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# Nonlinear EFWI via Plane Wave Source Extension

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# Conventional FWI

**Full waveform inversion** — deterministic model-based data fitting approach (Tarantola, 1984):

$$\min_m J_{OLS}[m],$$
$$J_{OLS}[m] = \frac{1}{2}\|F[m] - d_o\|^2,$$

- $m \in M$ earth model from space of admissible models
- $F : M \to D$ modeling operator
- $d_o \in D$ observed data

# Conventional FWI characteristics

Important aspects:

- Highly redundant data
- Lack of information (limited bandwidth, insufficient coverage)
- Discrepancy between “true” physics and idealized modeling
- Noise in the data

Non-uniqueness:

- No exact data fit
- Best fit with multiple models (up to noise, precision)

# Conventional FWI overview

Huge problem size leads to OLS minimization via local methods (CG, Gauss-Newton, etc.)

FUNDAMENTAL DIFFICULTY:

Numerous local extrema of $J_{OLS}$:

- Missing low frequencies the data
- Oscillatory nature of seismic signal
- Nonlinearity and sensitivity w.r.t. long-scale perturbations of the model $\rightsquigarrow$ cycle skipping

# Conventional FWI overview

Successful conventional FWI:

- Kinematically accurate initial guess; numerous examples:
  Gauthier et al., 1986; Bunks et al., 1995;
  Plessix et al., 1998; ...
- Impulsive source
  - Low-frequency energy carries long-scale model information
  - Some theoretical results in 1D (Symes, 1986),
  - Numerical evidence in ND (Sacks and Santosa, 1987; Bunks et al., 1995)

# Extended modeling concept

Introduce

- Extra degrees of freedom: $m \rightarrow \bar{m}$,
- Additional constraint: $A\bar{m} = 0$,
  where $\ker A$ consists of physically plausible models

**Goal:** "always" satisfy data fitting constraint
$\Rightarrow$ avoid cycle skipping problem altogether

# Surface-oriented extensions and NDSO

Setup:

- Bin data w.r.t. acquisition parameter $h$ (shot #, plane wave slowness, ...)
- Fit data for each $h$ separately $\to \bar{m} = \bar{m}(h)$
- Minimize incoherence of $\bar{m}(h)$ with DS-based annihilator:

$$A\bar{m} = \frac{\partial \bar{m}}{\partial h}$$

**Key idea:**

Combine MVA capability of producing macro model with FWI capability to account for nonlinearity and fit the data (Symes, 2008)

# Surface-oriented extensions and NDSO

DSO formulation:

$$\min_{\bar{m}} J_A[\bar{m}],$$
$$J_A[\bar{m}] = \|A\bar{m}\|^2,$$

$$\text{such that } \bar{F}[\bar{m}] \approx 0,$$

where $\bar{F}[\bar{m}](h) = F[\bar{m}(\cdot, h)]$.

**Key issue:**

How to parametrize the feasible set of extended models $S = \{\bar{m} : \bar{F}[\bar{m}] \approx 0\}$ ?

# Low-frequency control data

Parametrization of $S$ by low frequency control data (Symes, 2008; Sun, 2012)

Motivation:

- Solvability of the LS impulse response problem:
  - Unique,
  - Computationally tractable,
  - And robust LS solution
- Analogies:
  - Smooth control model in LEWI (Symes and Kern, 1994; Huang and Symes, 2015);
  - Control macromodel in MVA

# Parametrization

1. Introduce modeling operator $\bar{F}_f = \bar{F} + \bar{F}_c$ with full-bandwidth source, ($\bar{F}_c$ – with compimentary source).
2. Parametrize $\bar{m} = \bar{m}(m_c)$ with control model $m_c$ by solving

$$\bar{F}_f[\bar{m}] \approx F_c[m_c] + d_o.$$

3. Solve DS optimization:

$$\min_{m_c} J_A[\bar{m}(m_c)].$$

Extended functional:

$$J[m_c, \bar{m}; \alpha] = (1 - \alpha)\|\bar{F}_f[\bar{m}] - F_c[m_c] - d_o\|_D^2 + \alpha\|A\bar{m}\|_{\bar{M}}^2,$$

with $\alpha \in [0, 1]$

# Optimization problem

- Inner optimization problem for given $m_c$:

$$\bar{\mu} = \operatorname*{argmin}_{\bar{m}} J[m_c, \bar{m}; \alpha_I]$$

- Outer optimization over control variable $m_c$:

$$\min_{m_c} \mathcal{J}[m_c],$$
$$\mathcal{J}[m_c] = J[m_c, \bar{\mu}; \alpha_O]$$

