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Education

- **University of Chinese Academy of Sciences**, Beijing, China
  Ph.D. in Computational Mathematics 09/2009 - 07/2014
  Thesis: *Reverse time migration for inverse scattering problems*
  Thesis supervisor: Professor Zhiming Chen
- **Central South University**, Changsha, China
  B.S. in Information and Computing Science 09/2005 - 07/2009

Research Interests

- Source-based Extended Waveform Inversion
- Acoustic/Electromagetic/Elastic wave inverse scattering problem
- Phaseless data imaging and inversion

Matched Source Waveform Inversion: Space-time Extension

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Outline

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Seismic Inverse Problem

Acoustic wave eqn:

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} - \Delta u = \delta(\mathbf{x} - \mathbf{x}_s) f(t).$$

$u =$ pressure field, $v =$ velocity, $f =$ input src function and $\mathbf{x}_s =$ src position.

Forward modeling operator:

$$S[v]f = u(\mathbf{x}, t; \mathbf{x}_s)|_{\mathbf{x}=\mathbf{x}_r}.$$

$\mathbf{x}_r =$ receiver position.

Inverse Problem: Given $d(\mathbf{x}_r, t; \mathbf{x}_s)$, find v and f such that

$$S[v]f = d.$$

Full Waveform Inversion

- FWI via least square,

$$J_{\mathrm{FWI}}[v, f] = \frac{1}{2} \sum_{\mathbf{x}_r, \mathbf{x}_s} \int |S[v] f(\mathbf{x}_r, t; \mathbf{x}_s) - d(\mathbf{x}_r, t; \mathbf{x}_s)|^2 dt.$$

- FWI obj is quadratic w.r.t f, but highly nonlinear and nonconvex in v.
- Sensitive to frequency band.
- Local minima: cycle skipping problem (bad init model & low frequency missing).

Outline

Summary of Source-Receiver Extension

FWI via src-recv extn (SEG 2015, G. Huang and W. Symes)

- For "bad" v, assign different src func $f(f_{sr}(t))$ for each src-recv pair (more d.o.f)

$$G(\mathbf{x}_r, \mathbf{x}_s, t) * f_{sr}(t) = d(\mathbf{x}_r, \mathbf{x}_s, t).$$

- Fit data easily (only single trace fitting) $\Rightarrow$ no cycle skipping
- For true velocity, $f(t)^{-1} f_{sr} = \delta(t)$ focusing on $t = 0$.
- Minimizing non-focusing of f_{sr} multiplied by $A = t f(t)^{-1}$.

$$J_{\mathrm{MS}}[v] = \frac{1}{2} \sum_{\mathbf{x}_s, \mathbf{x}_r} \int |A f_{sr}|^2 dt.$$

See R. E. Plessix etc (2000), S. Luo and P. Sava (2011), L. Guasch and M. Warner (2014) for similar algorithm.

Summary of Source-Receiver Extension

- Good for single arrival (equiv to traveltime tomography).
- Fail if strong multipathing exists.

Reasons for failure:

- Ambiguity when fitting data from different branches;
- Slope of traveltime is lost (single trace fit);
- $G(\mathbf{x}_r, \mathbf{x}_s, t) * f_{sr}(t) = d(\mathbf{x}_r, \mathbf{x}_s, t)$ is NOT solvable in L_2 sense.

Outline

Motivation for New Extension

If we assume $G(\mathbf{x}_r, \mathbf{x}_s, t) * f_{sr}(t) = d(\mathbf{x}_r, \mathbf{x}_s, t)$ is solvable, it's equivalent to

$$G(\mathbf{x}_r, \mathbf{x}_s, t)^T G(\mathbf{x}_r, \mathbf{x}_s, t) * f_{sr}(t) = G(\mathbf{x}_r, \mathbf{x}_s, t)^T d(\mathbf{x}_r, \mathbf{x}_s, t)$$

NOTE:

- $G(\mathbf{x}_r, \mathbf{x}_s, t)^T d(\mathbf{x}_r, \mathbf{x}_s, t)$: "backpropagation" field (time shift of the data) in data domain (R. E. Plessix etc., 2000).
- How about backpropagation in the imaging domain?
- Extend the domain of f_{sr} to the imaging domain (put "src" everywhere in the whole domain).

