The IWAVE++ Inversion Framework

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The Rice Inversion Project

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Concept of IWAVE++

- Goal: make IWAVE capabilities - large-scale, parallel, extensible FD modeling - available for research on inversion/imaging driven by time-domain simulation
- Method: embed IWAVE in Rice Vector Library (RVL) Operator type, use RVL utilities (linear algebra, construction of least squares functions, etc.) and algorithms (CG, NLCG, LBFGS,...) to construct inversion applications.

Agenda

Simulation-driven Inversion

Waveform Inversion:

$$\min_{m \in \mathcal{M}} J[m] := \frac{1}{2} \|\mathcal{F}[m] - d\|_{\mathcal{D}}^2$$

Gradient $\nabla J[m] = D\mathcal{F}[m]^T (\mathcal{F}[m] - d)$

- $\mathcal{M}$: Model Space , $\mathcal{D}$: Data Space
- $\mathcal{F}$: $\mathcal{M} \to \mathcal{D}$ modeling operator
  ($\mathcal{F} = Su$, u wavefield, S sampling operator)
- $D\mathcal{F}[m]$: derivative map of $\mathcal{F}$ at m (Born operator)
- $D\mathcal{F}[m]^T$: adjoint operator of $D\mathcal{F}[m]$

Abstraction Discrepancy & Solution

Inversion involves two levels of abstraction:

- applications of $\mathcal{F}$, $D\mathcal{F}[m]$, $D\mathcal{F}[m]^T$

  involve solution of PDE's, depending on specific physics
  and numerical realization, ...

- optimization algorithm

  only involve generic linear algebra operations, independent of
  specific representation or modeling construction

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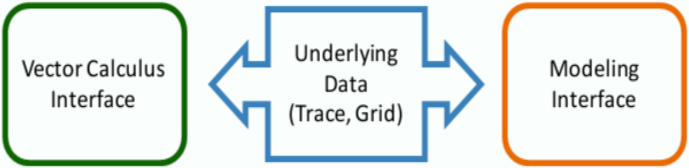
Abstraction Discrepancy & Solution

Object-oriented programming offers a solution to share information between vector calculus & modeling sofware



Both sides provide interfaces to reference the same common data structures for model grids, data traces, ... , which need not be part of either

In our case, the external data objects are structured disk files (RSF file structure for model parameter, SEGY for seismic traces)

- vector calculus (RVL) operations perform as disk-to-disk filters
- IWAVE has its own i/o functions and is regulated by parameter table

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Abstraction Discrepancy & Solution

A solution (WWS, DS & ME,10; DS & WWS, TR10-05, TR10-06):

- wrap $\mathcal{F}$ as a C++ class (RVL operator) for vector valued functions, with methods to
  - do modeling $\mathcal{F}[m]$
  - apply first derivative (action of Born Map $D\mathcal{F}[m]$)
  - apply adjoint derivative (adjoint action of Born Map $D\mathcal{F}[m]^T$)

- auxiliary classes:
  - `State` classes for $(m, u), (m, u, \delta m, \delta u)$ with necessary methods
  - `Sampler` classes to link $u, \delta u$ with data $d, \delta d$
  - `Stack` class to manage dynamic wavefields at specific time levels
  - `Algorithm` classes to update fields, read/write $m, \delta m$, initialize $u, \delta u$

  ...

Forward, Born and Adjoint Simulation

Model problem:

$$\frac{1}{\kappa(\mathbf{x})}\frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v} = f(\mathbf{x}, t)$$

$$\frac{1}{b(\mathbf{x})}\frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0$$

$$d = \mathcal{F}[m] := S\mathbf{u}$$

$\mathbf{u} = (p, \mathbf{v})$, S sampling operator ($Sp := p(x_r, t)$)

Simulation

SIM (1) $\mathbf{u} = 0$

SIM (2) *For* $n = 0, \ldots, N-1$ *do:*

SIM (2.1) *do:* $p \mathrel{+}= -\Delta t\, \kappa\, \nabla \cdot \mathbf{v} := L_0[m, u]$

$\mathbf{v} \mathrel{+}= -\Delta t\, b\, \nabla p := L_1[m, u]$

SIM (2.2) $\mathbf{u} \mathrel{+}= R\mathbf{f}$

SIM (2.3) $d \mathrel{+}= S\mathbf{u}$

Forward, Born and Adjoint Simulation

Born Simulation

BORN SIM (1) $\mathbf{u} = 0$, $\delta\mathbf{u} = 0$;

BORN SIM (2) For $n = 0, \ldots, N-1$ do:

BORN SIM (2.1) do:

$$\delta p \mathrel{+}= -\Delta t\ \kappa\ \nabla \cdot \delta\mathbf{v} := L_0[\mathbf{m}, \delta\mathbf{u}]$$

$$\delta p \mathrel{+}= -\Delta t\ \delta\kappa\ \nabla \cdot \mathbf{v} := L_0[\delta\mathbf{m}, \mathbf{u}]$$

