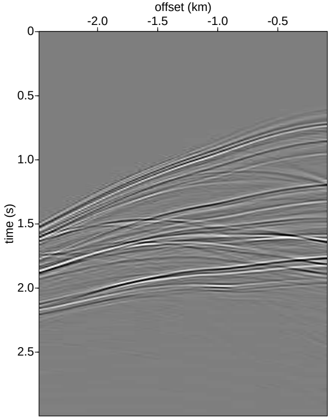
4. Localize in central region of model via tapered window.

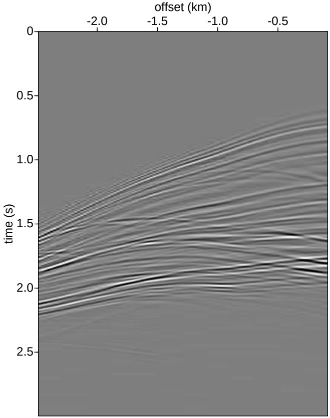
A Practical Approximate Inverse Method



Left: Velocity perturbation, difference of original Marmousi model and 40 m smoothing. Right: Approximate inversion via scaling and power of Laplacian.

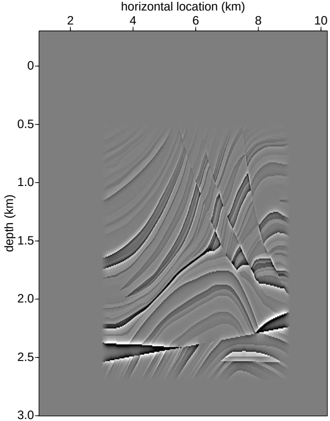
A Practical Approximate Inverse Method

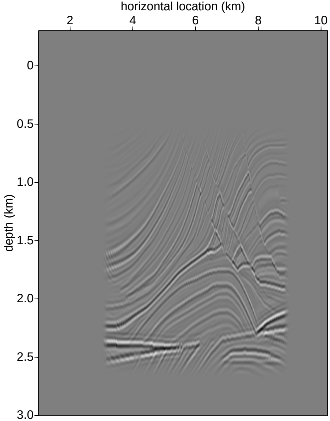




Shot at $\mathbf{x}_s = 7500$m. Left: Born simulation with exact model, truncated by spatial mute. Right: Born resimulation using scaling-filtering approximate inversion.

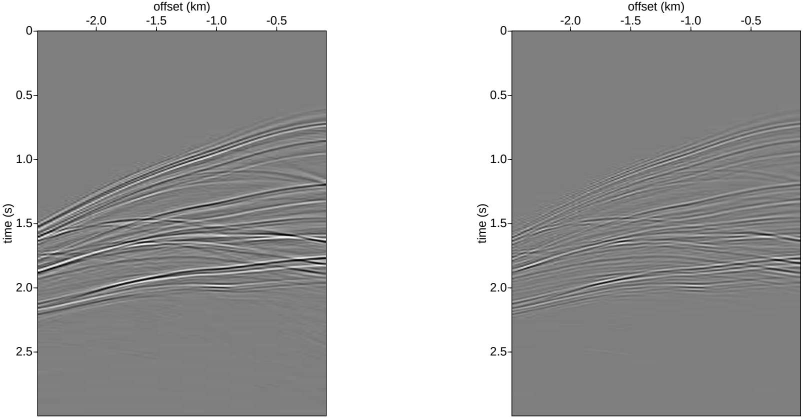
A Practical Approximate Inverse Method





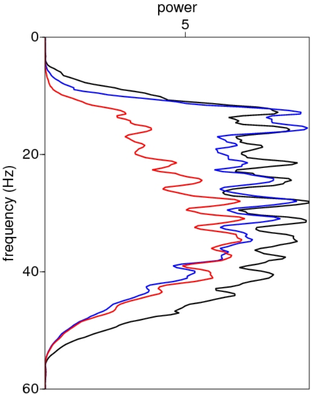
Left: Velocity perturbation, difference of original Marmousi model and 40 m smoothing. Right: Approximate inversion from scaling-only algorithm.

A Practical Approximate Inverse Method



Left: Born simulation with exact model, truncated by spatial mute. Right: Born resimulation using scaling-only approximate inversion.

A Practical Approximate Inverse Method



Trace power spectra for $\mathbf{x}_s = 7500$m, averaged over offset. Black = Linearized simulation with exact model, Blue = scaling-filtering approx. inversion, Red = scaling-only approx. inversion. Note missing linear-in-frequency trend in scaling-only result - equivalent to missing power of Laplacian!!!

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Conclusions

- Can understand scaling behaviour of migration in terms of its relation to modeling.
- Structure theorem for $\mathbf{A}$: normal operator $\mathbf{A}^T\mathbf{A}$ is (essentially, generically) pseudodifferential, acts as *frequency-dependent multiplier* on oscillatory wave packets.
- Structure theorem $\Rightarrow$ product of normal op and power of Laplacian acts as (frequency-independent) multiplier (i.e. scaling op) whereever dip is well-defined.
- Inversion of multiplier $\Rightarrow$ approx inverse for $\mathbf{A}^T\mathbf{A}$ $\Rightarrow$ effective and inexpensive approximate inversion *without ray-tracing* when migrated image consists largely of events with well-defined dip.

