TRIP in 2010

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The Rice Inversion Project

29 January 2009

Agenda

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Extended Modeling and Inversion

Project Organization

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Central goal: improve reliability and efficiency of waveform inversion

- roadmap based on *extended modeling* concept - common framework for WI and MVA
- complex of issues: beyond depth migration $\leftrightarrow$ nonlinear WI $\leftrightarrow$ multiple reflections $\leftrightarrow$ discontinuous models and model updates
- better modeling - improved accuracy in presence of interfaces, texture
- better inversion - improved optimization algorithms
- better infrastructure - modeling, optimization frameworks

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Extended Modeling and Inversion

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Extended Modeling and Inversion

The usual set-up:

- $\mathcal{M}$ = a set of *models*
- $\mathcal{D}$ = a space of (potential) data
- $\mathcal{F} : \mathcal{M} \rightarrow \mathcal{D}$ (forward map, modeling operator,...)

[These things are collectively "the model".]

The standard example: $\mathcal{M} = \{v(\mathbf{x})\}$, $\mathcal{D} = \{d(\mathbf{x}_r, t; \mathbf{x}_s)\}$, $\mathcal{F} =$ (solve acoustic wave equation with sources at $\mathbf{x}_s$, sample pressure field at $(\mathbf{x}_r, t)$)

Inverse problem: given $d \in \mathcal{D}$, find $m \in \mathcal{M}$ so that $\mathcal{F}[m] \simeq d$.

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Natural formulation: given $d \in \mathcal{D}$, choose m by

$$v = \mathrm{argmin}\,(\mathcal{E}[\mathcal{F}[m] - d] + \mathcal{R}[m])$$

$\mathcal{E}$ measures "energy" (eg. mean square) in fit error $\mathcal{F}[m] - d$ in which $\mathcal{R}$ (*regularization* functional) supplies additional stability.

Promoted heavily by Tarantola and others in the 1980's on grounds of Bayesian justification (maximum likelihood solution given Gaussian data error statistics)

Recently revived as major industry interest ("Full Waveform Inversion", FWI) - all-day SEG workshop with attendance > 300

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Upshot:

- FWI via iterative optimization method recovers very detailed subsurface models, at least in numerical tests with model data, when starting model is sufficiently accurate (Tarantola and coworkers 80's, 90's, Bunks 1995,...,Plessix et al, Albertin et al 2009)
- Fails when starting model is not sufficiently accurate (stalls at stationary point with poor data fit)
- Hard to tell what "sufficiently accurate" means - no *a priori* test
- Continuation from low to high frequency / depth permits convergence with less accurate starting model (Kolb et al 1986, Bunks 95, Pratt 2004, Shin & Min 2006, recently many others) - however no guarantees

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MVA - global changes in model

How to combine best features of MVA and FWI?

Seek hybrid approach retaining

- global convergence of MVA
- nonlinearity, physical fidelity of FWI

TRIP approach: inversion based on *extended modeling* - FWI, MVA under one roof

Extended Modeling and Inversion

Extended modeling operator:

- $\chi : \mathcal{M} \rightarrow \bar{\mathcal{M}}$ (extended model space)
- $\bar{\mathcal{F}} : \bar{\mathcal{M}} \rightarrow \bar{\mathcal{D}}$ (extended data space)
- $\phi : \bar{\mathcal{D}} \rightarrow \mathcal{D}$

so that

$$\begin{array}{ccccc} & & \bar{\mathcal{F}} & & \\ & \bar{\mathcal{M}} & \rightarrow & \bar{\mathcal{D}} & \\ \chi & \uparrow & & \downarrow & \phi \\ & \mathcal{M} & \rightarrow & \mathcal{D} & \\ & & \mathcal{F} & & \end{array}$$

commutes - that is,

$$\phi[\bar{\mathcal{F}}[\chi[m]]] = \mathcal{F}[v],\ m \in \mathcal{M}$$

Extended Modeling and Inversion

Example: extended acoustic modeling

$\bar{\mathcal{M}}$ = symmetric positive definite operators [Remark: action-at-a-distance]

$\bar{\mathcal{D}}$ = band-unlimited data

For $\bar{v} \in \bar{\mathcal{M}}$,

$$\bar{\mathcal{F}}[\bar{v}](t, \mathbf{x}_r; \mathbf{x}_s) = \frac{\partial u}{\partial t}(t, \mathbf{x}_r; \mathbf{x}_s)$$

u is causal solution of

$$\left(\bar{v}^{-2}\frac{\partial^2 u}{\partial t^2} - \nabla^2\right) u(t, \mathbf{x}; \mathbf{x}_s) = \delta(t)\delta(\mathbf{x} - \mathbf{x}_s)$$

Minor modification of Lions' construction $\Rightarrow$ well-posed when $\bar{v}$ acts as multiple of identity on functions supported near $\mathbf{x}_s$.