Extended Modeling & Annihilator

Extended Modeling:

$\bar{f}(\mathbf{x}, t; \mathbf{x}_s)$: extended model of $f(t)\delta(\mathbf{x} - \mathbf{x}_s)$

Extended modeling operator $\bar{S}\bar{f} = \bar{u}$:

$$\frac{1}{v^2}\frac{\partial^2 \bar{u}}{\partial t^2} - \Delta \bar{u} = \bar{f}(\mathbf{x}, t; \mathbf{x}_s).$$

Annihilator:

$A = |\mathbf{x} - \mathbf{x}_s|$: Penalize non-focusing energy around src position $\mathbf{x}_s$.

Do not need source function $f(t)$!

Matched Source Waveform Inversion

Extended waveform inversion:

$$J_\alpha[v] = \frac{1}{2\alpha} \sum_{\mathbf{x}_r,\mathbf{x}_s} \int |\bar{S}[v]\bar{f} - d|^2 dt + \frac{1}{2} \sum_{\mathbf{x},\mathbf{x}_s} \int |A\bar{f}|^2 dt$$

$$s.t. \qquad (\bar{S}^T \bar{S} + \alpha A^T A)\bar{f} = \bar{S}^T d.$$

Key feature: data fitting via $\bar{f} \Rightarrow$ no cycle skipping problem!

Why it works

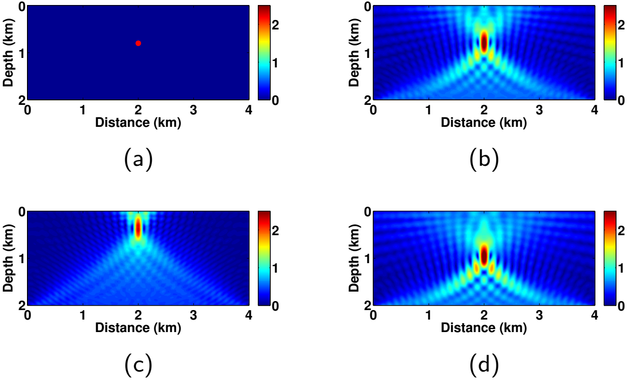

Figure: (a) True model; Amplitude of the backpropagation field ($\bar{S}^T d$) with (b) true velocity, (c) 10% low and (d) 10% high of true velocity

See Y. Zhang etc. (2008,2009) and R. Plessix etc (2010) for backpropagation-based waveform inversion.

Relation with WRI

Waveform Reconstruction Inversion (WRI, T. van Leeuwen and F. Herrmann (2013)):

$$
\begin{aligned}
J_{\mathrm{WRI}}[v] &= \min_{\bar{u}} \frac{1}{2} \sum_{\mathbf{x},\mathbf{x}_s} \int \left[\left(\frac{1}{v^2} \frac{\partial^2 \bar{u}}{\partial t^2} - \Delta \bar{u} \right) - f(t)\delta(\mathbf{x}-\mathbf{x}_s) \right]^2 dt \\
&+ \frac{1}{2\alpha} \sum_{\mathbf{x}_r,\mathbf{x}_s} \int \left(\bar{u}(\mathbf{x},t;\mathbf{x}_s) \Big|_{\mathbf{x}=\mathbf{x}_r} - d(\mathbf{x}_r,t;\mathbf{x}_s) \right)^2 dt. \\
&= \min_{\bar{f}} \frac{1}{2} \sum_{\mathbf{x},\mathbf{x}_s} \int |\bar{f} - f(t)\delta(\mathbf{x}-\mathbf{x}_s)|^2 dt \\
&+ \frac{1}{2\alpha} \sum_{\mathbf{x}_r,\mathbf{x}_s} \int |\bar{S}\bar{f} - d|^2 dt.
\end{aligned}
$$

Nonlinear annihilator: $A\bar{f} = \bar{f}(\mathbf{x},t;\mathbf{x}_s) - f(t)\delta(\mathbf{x}-\mathbf{x}_s)$.

Our source focusing annihilator: $A\bar{f} = |\mathbf{x}-\mathbf{x}_s|\bar{f}(\mathbf{x},t;\mathbf{x}_s)$.

Outline

Numerical Implementation

Forward modeling:

- 9-point FD method in frequency domain;
- Target source: zero-phased bandpassed source.

Data acquisition:

- Receivers: fixed spread geometry on the whole surface;
- Source: lesser dense sampling than receivers.