Prospects

- Actual migrated images often rife with conflicting dips. Must estimate full symbol σ, not just evaluation on phase normal $\bar{\sigma}$. Natural route: basis in which ΨDOs of order 0 are nearly diagonal - eg. curvelets (Moghaddam et al. SPMI 3.2) or Gabor repn (Margrave, Lamoureaux, Gibson), or spherical harmonics (Bao & WWS SIAM Sci Comp 96).
- Const-density acoustic amplitudes both unrealistic and uninformative - extend to "AVO" inversion for elastic moduli perturbations.
- Attenuation - is ΨDO representation still adequate?
- Structure Theorem *false* when background model has discontinuities, but these are essential in some cases (salt!) to maintain kinematic fidelity. Must extend to nonlinear scattering somehow...

Acknowledgements

- Jon Claerbout and students, for sparking my interest in this subject
- Sponsors of The Rice Inversion Project, for their support
- Total E&P USA for giving ED permission to participate in this study
- IFP for the Marmousi model
- BP and Frederic Billette for providing the BP 2004 velocity benchmark velocity model and data

For details, see: `www.caam.rice.edu`, TR 06-19 (*Geophysics*, in press).

Violating the Assumptions

Method hinges in principle on these assumptions:

1. data is primaries-only, i.e. Born data from constant-density acoustics;
2. background model is transparent, i.e. smooth on wavelength scale;
3. source is zero-phase bandpass, i.e. signature decon has been performed;
4. velocity model is known accurately.

Hard or impossible to satisfy for data acquired in the field!

How robust is method under violations of these assumptions?

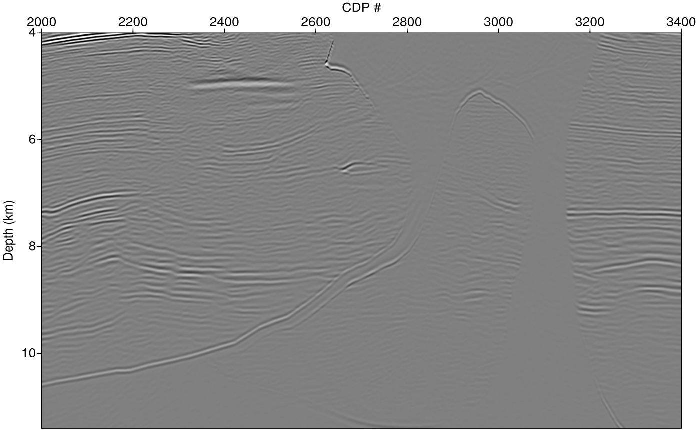
Violating the Assumptions

BP benchmark 2D data (Billette and Brandsberg-Dahl 2004):

1. data results from full-wave variable-density acoustic simulation, including free surface multiples (but SRME applied); sedimentary reflection modeling via density fluctuations;
2. background model includes rugose salt bodies and other non-transparent features;
3. source is not zero-phase bandpass filter, and moreover is unknown (to us);
4. but - thanks to BP - we know the velocity precisely.

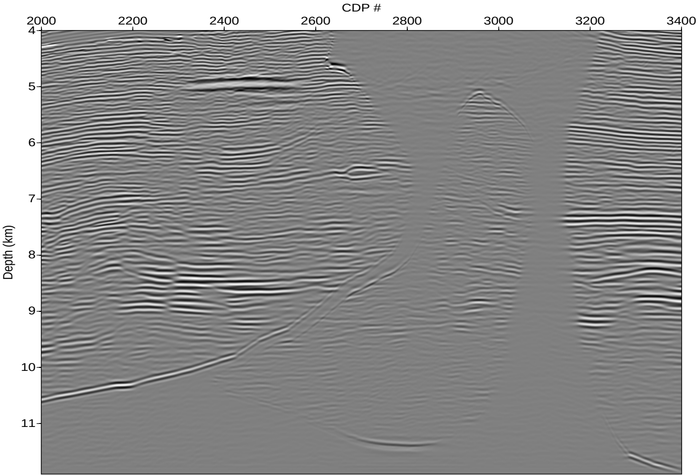
First pass: apply method explained above, ignoring violation of assumptions 1-3.

Violating the Assumptions



Window of migrated image, intersalt sediment zone.

Violating the Assumptions



Same window, approximate inversion via scaling-filtering method.

Violating the Assumptions

Discussion:

- Amplitudes *a priori* meaningless, because model is wrong - but they are more reasonable.
- Sedimentary fluctuations much more comparable to salt boundary reflections - the latter come only from "imprint" of migration velocity model on image, as data is *residual* (salt reflections due to velocity contrast removed).
- Some other aspects seem more reasonable - "eyelet" feature under flank has comparable top/bottom amplitudes after filtering/scaling, etc.
- How to do this right: use variable density acoustics! Normal operator is 2×2 matrix of ΨDO's, yields invertible 2×2 scaling *matrix* where dip is well defined. Inversion (cheap!) yields estimates of velocity *and* density perturbations. Also vital: estimate, deconvolve source wavelet (see Minkoff & S., *Geophys.* 97).