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$$
\begin{array}{ccccc}
 & & \bar{\mathcal{F}} & & \\
 & \bar{\mathcal{M}} & \rightarrow & \bar{\mathcal{D}} & \\
\chi & \uparrow & & \downarrow & \phi \\
 & \mathcal{M} & \rightarrow & \mathcal{D} & \\
 & & \mathcal{F} & &
\end{array}
$$

$\chi[m]$ = operator (multiply by $1/v^2$) ("diagonal", no action at distance - "physical" models)

ϕ = filter by source wavelet - turns impulse response into bandlimited data

[*depth-offset* extension - not the only possibility]

Extended Modeling and Inversion

Invertible extension: $\bar{\mathcal{F}}$ has approximate left inverse $\bar{\mathcal{G}}$ (on $\mathcal{R}(\bar{\mathcal{F}})$) - *impulse response inversion*

Example: considerable numerical evidence (but little theory, except for space dimn $= 1$ and layered media) strongly suggests that example extension is invertible.

Extended Modeling and Inversion

Optimization formulation via annihilator $W : \bar{\mathcal{M}} \to \bar{\mathcal{M}}$ annihilates range of χ:

$$W[\bar{m}] = 0 \text{ if } \bar{m} = \chi[m]$$

Define

$$A[\bar{d}] = W[\bar{\mathcal{G}}[\bar{d}]]$$

Then if $\bar{d}$ "agrees" with d ($\phi[\bar{d}] = d$),

$$A[\bar{d}] \simeq 0 \Rightarrow \bar{\mathcal{G}}[\bar{d}] = \chi[m] \Rightarrow d \simeq \phi[\bar{\mathcal{F}}[\bar{\mathcal{G}}[\bar{d}]]] = \bar{\mathcal{F}}[\chi[m]] = \mathcal{F}[m]$$

Thus inverse problem equivalent to "nonlinear differential semblance":

search over $\{\bar{d} : \phi[\bar{d}] = d\}$ to minimize $\mathcal{E}[A[\bar{d}]]$

Extended Modeling and Inversion

Why should you care -

if you replace

- modeling ($\mathcal{F}$) with Born modeling ($D\mathcal{F}$)
- search over out-of-band data components ($\phi^{-1}[d]$) with search over smooth models (correspondence via geometric optics)

obtain various versions of optimization-based MVA, depending on precise modeling assumptions and choice of extended modeling -

"differential semblance" - TRIP 90's, Chauris & Noble 01, Plessix & Mulder 02, Shen et al 03, 05, Foss et al. 05, Khoury 06, Kabir et al. 06, 07, Li & WWS 07, Shen & WWS 08, Fei 09

This works! Simplest versions even have theoretical results: no local mins!!!

Extended Modeling and Inversion

Identifying an annihilator for depth-offset extension: range of χ consists of multiplication ops by L^∞ functions, which commute with other multiplication ops - so can choose

$$W[\bar{m}] = [\bar{m}, \mathbf{x}]$$

in which $\mathbf{x}$ represents multiplication by coordinate vector.

Write $\bar{m}$ formally as integral operator with kernel $\bar{m}(\mathbf{x}, \mathbf{y})$. Then

$$W[\bar{m}]u(\mathbf{x}) = \int d\mathbf{y} \bar{m}(\mathbf{x}, \mathbf{y})(\mathbf{x} - \mathbf{y})u(\mathbf{y})$$

multiplication of $\bar{m}$ by *offset* $\mathbf{x} - \mathbf{y}$ (cf. Shen 03 and descendants)

Extended Modeling and Inversion

What is needed?

- accurate modeling in presence of discontinuities (else no need for nonlinear inversion!)
- good modeling framework, including parallel execution, standard i/o, etc. etc.
- efficient extended modeling (no full matrix multiply at every time step)
- good optimization framework
- interface between modeling and optimization
- preconditioners, other accelerators to enhance convergence of iterative optimization

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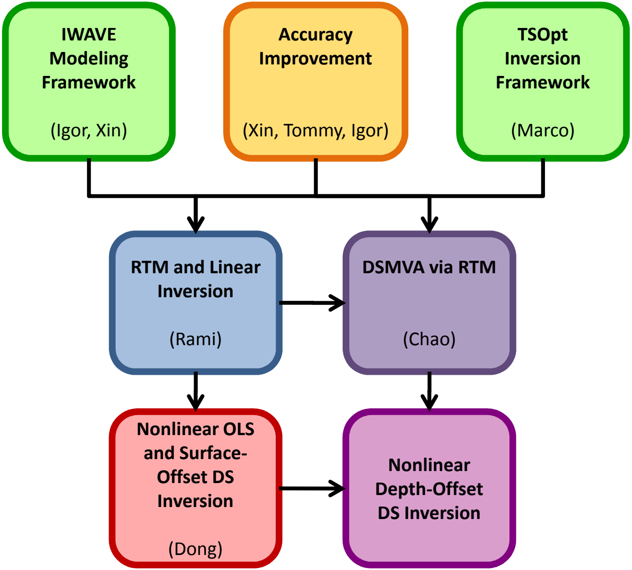

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