Inversion:

- Subproblem: direct solver to guarantee the accuracy of gradient;
- Optimization method: LBFGS with backtracking line search;
- Avoid inverse crime: coarse mesh grid for inversion, fine mesh grid for recording data.

Gaussian Model: Multipathing

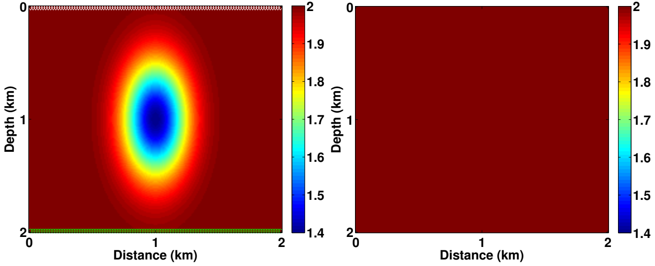

Figure: Transmission configuration: true model and initial model

Simulated Data

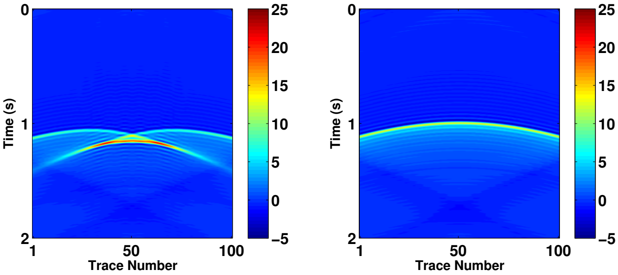

Figure: Recorded data and simulated data with initial model at center shot $x_s = 1$ km

Inverted Results

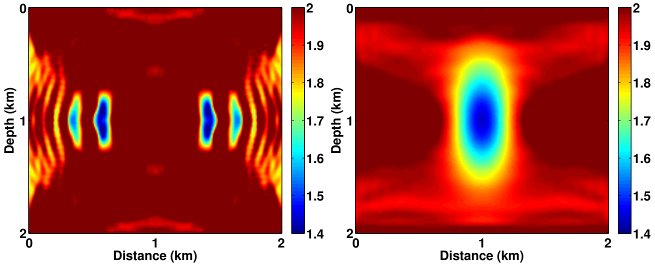

Figure: Inverted velocity by FWI and MSWI with (6, 10, 14, 18) Hz data

Diving Wave

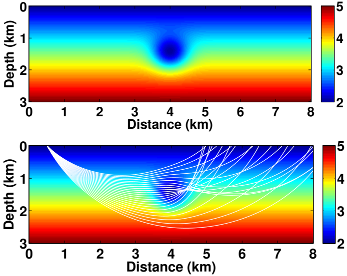

Figure: True model and ray tracing on model for shot $x_s = 0.5$ km

Comparison of Results

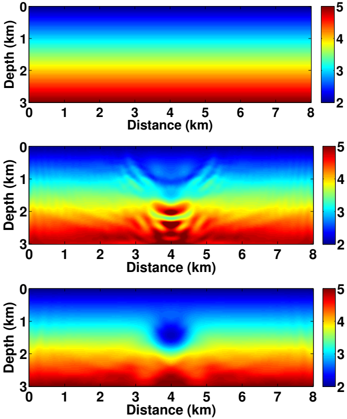

Figure: Top: initial model; inverted model by FWI (middle) and MSWI (bottom) using (6,7,8,9,10) Hz data

Comparison of Results

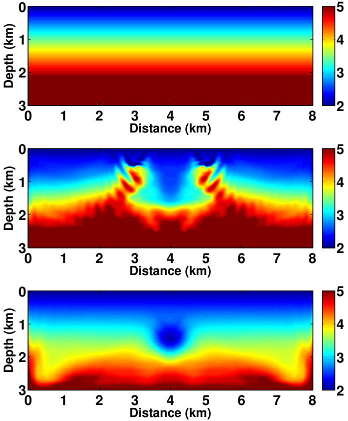

Figure: Top: initial model; inverted model by FWI (middle) and MSWI (bottom) using (5,6,7,8,9,10) Hz data