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$$p \mathrel{+}= -\Delta t\ \kappa\ \nabla \cdot \mathbf{v} := L_0[\mathbf{m}, \mathbf{u}]$$

$$\delta\mathbf{v} \mathrel{+}= -\Delta t\ b\ \nabla \delta p := L_1[\mathbf{m}, \delta\mathbf{u}]$$

$$\delta\mathbf{v} \mathrel{+}= -\Delta t\ \delta b\ \nabla p := L_1[\delta\mathbf{m}, \mathbf{u}]$$

$$\mathbf{v} \mathrel{+}= -\Delta t\ b\ \nabla p := L_1[\mathbf{m}, \mathbf{u}]$$

$$\mathbf{v} \mathrel{+}= -\Delta t\ b\ \nabla p := L_1[\mathbf{m}, \mathbf{u}]$$

BORN SIM (2.2) $\mathbf{u} \mathrel{+}= R\mathbf{f}$

BORN SIM (2.3) $\delta\mathbf{d} \mathrel{+}= S\delta\mathbf{u}$

Forward, Born and Adjoint Simulation

Adjoint Simulation

ADJ SIM (1) $\lambda\mathbf{u} = (\lambda p, \lambda\mathbf{v}) = 0$

ADJ SIM (2) *For* $n = N, \ldots, 1$ *do:*

set time in $\mathbf{u}, \mathbf{d}$ *to* $n-1$*;*

ADJ SIM (2.1) $\lambda\mathbf{u} \mathrel{+}= S^T\lambda\mathbf{d}$;

ADJ SIM (2.2) *do:*

$$\lambda b \mathrel{+}= -\Delta t\ \lambda\mathbf{v}\ \nabla p := L_1[\lambda\mathbf{u}, \mathbf{u}]$$
$$\lambda\mathbf{v} \mathrel{+}= \Delta t\ b\ \nabla\lambda p := L_1[m, \mathbf{u}]$$
$$\lambda\kappa \mathrel{+}= -\Delta t\ \lambda p\ \nabla\cdot\mathbf{v} := L_0[\lambda\mathbf{u}, \mathbf{u}]$$
$$\lambda p \mathrel{+}= \Delta t\ \kappa\ \nabla\cdot\lambda\mathbf{v} := L_0[m, \lambda\mathbf{u}]$$

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Agenda

1. From Modeling to Inversion
2. Numerical Experiments & Discussion
3. Summary and Future Work

Numerical Verifications with the "Camembert"

Principle example in O. Gauthier, J. Virieux and A. Tarantola (1986):
Two-dimensional Nonlinear Inversion of Seismic Waveform: Numerical Results
(first published exploration of iterative FWI with multi-D data and multi-D models)

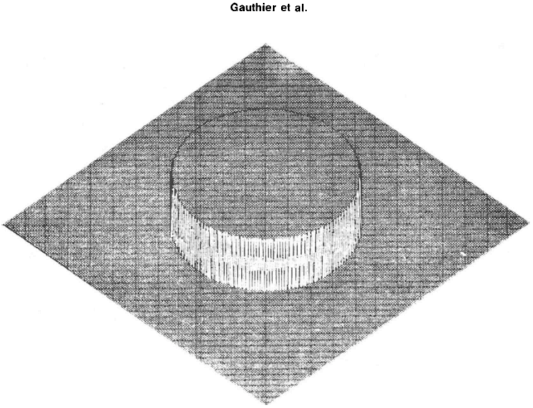


FIG. 5. The model is now a circular inclusion (the "Camembert") in a homogeneous medium. The size of the Camembert is about 10 wavelengths. The model is numerically defined in a grid with 200 × 200 points, so the model contains 10^4 parameters (unknowns for the inversion).

- 1 km $\times$ 1 km domain
- background: v = 2.5 km/s, $\rho = 4$ g/cm^3
- peak frequency $\sim$ 50 Hz
- circular bulk modulus anomaly, diam = 0.5 km, "small" = 2% or "large" = 20%
- "nonlinear saturation" at 10% - full wavelength traveltime perturbation
- 5 m grid - 10 gridpts / (peak) wavelength
- absorbing BCs on all sides

Numerical Verifications with the "Camembert"

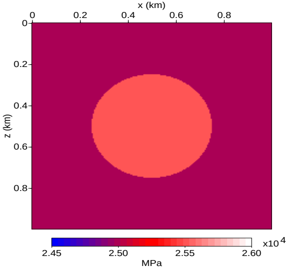
Numerical Verification of $D\mathcal{F}[m]$ with reflection configuration:

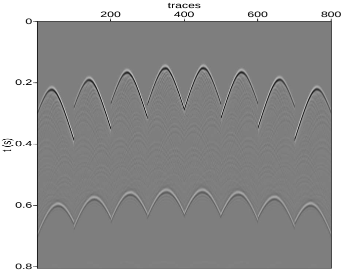
- 100 receivers (fixed spread), $z_r = 80, x_r = 0, 10, ..., 990$ m
- 8 sources, $z_s = 40, x_s = 110, 220, ..., 880$ m

$$\left\| \frac{\mathcal{F}(\mathbf{m}+\mathbf{h}\delta\mathbf{m})-\mathcal{F}(\mathbf{m}-\mathbf{h}\delta\mathbf{m})}{\mathbf{2h}} - \mathbf{D}\mathcal{F}[\mathbf{m}]\delta\mathbf{m} \right\| / \|\mathbf{D}\mathcal{F}[\mathbf{m}]\delta\mathbf{m}\|$$

h	relative error	conv rate
1	0.275	—
0.9	0.233	1.547
0.8	0.193	1.631
0.7	0.153	1.708
0.6	0.117	1.779
0.5	0.083	1.839
0.4	0.055	1.891
0.3	0.031	1.934
0.2	0.014	1.965
0.1	0.004	1.974

small anomaly (2%)





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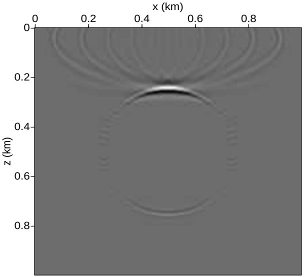
Numerical Verifications with the "Camembert"

Numerical Verification of $D\mathcal{F}[m]^T$ with reflection configuration

Let $\mathcal{J} := D\mathcal{F}[m]$, adjoint relation holds if

$$\texttt{AdjErr} := \frac{\left|(\mathcal{J}\delta m\,,\,\delta p) - (\delta m\,,\,\mathcal{J}^T\delta p)\right|}{\|\mathcal{J}\delta m\|\|\delta p\|} < 100\texttt{*macheps}$$



$(\mathcal{J}\delta m\,,\,\delta p)$	3.89868e+06
$(\delta m\,,\,\mathcal{J}^T\delta p)$	3.89868e+06
$\|\mathcal{J}\delta m\|\|\delta p\|$	3.89867525e+06
AdjErr	5.32232070e-06
$100 * \texttt{macheps}$	1.19209290e-05

Dot-product Test with Random Vector

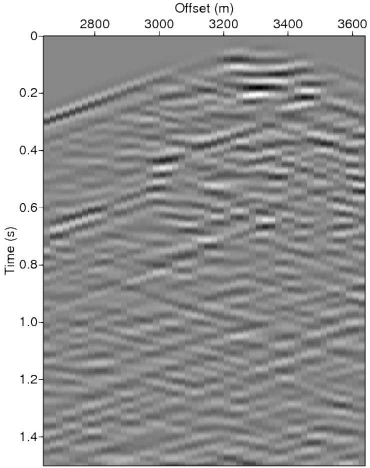


Figure: Born modeling acoustic synthetic shot gather ($DF[m]\delta m$) resulting from homogeneous background m and random δm.

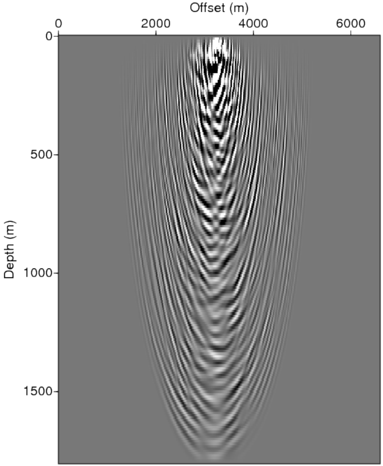


Figure: Migrated bulk modulus (component of $DF[m]^T\delta d$) with random δd.

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Dot-product Test with Random Vector

$\langle DF[m]\delta m\, ,\, \delta d\rangle_D$	-2.91666225e+06
$\langle \delta m\, ,\, DF[m]^T \delta d\rangle_D$	-2.94833250e+06
$\|DF[m]\delta m\|_M \|\delta d\|_D$	3.05501747e+09
$\frac{\langle \left\vert (DF[m]\delta m\, ,\delta d\rangle_D - \langle \delta m\, ,DF[m]^T \delta d\rangle_M \right\vert}{\|DF[m]\delta m\|_M \|\delta d\|_D}$	1.03666343e-05
$100 * \texttt{macheps}$	1.19209290e-05

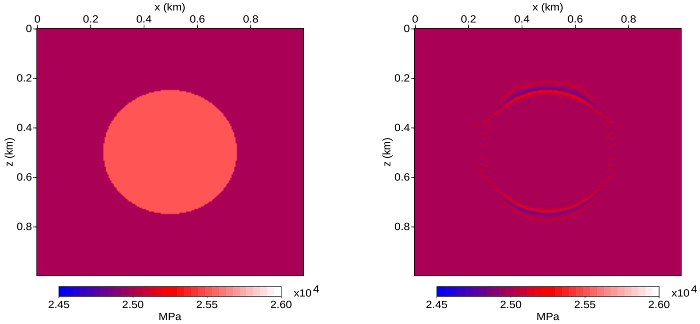
Table: Standard RVL test for accuracy of adjoint operator pair: adequate quality if model space and data space inner products differ by less than a modest multiple of machine precision, relative to data-space norms of input and output data perturbations.