# Gradients

- Inner problem gradient:

$$\nabla_{\bar{m}} J[m_c, \bar{m}; \alpha_I] = (1-\alpha_I) D\bar{F}_f[\bar{m}]^* (\bar{F}_f[\bar{m}] - F_c[m_c] - d_o) + \alpha_I A^* A \bar{m}$$

- Outer problem gradient:

$$\nabla_{m_c} \mathcal{J}[m_c] = -(1-\alpha_O)(\bar{F}_f[\bar{\mu}] - F_c[m_c] - d_o) + DF_c[m_c]^* D\bar{F}_f[\bar{\mu}]\, Q[\bar{\mu}, \bar{F}_f[\bar{\mu}] - F_c[m_c] - d_o]^{-1} \nabla_{\bar{m}} J[m_c, \bar{\mu}; \alpha_O],$$

where self-adjoint linear operator $Q$ is

$$Q[\bar{m}, d]\, \bar{m}_1 = \left\{ N[\bar{m}] + W[\bar{m}, d] + \alpha_I/(1-\alpha_I) A^* A \right\} \bar{m}_1,$$

and normal operator $N$ and tomographic operator $W$ are

$$N[\bar{m}]\, \bar{m}_1 = D\bar{F}_f[\bar{m}]^* D\bar{F}_f[\bar{m}]\, \bar{m}_1,$$
$$W[\bar{m}, d]\, \bar{m}_1 = (D^2 \bar{F}_f[\bar{m}]\, \bar{m}_1)^* d$$

N.B. $D\bar{F}_f[\bar{m}]^*, N, W$ are computable variants of the adjoint state.

# IVA and VP

- Inner problem for given $m_c$: $\bar{\mu} = \text{argmin}_{\bar{m}} J[m_c, \bar{m}; \alpha_I]$
- Outer problem over $m_c$: $\min_{m_c} J[m_c, \bar{\mu}(m_c); \alpha_O]$

Choice of weights $\alpha_I$ and $\alpha_O$?

- Inversion velocity analysis: $\alpha_O = 1$, $\alpha_I = 0$
  "pure data fitting" inner problem
  non-physicality penalizing outer problem
- Variable projection (Golub and Pereyra, 1973): $\alpha_O = \alpha_I$
  stationary point of the inner problem leads to

$$\nabla_{m_c} \mathcal{J}[m_c] = -(\bar{F}_f[\bar{\mu}] - F_c[m_c] - d_o)$$

# IVA and VP

Pros and cons:

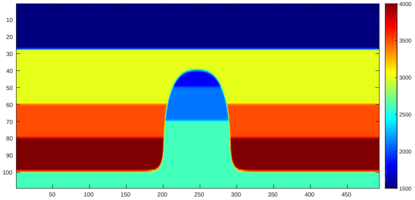
- IVA
  - $+$ No compromise parameter $\alpha$
  - $+$ Conventional LS inner problem
  - $+$ Embarrassingly parallel w.r.t. model extension parameter
  - $-$ Expensive outer problem
- VP
  - $+$ Simplified and cheap outer problem
  - $-$ Choice of $\alpha$

? How errors in the inner problem solution affect the outer problem (gradient computation)

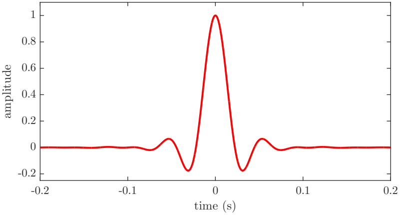
# Dome example

- 2D dome model: 101 $\times$ 495 g.p., 10 m spacing
- Absorbing boundary conditions
- Plane wave source: Ormsby 0-0-15-30 Hz
- 495 receivers at the top of the domain ($z = 0$) with 10 m spacing
- 4 seconds trace length
- 2-8 finite difference scheme

Velocity profile




Source wavelet



# Dome example

Inversion:

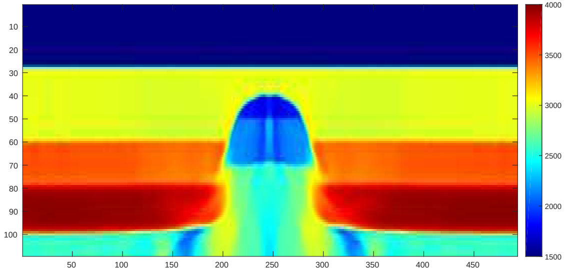
- Frequency continuation technique, from 1 Hz low-pass filter
- Homogeneous (water) initial model
- Polak-Ribière CG limited by 80 iterations
- No regularization, preconditioning, etc.
- Independent inversions for plane wave incidence angle in $[-30^\circ, 30^\circ]$ range

Goal:

Assess some potential difficulties of IVA & VP approaches

# Dome example: inversion results

Horizontal plane wave inversion:
Relative objective function error $\approx 0.0001$ (1% fitting error)

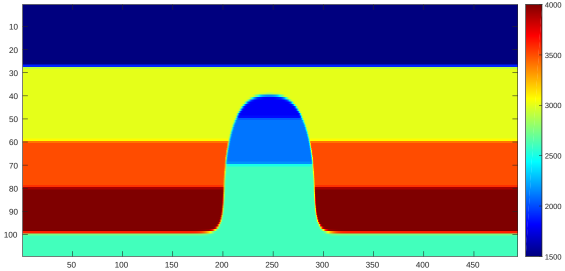



Conclusion:

- Good data fit
- Long-scale model recovered (expected for impulsive source)

# Dome example: inversion results

Horizontal plane wave inversion:
Relative objective function error ≈ 0.0001 (1% fitting error)

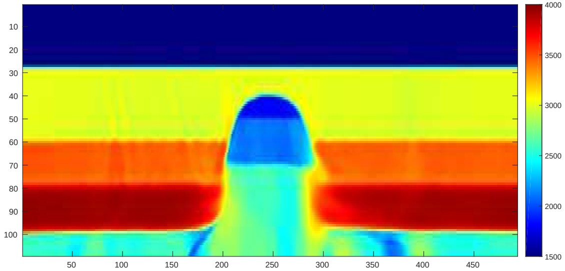



Conclusion:

- Good data fit
- Long-scale model recovered (expected for impulsive source)

# Dome example: inversion results

10° plane wave inversion:
Relative objective function error ≈ 0.0001 (1% fitting error)

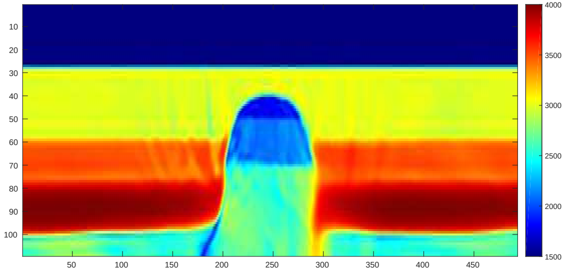



Conclusion:

- Good data fit
- Long-scale model recovered (expected for impulsive source)

# Dome example: inversion results

20° plane wave inversion:
Relative objective function error ≈ 0.0001 (1% fitting error)



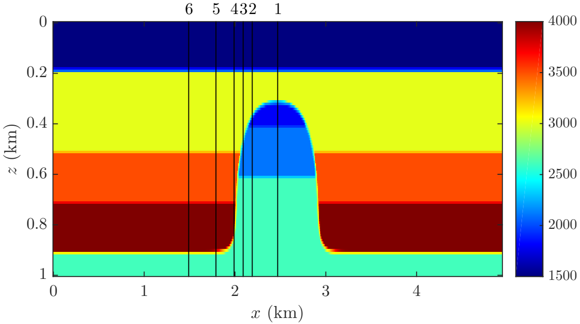

Conclusion:

- Good data fit
- Long-scale model recovered (expected for impulsive source)

# Dome example: model gathers

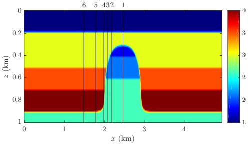
Initial assessment: how inner problem solution accuracy may affect the outer problem

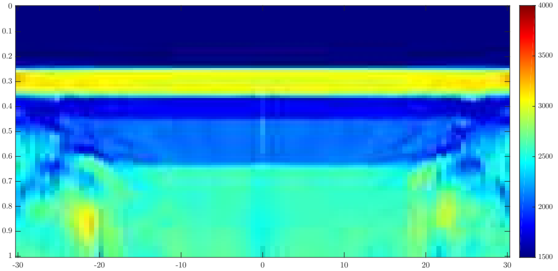
Six gather locations:



# Dome example: model gathers

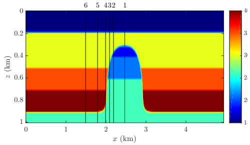
Location 1:

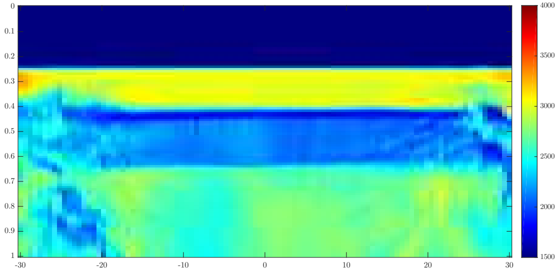


# Dome example: model gathers

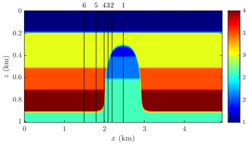
Location 2:

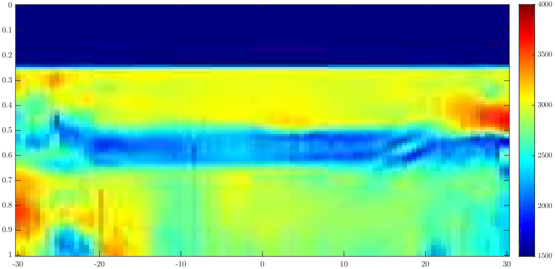


# Dome example: model gathers

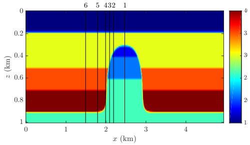
Location 3:

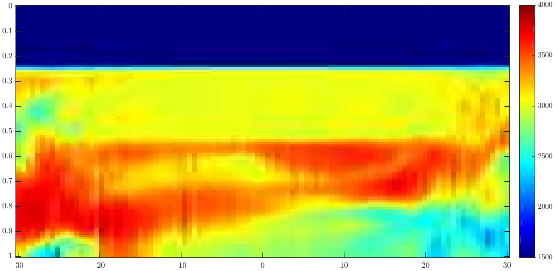


# Dome example: model gathers

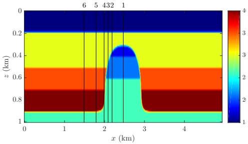
Location 4:

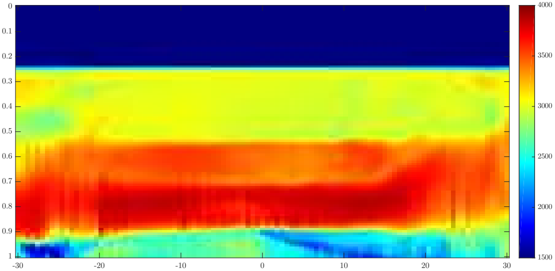


# Dome example: model gathers

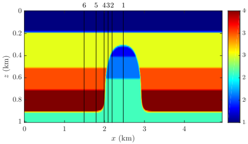
Location 5:

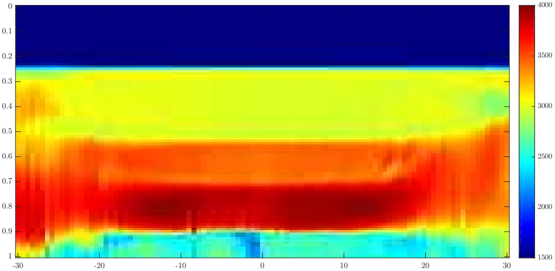


# Dome example: model gathers

Location 6:



# Dome example: model gathers

Conclusions:

- Noisy behavior of the gathers $\Rightarrow$ IVA likely to fail without good inner problem regularization
- For VP, annihilator term itself serves as a stabilizer
- Annihilator weight $\alpha$ will play important role and may need to be adaptive

# Research summary

- Implementation:
  - Constant density acoustics
  - Framework for surface-based extended modeling
  - Outer gradient computation
  - Fast and robust optimization for inner problem (TR Newton-type, preconditioners, regularization)
- Questions:
  - IVA vs VP: inner problem solution accuracy, choice of $\alpha$ in VP, etc.
  - Robustness w.r.t. noise
  - Robustness w.r.t. physics discrepancy
- Numerical results:
  - Marmousi test
  - Real data 2D

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