Marmousi Model

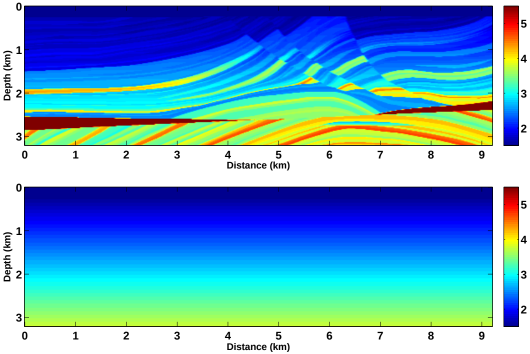

Figure: Marmousi model and 1D initial model

Comparison of Results

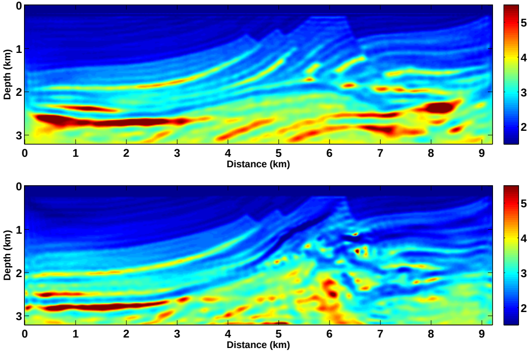

Figure: Inverted velocity by MSWI and FWI with (4,5,6,7,8) Hz data

BP Benchmark Model

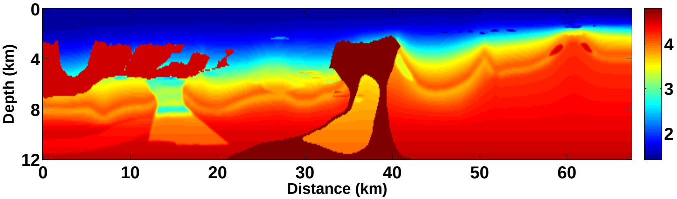

Figure: BP model

Inverted Result

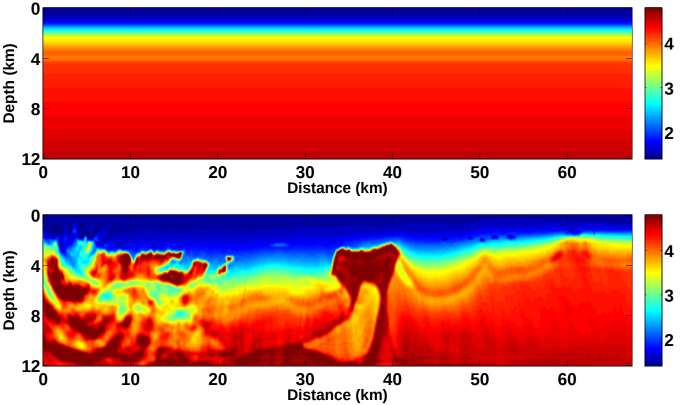

Figure: Initial model and inverted model using (1,2,3,4) Hz data

BP Benchmark Model

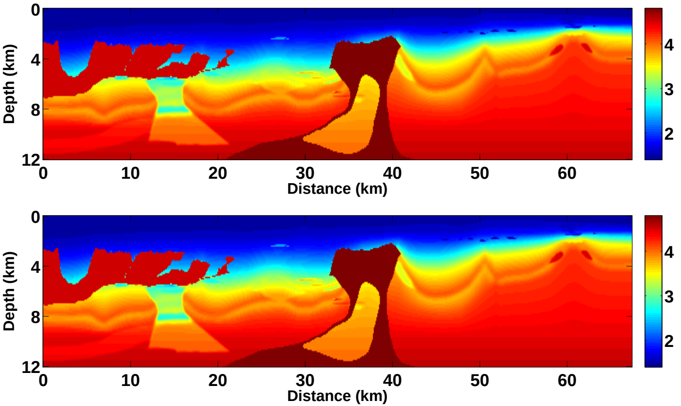

Figure: BP model

Conclusion

- Nonlinear extended waveform inversion can handle any kinds of waves including transmission wave and reflection without separation of data.
- Lower the requirement of low frequency and initial model.
- Source wavelet function is not required.
- Straightforward extension to multi-parameter inversion and elastic wave inversion.

Main obstacles:

- Limitation to 3D Helmholtz eqn solver.
- Storage requirement of extended model $\bar{f}(\mathbf{x}, \mathbf{x}_s, t) \in \mathbb{R}^6$ in 3D.

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