Various Inversions with the "Camembert"

inversion - reflection - band-limited data with central frequency $\sim$ 50 Hz:

small anomaly (2%)

Initial MS resid = 3629; Final after 5 LBFGS steps = 254



Bulk modulus: Left, true model; Right, inverted model

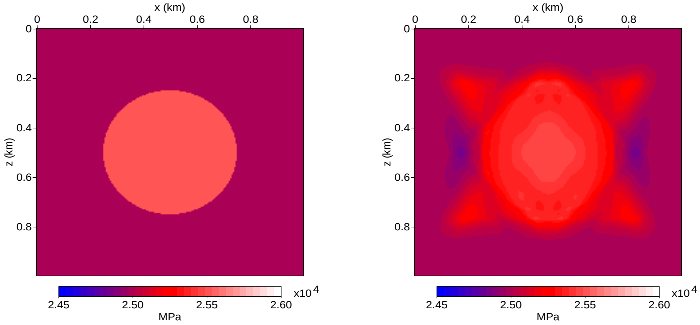
Various Inversions with the "Camembert"

transmission configuration:

- 400 receivers - each side like top in reflection
- 8 sources at corners and side midpoints

small anomaly (2%)- band-limited data

Initial MS resid $= 2.56\times10^7$; Final after 5 LBFGS steps $= 2.6\times10^5$



Bulk modulus: Left, true model; Right, inverted model

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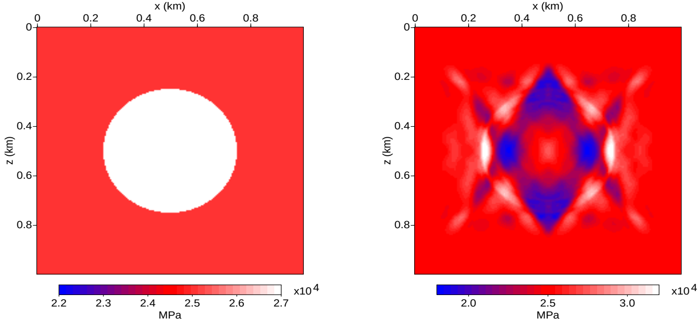
Various Inversions with the "Camembert"

Large anomaly (20%)- transmission – band-limited data

Initial MS resid $= 2.14\times10^8$; Final after 5 LBFGS steps $= 1.44\times10^8$



Bulk modulus: Left, true model; Right, inverted model

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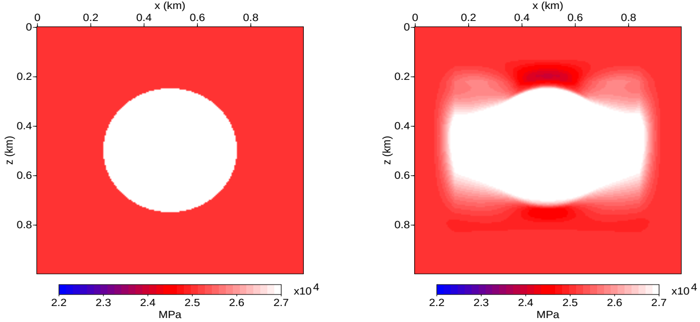
Various Inversions with the "Camembert"

Impulsive Inversion:

Large anomaly (20%)- reflection - **impulsive data, 0-60 Hz**

Initial MS resid $= 3.45\times10^{13}$; Final after 5 LBFGS steps $= 4.78\times10^{11}$



Bulk modulus: Left, true model; Right, inverted model

Agenda

Summary and Future Work

IWAVE++ Inversion Framework:

- demonstrates a design principle for straightforward incorporation of sophisticated modeling techniques in inversion software
- provides a platform for various migration and inversion research and applications
- separates simulation from optimization, which facilitate other research projects

Summary and Future Work

Future Work

- complement the implementation of IWAVE++ for extended inversion and documentation
- explore LS inversion performance enhancing techniques and add more built-in options to IWAVE++

  ✳ various regularization strategies

  ✳ adaptive scaling

  ... ...

- explore different optimization methods

  ✳ trust-region ...

  ✳ beyond L_2 norm (methods from other communities)

  ... ...

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Thank you!

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